FINITE ELEMENT SOLUTION OF THE WARPING TORSION USING THE EQUIVALENT BEAM-COLUMN ANALOGY IN MULTI-SPAN BEAMS

By

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**ABSTRACT**

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A small, powerful, versatile and straightforward software application will be introduced to analyze problems of warping torsion by means of the analogy between the straight beam-column under mixed transverse and axial loading and the straight beam under restrained warping. This software, originally developed to analyze multi-span beam-columns immersed on elastic foundations, resorts to a high order finite element. Validation against a number of already solved cases will be provided, including, but not limited to the cases in the AISC-Design Guide 9. Additionally, the present work will provide a brief account on the first order general torsion theory, a discussion on the roles analogies play in sciences and engineering, an account on the equivalent beam-column analogy as special case of the equivalent beam on elastic foundation, some details of the aforesaid high-order-finite element, and a contribution of a novel convergence study regarding the axial tensile parameter.

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# LIST OF SYMBOLS

| | |
|---|---|
| 3DOF RE | rod element with 3 degrees of freedom, |
| 6-DOF BC | beam column with 6 degrees of freedom, also 6-DOF BE |
| 6-DOF BE | beam element with 6 degrees of freedom, |
| 9-DOF BC | beam column with 9 degrees of freedom, also 9-DOF BE |
| 9-DOF BE | beam element with 9 degrees of freedom, |
| $a$ | $L/2$, semi-length of finite element, or $\sqrt{\ ECw/GJ)} = 1/\lambda$ according to the AISC DG 9 nomenclature for TWB, in length units, |
| $A$ | cross section area |
| AISC | American Institute of Steel Construction, |
| AISC-DG-9 | Steel Design Guide Series No. 9, |
| $b$ | a characteristic dimension of a TWB cross-section, width or height. The convex contact width in a Winkler problem, in, |
| $B$ | bimoment, an abstraction in TWB theory, force·length$^2$, |
| [B] | strain matrix defined by the generic curvature, it is the 2nd derivative of the shape function matrix [N], |
| BEF | beam on elastic foundation, |
| BC | beam-column, |
| BMCOL, BMCOLG | original name of beam-column software renamed BMTORSWP to undertake restrained warping problems, |
| BMTORSW | after beam-torsion-warping, 32-bit-software application, |
| BMTORSWP | beam-torsion-warping with Portland adaptation for 64 bits, |
| $C_w$, $Cd$ | warping constant for the cross-section, length$^6$, |
| $d$ | a characteristic dimension of a TWB cross-section, width or height, |
| $Ce$ | dimensionless factor in the 9-DOF- BC matrix |
| DOF | degree of freedom, |
| DG-9 | Steel Design Guide Series No. 9, |
| $E$ | modulus of elasticity of material, force/length$^2$, |
| EBEF | equivalent beam on elastic foundation, |
| {g} | generic transversal field of displacements, |
| $G$ | shear modulus of elasticity of material, force/length$^2$, |
| $I$ | cross section inertia, length$^4$, |
| $I_f$ | moment of inertia of one I-beam flange around the cross section minor axis, |
| $J$ | torsional constant for the cross-section, length$^4$, |
| $JBW$ | Half band width in a matrix, |
| $k$o | soil normal modulus, a parameter for the BC, force/length$^3$, |
| $k$ | soil normal modulus multiplied by the convex contact width $b$, force/length$^2$, |
| $l$ | $\pi/\lambda$ characteristic wave length for a EBEF used in rail trucks, |
| $L$ | member length, |
| $Le$ | maximum element length = $2\sqrt{(ECw/GJ)}$, in length units, |
| $M$ | bending moment, |

| | |
|---|---|
| N, NI, NJ | node, near node, far node, where I < J, |
| $N$ | axial force always in the direction of the original straight BC, |
| [N], [N,x], [N,xx] | shape function matrix and 2 derivatives along the axial coordinate x in the finite element formulation, |
| $Ne$ | minimum number of elements in a given span of length $L$, an integer > $(L/Le) = 0.5L\sqrt{(GJ/ECw)}$ in the TWB, |
| $p$ | applied distortional load in force/length units, |
| $q$ | transverse loading in the BC, force/length, |
| $q_i$ | degree of freedom with the specific number $i$, |
| {q} | vector of specific (reference) displacements along the DOFs in finite element formulation containing all the $q_i$, |
| $Q$ | applied shear in the BEF under mixed loading, force units, |
| $Q_v$ | vertical shearing force, always perpendicular to the original straight BEF, |
| $Q_n$ | shearing force normal to the deformed BEF elastic line, |
| $r$ | a distance from the bar axis in a cylindrical bar, |
| $R$ | radius of a cylindrical bar, length, |
| $T_w$, $T_2$ | resisting moment due to restrained warping of the cross-section, force-length, |
| $T_t$, $T_1$ | resisting moment of unrestrained cross-section, force times length, |
| TWB | thin-walled beam or bar, |
| TWM | thin-walled member, |
| $u$, $u_s$ | displacement along the contour, lateral in the case of an I-beam flange, length, |
| $u$, $v$ | generic axial and transverse displacements in the 9DOF element, |
| $u_r$ | displacement perpendicular to the contour, length, |
| $V_f$ | the shear occurring in each flange of an I-beam, force, |
| $w$ | distributed constant load in lb/in or force/length units, |
| $W$ | vector with powers of varying distributed loads in lb/in, a measure of distortion in box girder analysis, |
| $x$ | $a\xi$, member longitudinal coordinate, |
| $y, y', y'', y''', y''''$ | longitudinal coordinate, and four derivatives, |
| $z$ | longitudinal coordinate, length, |
| $\alpha i$ | generalized displacements, coefficients of the polynomial expressing the field of the generic displacements used in the finite element formulation, |
| $\delta$ | shell thickness, length, |
| $\varepsilon$ | a unit axial tensile strain, dimensionless, |
| $\theta$, $d\theta/dz$, $d^3\theta/dz^3$ | angle, mostly torsional, and 1st and 3rd derivatives with respect to $z$, |
| $\kappa$ | deformational stiffness of the girder per unit length in force units, |
| $\lambda$ | a characteristic BEF constant = $(k / 4E{\cdot}I)^{1/4}$, 1/in, a characteristic dimensionless BC constant = $L(N\ /E{\cdot}I)^{1/2}$, a characteristic dimensionless TWB constant = $L(GJ/E{\cdot}Cw)^{1/2}$, |

| | |
|---|---|
| $\rho,\ \rho_1$ | initial and final radius of curvature of cross section contour, length, generalized distortional load per unit length in box girder analysis |
| $\sigma_y$ | axial stress, force/length$^2$, |
| $\tau$ | shearing stresses, force/length$^2$ |

# CHAPTER 1

# INTRODUCTION

St. Venant torsion theory does not suffice to predict the behavior of thin-walled members with open cross sections, commonly used in steel industry, subjected to unavoidable torsional loads. Thin-walled members behave according to Vlasov's treatise on $1^{st}$ order general torsion and must be checked for restrained warping according to the AISC. Calculations of the warping effect are tedious and complex, even if using AISC design aids; and more so when designing multi-span beams under mixed torsion.

## 1.1 The Restrained Warping Problem

A cross section of a straight bar will rotate freely when undergoing two equal and opposite torques at its end nodes. The twist per unit length will be constant along the bar; this is known as uniform torsion, which can be studied separately from flexure and their effects superposed. However in the XX century there was a new advent in mechanics of materials: The first order general theory of torsional flexure. It is a relatively modern advance in mechanic of materials that combines the study of both uniform and non uniform torsion (Gjelsvik, 1981 and Rhodes, 1984.) See Figure 1.

Thin-walled bars under torsional loading with node restraints are mostly susceptible to develop axial stresses. In addition, applied loads are particularly transmissible quite a distance along the length of the bar, flouting St. Venant principle as shown in Figure 2 and Figure 3. First order general torsion analysis involves even more complicated studies of cross section properties, and loading effect allocation.

In fact, the analysis gets more complicated in multi-span thin-walled members with open cross sections, and the number of calculations required for a comprehensive scrutiny of these torsional effects is paramount.

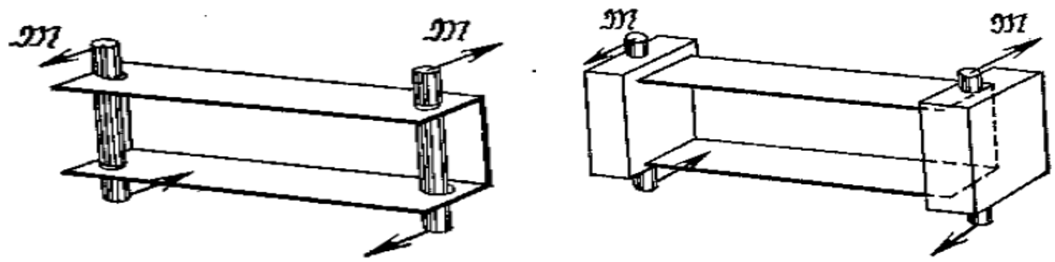

Figure 1. Free (Left) and Restrained (Right) Warping

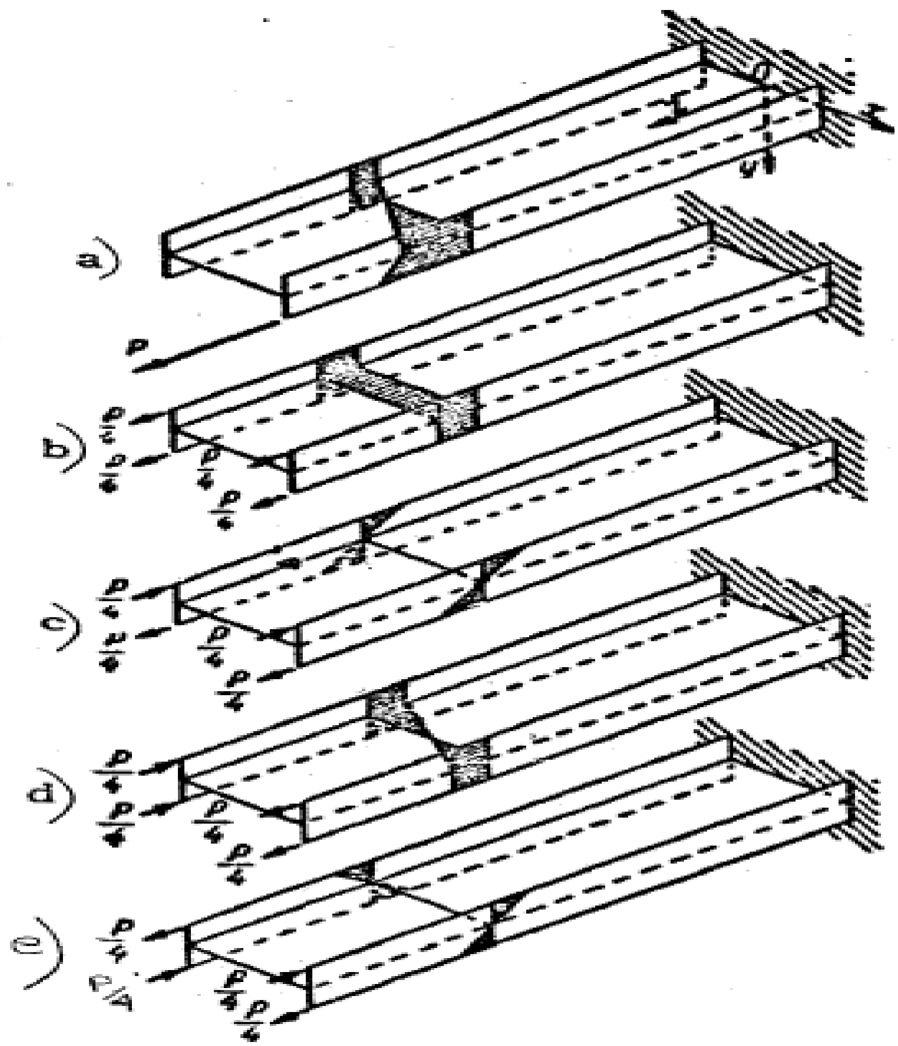

Figure 2. Load Location Exerts Influence along the Beam

Vlasov (1961) is credited as the author of the seminal work on modern 1st order general torsion theory. He found that, due to their peculiar geometry, thin-walled bars should be studied like a particular case of shells, whose geometry influences the acting shear and its propagation. For example even a centered axial resultant load applied to a member could produce transverse lateral bending with a zone of non uniform distribution of stresses. This effect is important around the cross section minor axis for thin-walled elements like the I-beam in Figure 3 taken from Feodosiev (1972).

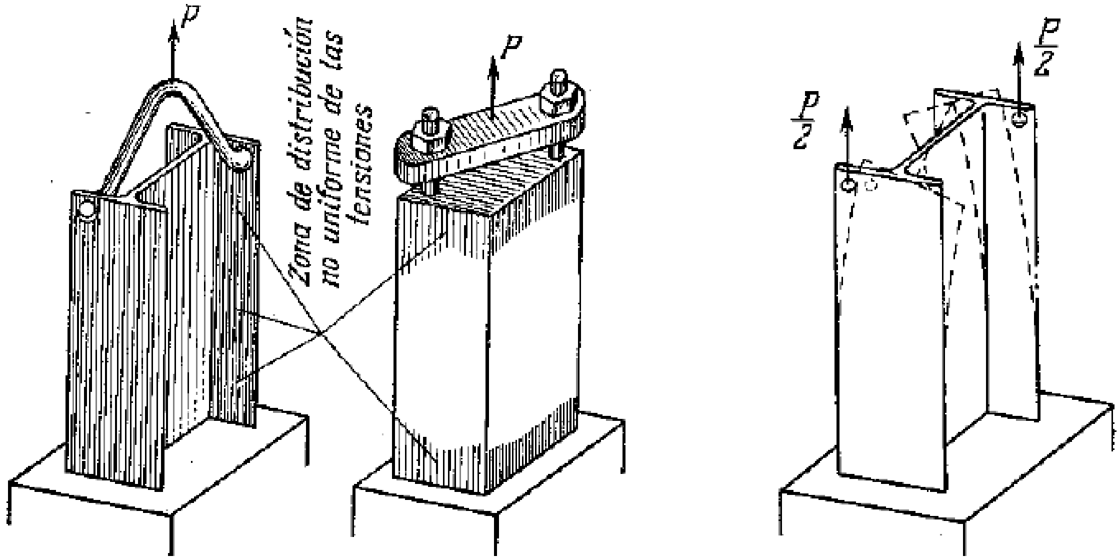


Figure 3. Restrained Warping due to a Centered Axial Tensile Load

The uniform or Saint Venant torsion Equation (1) is shown below; where $G \cdot J$ is the cross section torsional rigidity and $\theta$ is the angular twist around the longitudinal axis. The resisting moment for an unrestrained cross-section is:

$$T_t = G \cdot J \frac{d\theta}{dz} . \tag{1}$$

For a better understanding of the non-uniform torsion, it is useful to consider the particular case of a structural steel W shape whose lateral flanges undergo opposite curvatures due to a pair of lateral opposite shearing forces in the flanges. In the lateral direction, these forces are in equilibrium and make a force couple with a moment arm *h*, the distance between flanges, which is a torque $T_w$ around the longitudinal axis *z* as expressed in Equation (2):

$$T_w = V_f h \,. \tag{2}$$

Moreover, the shear in each flange *Vf* may be derived by considering the beam lateral displacement *u*, parallel to the original major axis. Due to the fact that the beam cross section is symmetric and its rotation is relatively small, the lateral displacement $u = (h/2)\,\theta$ and its first derivative $u' = (h/2)\,\theta'$ are shown in Figure 4 and Equation (3):

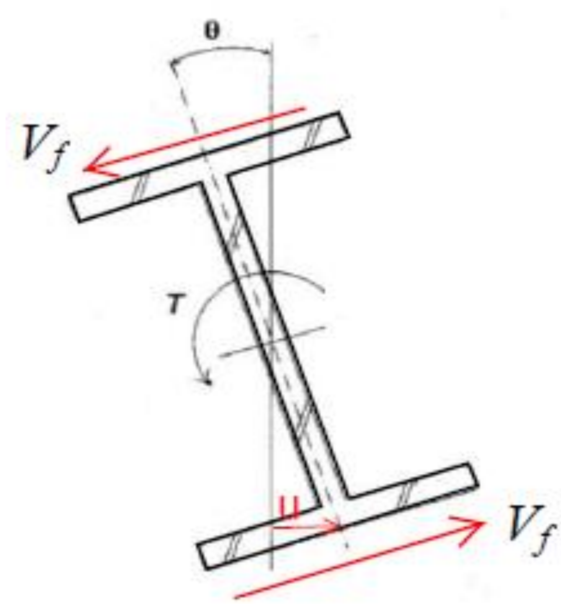


Figure 4. Lateral Transverse Displacement *u* due to Torsion

$$\frac{du}{dz} = \frac{h}{2}\frac{d\theta}{dz} \,. \tag{3}$$

Due to the opposite lateral displacement of each flange, the system of lateral bending moments *B* and shears $V_f$ in each flange produced by the lateral curvatures in each flanges is in equilibrium because those forces are opposite and equal in magnitude.

Therefore, the system is statically equivalent to zero. Equations (4) and (5) are shaped using the curvature formula and the appropriate sign convention. Shear and normal stresses shown in Figure 5 will appear in the member cross section. See Ugural (1987) and Timoshenko (1978).

$$B = -EI_f \frac{d^2u}{dz^2} = -EI_f \frac{h}{2}\frac{d^2\theta}{dz^2}, \quad (4)$$

$$V_f = \frac{dB}{dz} = -EI_f \frac{d^3u}{dz^3} = -EI_f \frac{h}{2}\frac{d^3\theta}{dz^3}, \quad (5)$$

where $I_f$ is the moment of inertia of one flange around the cross section minor axis. Hence, Equation (2) will be transformed into Equation (6), and the total torque will be that of Equation (7):

$$T_w = -EI_f \frac{h^2}{2}\frac{d^3\theta}{dz^3}, \quad (6)$$

$$T = T_t + T_w = G \cdot J \frac{d\theta}{dz} - E \cdot I_f \frac{h^2}{2}\frac{d^3\theta}{dz^3}. \quad (7)$$

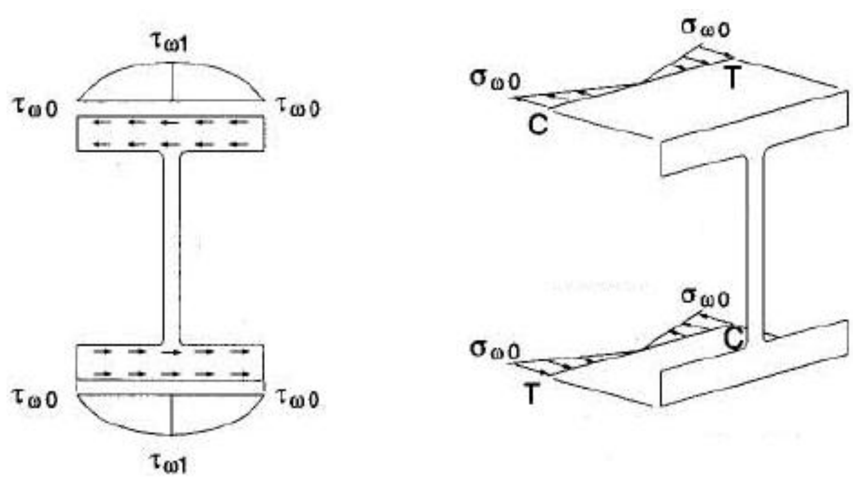


Figure 5. Shear and Normal Stresses due to Restrained Warping from DG-9

### 1.2 Significance of this Work

It is desirable, but not always possible, to avoid torsion: in slender structures, as steel structures, most structural members under torsion are not free to warp. In addition, even when the ends are free to warp, internal restraint of warping is produced by torsional loading variations along the span (Bresler, 1963). In fact, structures undergo torsion quite frequently and open profiles are prone to suffer restrained warping stresses.

Open profiles are common in structural steel, crane girders, airplanes, naval structures, land vehicle with long frames including trucks and buses, box-girder bridges, and shear walls in high rise buildings including towers or elevator shafts. An I fact, restrained warping normal stresses might be critical design components.

Current practice calls for protective measures against the adverse effects of non-uniform torsion in case of members with modest resistance to restrained warping (AISC-DG-9). In thin-walled beam analysis, end restraints or changes in the internal torque along the length produce normal and shear stresses due to restrained warping (Figure 5). The problem of bars under mixed axial and transverse loads is a real predicament in structural engineering; particularly in the case of multi-span bars under mixed torsion.

### 1.3 Aims of this Work

The following is an account of the tasks that will be undertaken in the first two chapters of this work: First, a straightforward software application able to solve multi-span TWB originally designed to analyze multi-span BEF subjected to three-parameter-foundations plus the geometric parameter of a tensile load will be selected. Second, a brief account on the first order general torsion theory and some of its major contributions will be provided. Third, the roles analogies play in engineering will be discussed including an account on the equivalent beam-column analogy. Fourth, some details of the high-order-finite element built in the software (Figure 6) will also be presented and discussed.

Afterwards, a convergence study of the size of the finite element to be used in the software application will be undertaken for the first time regarding two aspects: First, the effect of the geometric parameter of a tensile load will be studied for the first time, and second, the finite element stiffness matrix with 6 DOF used by the software will be studied. Then, a maximum finite element size will be recommended and used. Previous studies were made on the soil parameter, not in the geometric one, and a finite element stiffness matrix with 4 DOF was used as an upper limit for accuracy, instead of the original finite element stiffness matrix with 6 DOF.

Already analyzed single-span and multi-span beams in mixed torsion under distributed and concentrated torques with diverse boundary conditions will be reanalyzed to validate this software. The examples will be taken from different sources, including all the examples of restrained warping torsion contained in the AISC- Design Guide 9. Numerical data and curves will be provided. A performance experiment will be undertaken and analyzed with the software. Finally, user friendly adjustments to interact with the software will be implemented, and conclusions and recommendations will be presented. This study is not concerned with the analysis of box girders. However, one of the three soil parameters handled by the software would suffice to analyze box girders problems.

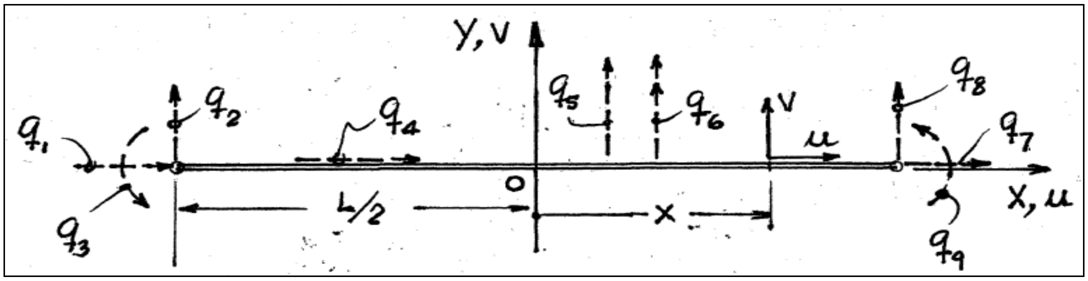

Figure 6. The 9-DOF-Beam Element by Deschapelles

## CHAPTER 2

## REVIEW OF RELEVANT TOPICS AND LITERATURE

The torsional flexure theory is only an approximate theory but supposes a great advance for engineering. Shear strains due to lateral flexure are neglected; just as the Navier hypothesis neglects shear strains in traditional bending theory, but shear strains due to torsion are indeed included (Gjelsvik, 1981).

### 2.1 Significance of Analogies, Metaphors and Equivalences

It is convenient to take into consideration that analogy is the only one of those introductory terms accepted in science language without a debate. The other terms could be considered rhetoric language, i.e., the word metaphor (the pursuit of the ability to use language or representations to generate the intended results) was borrowed from rhetoric.

The simple engineering practice known since Kirchoff times of denoting any complex number $r + ix$ by a single symbol expressing in one vector both the real and the imaginary fields, (Kron, 1939) might be considered as an analogy, a metaphor or an equivalence. In many fields, including software engineering, metaphor refers to a somewhat sophisticated system resemblance, but positivists reject the use of rhetoric language in research[1] contrary to constructionists.

#### 2.1.1 Concepts on Analogies, Metaphors and Equivalences

Analogy is a way of inferring properties of one lesser-known item from a similar better-known item on the basis of an acknowledged similarity between both of them. Modeling is the simplest statement or intellectual depiction of a process, concept,

[1]According to dictionaries, constructionists find in the word rhetoric a useful meaning, because they construe or interpret. Positivist use the word rhetoric in the sense of unnecessary, because they believe in positivism, a thinking system founded by Comte, concerned with positive facts that rejects speculation.

operation or system. The modeling of an item or domain is the first step to identify analogies or equivalences. Thus, analogies and modeling are intertwined.

Analogies have been proven historically effective by making possible the findings and achievements in complex problems. Insights into two analogous systems are obtained by studying the easiest to conceive; thus improvements in the technique of solving problems in one mastered field favor the findings in the novel analogous field (Hetényi, 1966).

Analogies can be improved when basic assumptions are refined and redefined. For example, behavior (of beings) and computer output are not straightforward analogous for Cisek (1999), who thinks that behavior is more analogous to "a control process where actions are performed in order to affect perceptions..." Herbsleb (1999) states that metaphors, as opposed to straightforward analogies, help to recognize new or complex domains as if they were already understood.

Metaphors set up implicit conceptual foundations for the lesser-known domain, hence enlightening important unperceived characteristics but neglecting options and aspects of objects under scrutiny. Herbsleb (1999) considers analogies as a subgroup of metaphors. Conversely, Deschapelles (1987) conceives the bimoment as a metaphor included in the beam-column analogy. In fact, the use of this rhetoric language in research is always useful but not standard throughout interdisciplinary borders.

Analogies are not only useful but are also the core of the discipline of analog computer simulation. Mathematical models are analogous representations of reality or of other models that can be scrutinized not only by classical analytical and numerical methods or digital computers, but also by means of analog computer simulation (James, 1971, Carlson, 1967).

### 2.1.2 Examples of Analogies

A good example of the practicality of analogies in engineering is the significant work by Wilby (1963) on the use of electronic analogy for the analysis of elastic shells. Another paradigm is the work of Kron (1939) on the analogy between electrical and mechanical networks (3D frames), which provides a complete algorithm in matrix form solving 3D frames: "Whatever the advantages of steel frames offer for the construction of buildings, analogous advantages are offered by the use of tensor spinor[2]."

According to a work by Mindlin and Salvadori that appears in Hetényi (1966), in Kron analogy, currents represent stresses and voltage drops represent strains in the electric circuit. The equations of equilibrium in nodes are analogue to Kirchhoff's current law[3]. The compatibility conditions are analogue to Kirchhoff's voltage law[4]. Finally, Hooke's law is analogue to "Ohm's law $I = Y \cdot E$, so that the elastic constants are represented by lumped admittances $Y$."

Kelvin's fluid-flow analogy was very useful in picturing Saint Venant shear stress flux in terms of streamlines of an ideal fluid circulatory flow around the vortex inside a container tube with analogous cross section contour. Prandtl's analogy was very useful in studying the torsion problem (Den Hartog, 1952). Weibel's analogy helped to study the stress concentration at cross section fillets. Jacobsen's analogy assisted to study the torsion in a shaft of variable diameter, etc. (Timoshenko, 1953).

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[2] Spinor is a quantity resembling a tensor used to represent the spins of fermions (particles that obey the exclusion principle and Fermi-Dirac statistics whose spins are half an odd integer: 1/2, 3/2, 5/2, (Webster).
[3] The summing up of all currents that enter a node is zero at every node.
[4] The sum of voltages drops $E$ around every closed loop or mesh in the circuit must equal zero.

Still nowadays same British consider that epoch-making moment distribution method of flexure structural analysis by Cross (1930, 1932, and 1954) and Grinter (1932, 1933) is a particular case of the systematic relaxation of constraints advanced later by Southwell (1935).

Samuelsson and Zienkiewicz (2006) consider that the Framework Method by Hrennikoff (1941) and the Lattice Analogy Method by McHenry (1942, 1943) are analogies which solve linear elasticity problems by modeling a continuum elastic media with "elastic lattices of suitably defined discrete components." Because the one who first coined the term finite element was Clough (1960) in an epoch-making paper, historically, Hrenikoff and McHenry may well be considered the grand-parents of the finite element method. The theory of open thin-walled beams shares with the finite element method the condition of latecomers in mechanics. And now, thanks to the development of digital computers, key advances in finite element method have taken place since the middle of the XX century.

However, the finite element genesis may well be dated back to the work of Lord Rayleigh in 1870s, later generalized by W. Ritz in 1900s. That is, to the times of the formulation of the variational Rayleigh-Ritz principle and the weighted-residual method by B. G. Galerkin (1915). In addition, the general theory of 3-D systems to solve the problem of a "self-contained space truss with the form of a closed polyhedron" was initiated by Möbius and independently developed by engineers like Föppl who published his works in 1892 calling these structures as lattice structures (Timoshenko, 1953).

The column analogy is used to solve many problems of mechanics. Magnitudes and directions can be modeled graphically like it is done with the old polygon of forces and the funicular polygon—known since Varignon's times (in 1720s). The conjugated beam is an analogy to calculate beam deflections from the bending moments created by a made up M/EI loading. Mohr's circles are analogies that relate certain physic properties with the circumference geometry.

In page 144 of his book, Vlasov (1961) first proposed the use of the beam-column analogy by stating that the equation of a beam under mixed torsion is "analogous to the equation of the theory of transverse bending of an initially extended beam." He also listed the matching analogous terms between his law of sectorial areas relevant to torsional bending and the law of plane sections applicable to flexural bending as shown in Table 1.

Table 1. Flexure and Warping Analogies

| Bending in the plane ***Oyz*** (law of plane sections, Figure 87a) | Restrained torsion (law of sectorial areas, Figure 87b) |
|---|---|
| $J_x = \int_F y^2\, dF$ | $J_\omega = \int_F \omega^2\, dF$ |
| $S_x = \int_F y\, dF$ | $S_\omega = \int_F \omega\, dF$ |
| $\eta = \eta(z)$ | $\theta = \theta(z)$ |
| $\eta' = \frac{d\eta}{dz}$ | $\theta' = \frac{d\theta}{dz}$ |
| $M_x = -EJ_x\eta''$ | $B = -EJ_\omega\theta''$ |
| $Q_y = M'_x = -EJ_x\eta'''$ | $H_\omega = B' = -EJ_\omega\theta'''$ |
| $\sigma_x = \frac{M_x(z)}{J_x} y(s)$ | $\sigma_\omega = \frac{B(z)}{J_\omega}\omega(s)$ |
| $\tau_x = -\frac{Q_y(z) S_x(s)}{J_x \delta(s)}$ | $\tau_\omega = -\frac{H_\omega(z) S_\omega(s)}{J_\omega \delta(s)}$ |
| $q_y = q_y(z)$ | $m = m(z)$ |

In addition, the solution of multi-span bars by means of the 3-bimoments-equation, an analog version of the 3-moments-equation or Clapeyron's equation was first proposed by Vlasov (1961) in page 145. Nevertheless, Medwadowski (1985) states that

Kollbrunner and Basler were the ones who proposed the 3 warping moment method in 1965 to solve the first order general torsion problem.

Boothby (1984) and Medwadowski (1985) works will be discussed later with more detail. They both acknowledged the beam-column-analogy to solve general torsion problems resorting to what they called a Bimoment Distribution Method and a Warping Moment Distribution Procedure respectively. These methods rely on principles similar to the moment distribution method of flexure structural analysis by Cross (1930, 1932, and 1954) and Grinter (1932-33).

In fact, the theory of beams on Winkler elastic foundations is an analogy between a continuous and a discrete model originally formulated to describe the discrete railroad track behavior. Hetényi (1961) states that this analogy is proficient if at least 4 separate elastic supports are provided to cover the characteristic wave length $l = \pi / \lambda$; where $\lambda = (k / 4E \cdot I)^{1/4}$, and $k$ (lbs/in$^2$) is the equivalent continuous normal modulus of the foundation $k_o$ times its contact width $b$.

The analysis of beam grillages or grid-works was developed as a consequence of the analysis of the railroad tracks, where the rails and the cross-ties intertwine similarly. An important application of beam grillages is found in ship structures. So beam grillages were analyzed by Muckle and Timoshenko (Heténti, 1966).

Vierendeel girders may be analyzed as analog grid-works. Beams unrestrained against deflection but restrained against angular rotations may be regarded as a special case of EBEF. In addition, Hetényi (1966) states: "…the effect of distributed rotational elastic restraint" is analogous to "an equivalent axial tensile force."

Other examples of the "Winkler problem" or BEF analogies are the floor-beam elastic interaction and the wall of a circular cylindrical tank with radius *r* and thickness *t* with a vertical axis of symmetry under axially symmetric loading and support (Lightfoot, 1961). Wright (1968) used the BEF analogy to analyze distorted box girders. In the BEF method, solutions are obtained either analytically or numerically by the Fourier series or by moment distribution methods as in Lightfoot (1961).

Hsu (1995) introduced a finite element formulation in the BEF analogy, so creating the EBEF analogy, and applied to the solution of multi-span beams. Spans are considered flanked by supports or internal restraints, namely diaphragms that might provide or not shear and/or warping restraints as shown in Table 2.

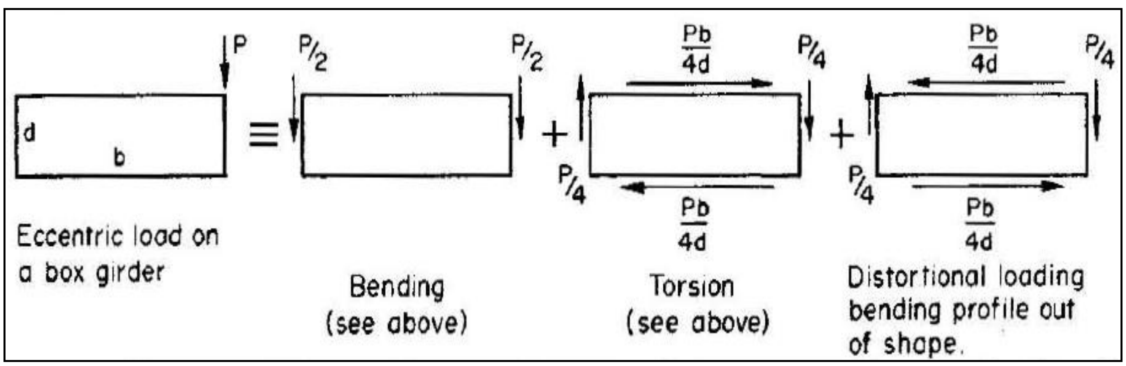
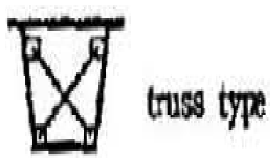
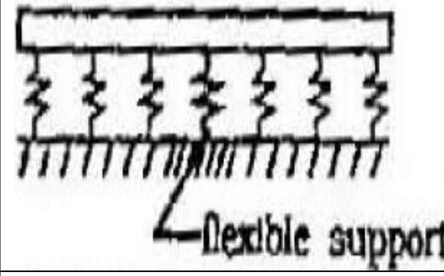
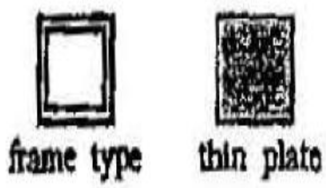
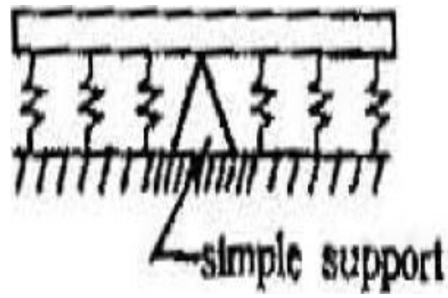
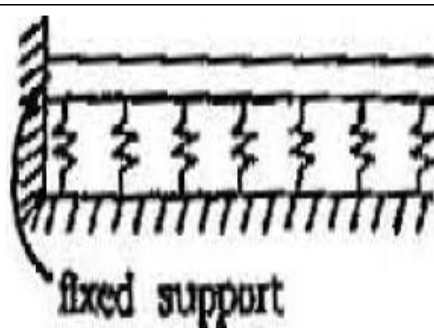



Figure 7. Distortion in Box Girders

It is important to notice that the fundamental differential equation of curved box girders, as closed TWB, is not analogous to the beam-column under mixed axial and transverse loading. Box girders undergo distortions that put cross section profiles out of shape due to the force system shown in Figure 7.

The differential equation of the beam on elastic foundation is $E \cdot I \cdot y'''' + k \cdot y = q$ (Miranda, 1966), where *E*, *I*, *y*, *k*, and *q* are the modulus of elasticity, the cross section inertia, the deflection, the soil modulus, and the intensity of the distributed transverse load at that point respectively.

Table 2. Internal Diaphragms in Box Girders and Analog Supports

| Internal Diaphragm | | | Support in BEF |
|---|---|---|---|
| Deformation Allowed | | Possible Configuration | |
| Shear | Warping | | |
| Yes | Yes |  |  |
| No | Yes |  |  |
| No | No |  |  |

The box girder (closed TWB) governing differential equation has the form $E \cdot C_d \cdot W'''' - \kappa \cdot W = \rho$; therefore, it is analogous to the BEF differential equation according to Wright (1968). Where $W$ is a measure of distortion, $C_d$ is a cross section property related to warping, $\kappa$ is a measure of the deformational stiffness of a unit length of the box cell, $\rho$ is the applied generalized distortional load per unit length, $E$ is the Young's modulus of the box girder (Wright, 1968).

### 2.2 Contrasts between Saint Venant and Vlasov Torsions

When a member is subjected to opposite torques at its end nodes, there is a stress state in which only tangential stresses appear over the faces of the cross section: the free warping, (Feodosiev, 1972), also called Saint Venant's uniform torsion. Each cross section at ends is a free surface. Member longitudinal fibers are not restricted to elongate or contract as shown in Figure 8, and shear stresses are generated as in Figure 14.

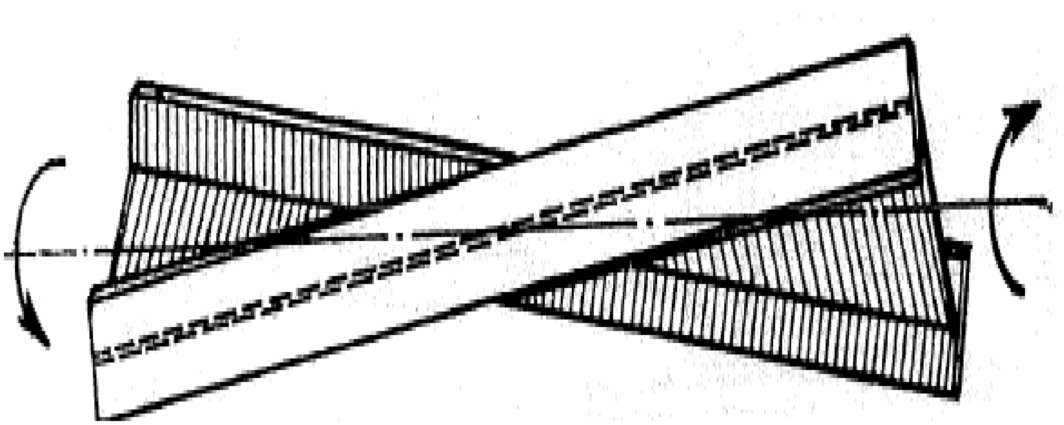


Figure 8. Free Warping or Uniform Torsion

Conversely, in the case of restrained warping, at least one of the end planes of the element is not free to warp[5] (Vlasov, 1961). The longitudinal fibers do elongate while the boundary restraints prevent the section rotation; thus normal and shear stresses appear as shown in Figure 9 (Rhodes, 1987), Figure 10 and Figure 15 (Hoogenboom, 2005).

St. Venant theory could be applied both to uniform and non uniform torsion, since the cross sections are free to deform out of its plane during torsion, a phenomenon referred to as the free warping of the cross section, which is a crucial point considering the later development of the general torsion theory according to Gjelsvik (1981).

---

[5] It is understood that it will suffice that any cross section, not necessarily the end planes, is not free to warp. i.e., a beam with two free ends with an intermediate fork subjected to a concentrated torque at any point suffers warping. This problem could be modeled as a two span beam.

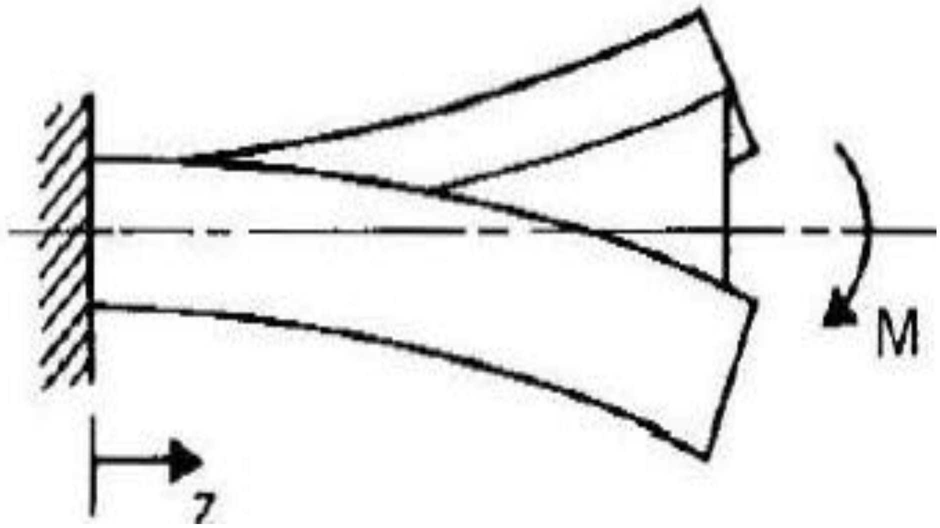



Figure 9. Warped Cantilevered Beam with Torque at Far End

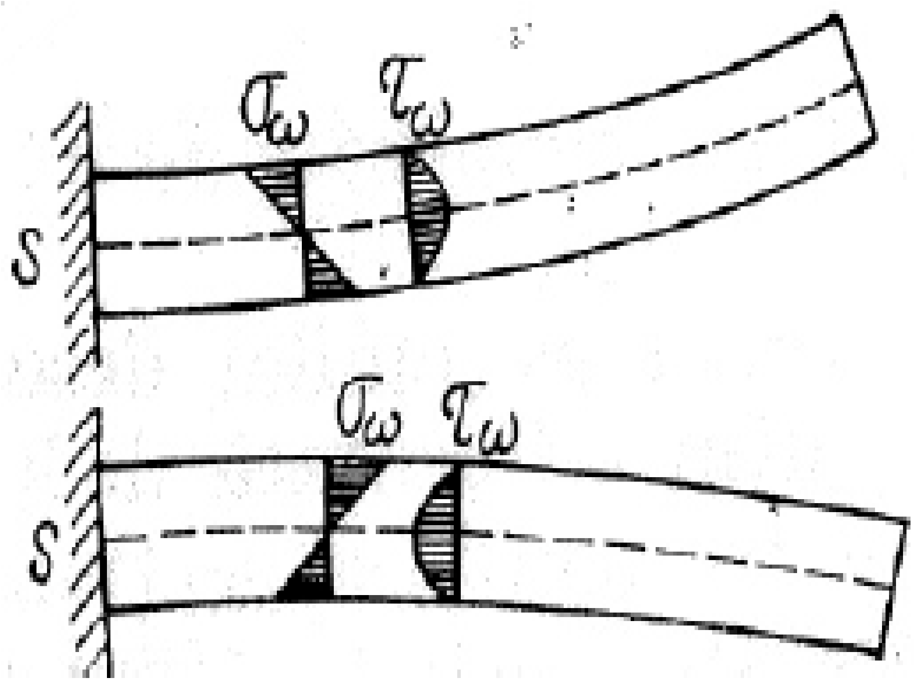



Figure 10. Shear and Axial Stresses in Cantilevered Beam under a Far- End Torque

**2.3 Definition of Shear Center**

The first shear center sound study is attributed to Maillart. The shear center axis is a line parallel to the beam generator (Figure 11). When the external transverse load passes through the shear center, pure transverse bending is produced. That is, a bending deformation for which the cross sections will remain plane and only suffer transverse vertical or lateral displacements as it is shown in right side of Figure 12.

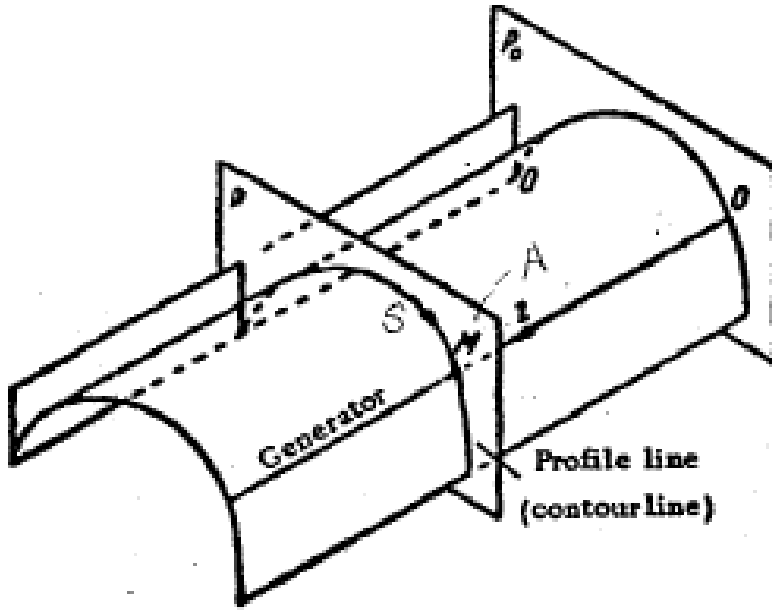



Figure 11. Generator Definition for Vlasov

If the resultant transverse load does not pass through the shear center, the beam will suffer restrained warping due to the presence of cross section restraints or changes in the torsional loading. Balancing stresses of flexural torsion (Figure 10) determined by the law of sectorial areas will materialize in the bar cross sections in addition to the pure bending stresses as shown in Figure 12 from Lundquist (1937).

As a rule of thumb, it is useful to know that: First, the shear center is located in the symmetry axis in symmetric cross-sections. Second, the centroid and shear center coincide in double symmetric cross-sections. Third, the shear center lies in the axis of symmetry but not necessarily coincides with the cross-section centroid in singly symmetric cross-sections (AISC-DG 9, 1997). Fourth, the centroid and shear center coincide in profiles composed of plate elements whose centerlines intersect at a common point such as structural T or L (Heins, 1963).

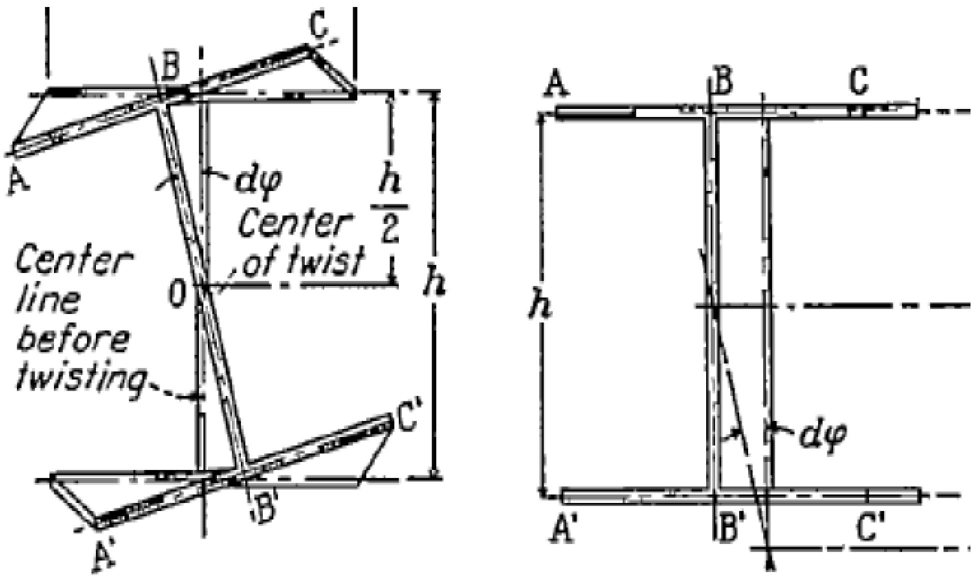


Figure 12. Bending and Bending with Torsion Responses

### 2.4 Reclassification of Structural Elements as per TWB-Theory

Before Vlasov, traditional structural engineering classified bodies into three groups: First, massive bodies with three similar dimensions. Second, plates and shells whose thickness is small compared with the other two sharing the same order of magnitude. Third, beam-columns with two dimensions of the same order of magnitude, which are small compared with the third dimension or length. Vlasov introduced a new fourth class, thin-walled bars (TWBs) having the shape of long prismatic shells and a thickness much smaller than a representative dimension of the length of its cross section contour. In addition, thin-walled bars are subdivided according to whether their cross section contours are open or closed.

TWBs have cross-sections comprised by single long shells or combination of plates with proportions that satisfy Equation (8). The component plates of these shells are rigidly jointed along their contact lines; so that, no plate is free to move with respect to its neighbor at any point of the joint.

According to Vlasov (1961), if $\delta$ is the shell thickness, *d* is any other characteristic dimension of the cross-section (its width or height) and *L*, its length, the proportions must satisfy:

$$\frac{d}{L} \leq 0.10 \ldots \vee / \wedge \ldots \frac{\delta}{L} \leq 0.10 \qquad (8)$$

However, this somewhat arbitrary but reasonable standard is not a rule of thumb and could be misleading. Particular conditions like loading and mechanic characteristics should be more carefully analyzed (Vlasov, 1961).

### 2.5 Evolution of Approaches to the Restrained Warping Problem

Same rudiments of restrained warping were identified even before Timoshenko (1934), Wagner (1929), Goodier (1942), or Vlasov (1961.) The Euler-Bernoulli hypothesis stating that plane cross sections remain plane before and after bending was known not to be always accurate. The restrained warping in bars was already noticed by scientists like Young, who lived between 1773- 1829, the early stages of the primitive restrained warping theory.

Young delivered an approach for the twist of a cylindrical bar of radius *R* with a rectangular element *abcd* on its surface at a distance *r* of the bar axis (Figure 13). After twisting *bc* with respect to *ad* for a unit angle $\theta'$ (angle of twist per unit length *ab* = *cd*) around the bar axis, the parallelepiped $ab_1c_1d$ is created. The shearing strains $\gamma$ in *ab* and *cd* are $r\theta'$; thus their corresponding shearing stresses $\tau = Gr\theta'$ produce a torque around the bar axis expressed in Equation (9). However, if the unit distance *ab* (between the each cross section segment *ad* and *bc*) is not allowed to shorten, the fiber *ab* experiences a unit axial tensile strain equal to $\varepsilon = [(1 + r^2 \theta'^2)^{1/2} - 1] \approx ½ (r^2 \theta'^2)$ that is, a higher order small magnitude. From Timoshenko (1953) with notation adjusted.

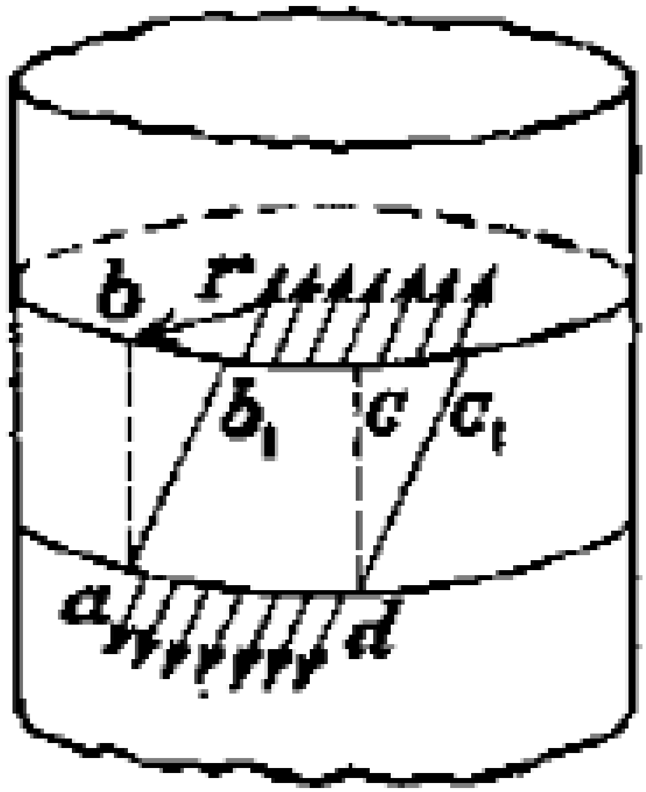



Figure 13. Young's Approach

The generated tensile stresses $Er^2\theta'^2/2$ have a tangential component that produces an additional torque about the bar axis that, applying minor deformation theory, is *atan (rθ'/1) ~ asin rθ' ~ rθ'*; thus the expression in Equation (10) is shaped. Young found that the total expression for torque consists of two terms: One proportional to the angle of twist per unit length *θ'* corresponding to

$$T_1 = \int_0^R Gr\theta' \cdot r \cdot 2\pi r dr = \frac{\pi}{2} R^4 G\theta' = GJ\theta', \tag{9}$$

and another proportional to $(\theta')^3$ multiplied by a constant expressed in length[6] related to the fact that axial elongation was prevented or restrained

$$T_2 = \frac{1}{2}\int_0^R Er^2\theta'^2 \cdot r\theta' \cdot 2\pi r^2 dr = \frac{1}{6}\pi R^6 \theta'^3 E, \tag{10}$$

where the axial stress projection in the transversal direction $T_1$ is now known as the circulatory or Saint Venant torsion and $T_2$ is the torsion produced by the longitudinal stresses preventing changes of distances between cross sections, now known as Vlasov torsion. In 1939, Biot probed that Young method provides the correct answer for circular shafts according to Timoshenko (1953).

In 1853, Saint Venant presented the first sound analytical study of free warping before the French Academy Committee solving the torsion problem of unrestrained cross sections for a variety of prismatic bars, including the elliptic shaft (Timoshenko, 1953). Free warping theory created a pathway for the restrained warping theory development.

At the end of the XIX century Bredt formulated his equations on torsion, one of them named by August Föppol the first Bredt equation. Bredt also proposed a 2nd equation for thin-walled hollow sections with constant thickness.

The work of Bredt in 1896 on tubes and the membrane analogy further developed by Prandtl in 1903 made it possible to apply the St. Venant torsion theory to the thin-walled engineering shapes of those times (Gjelsvik, 1981).

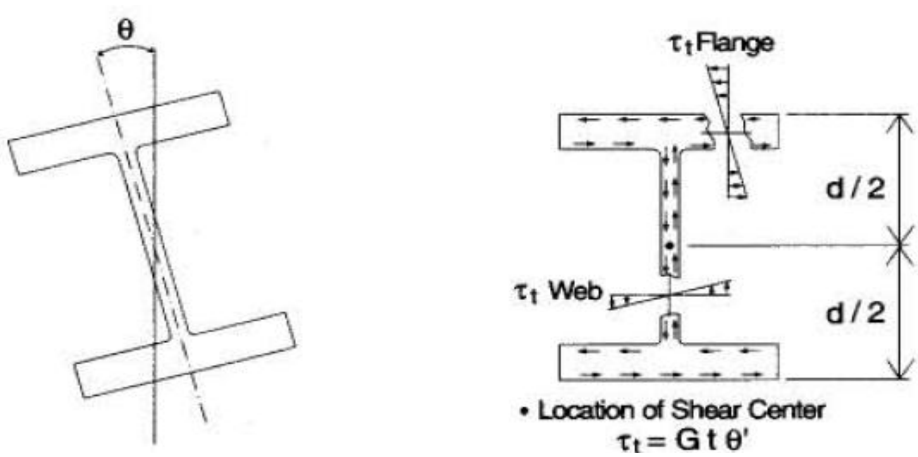



Figure 14. Shear Flux in Saint Venant Torsion

The early stages of the 1st order modern torsion theory for thin-walled bars with open profiles dates back to 1905-06, when Timoshenko published an analysis for I-section bars finding a previously unpredicted torsional resistance different from St. Venant's. Goodier also developed separately general theories on torsion at Cornell University (Ojalvo, 1990, and Gjelsvik, 1981).

The general torsion equation was first deduced for I beams of bisymmetrical cross section and the shear center was already discussed by Timoshenko in the Bulletin of the Institute of Engineers of Ways of Communications, St. Petersburg in 1913 (Pettersson, 1955, Timoshenko and Goodier, 1951). However, R. Maillart is credited with the finding of the shear center. In 1921[6], Maillart published a work on a cross section point he had found through which the shear must pass to avoid the cross section twisting. He also provided the method to locate this point in open profiles (Ojalvo, 1990, and Gjelsvik, 1981).

Wagner (1929) presented the general torsion differential equation of thin-walled bars with open profiles with arbitrary shapes approaching the issue of combined bending and twisting[7] that influenced subsequent works including Vlasov's. Ojalvo (1981, 1982 and 1990) has criticized the application of Wagner's multifilament model to buckling studies.

Ojalvo considers this model inconsistent with the single filament model used in current general torsion theory; therefore, the Wagner hypothesis would be unnecessary for a general theory of rods. See also Falgoust (2004).

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[6] Ojalvo, 1990, mistakenly states that it was in 1924, but both Timoshenko, 1953, and Gvelski, 1981, state that it was in 1921

[7] Referring to restrained warping, the translator of the N .A. C. A. Technical Memorandum No. 807 S. Reiss used either combined bending and twisting or bending accompanying the torsion.

In addition to his previous studies in TWB, Timoshenko (1945) analyzed the bending of thin-walled members of open cross section demonstrating the identity of shear center and the center of twist by using Maxwell's reciprocal theorem.

In the 1930s, Vlasov (1961) developed the general theory of thin-walled bars, from his earlier engineering theory on orthotropic cylindrical and prismatic shells. This work is credited as the seminal contribution to the 1$^{st}$ order general torsion theory that put under the scope the restrained warping phenomenon. Vlasov's work was compiled in a book and first published in Russian, Moscow, 1940 (Timoshenko, 1953).

In order to solve problems of multi-span beams under mixed torsion, Pettersson (1955) advanced a wide-ranging of torsion cases to be solved by means of the Hardy Cross method of successive approximations to find the bending moments acting on the flanges of I-beams, which later was named as bimoment by Vlasov (1961). Possibly due to the year of publication, he did not mention either Vlasov or the moment-bimoment analogy. On the other hand, Pettersson did acknowledge the contributions of Wagner (1929) on the governing differential equation of the general torsion theory and the compilation of solutions of torsional fundamental cases by Bornscheuer in 1953.

In a more detailed analysis, Dabrowski (1960) found that few essential terms were missing in Vlasov's equations, which could have conducted to errors. His work on Curved Thin-Walled Girders published in 1968 is frequently quoted by many torsion theorists and practitioners as Heins (1975).

Ligthfoot (1961) and Gere (1963) used a comprehensive approach of the moment distribution method to undertake the problem of beam-columns under mixed axial and transverse loads. Stiffness coefficients and carry over factors for uniform beams axially loaded in compression or tension and with their far end fixed or hinged can be found in these works, as well as influence lines for fixed end moments.

Heins (1963) and Seaburg prepared a manual to be used as design data for the Bethlehem Steel Corporation. This manual provides charts of torsional function curves, tables of torsional properties, case charts listings for various torsional loading and end conditions, case charts of torsional function curves with examples on general torsion, etc. All this material has been widely used by practitioners and even assimilated for other publications as the AISC-DG-9 by the very Seaburg (1997) and Charles J. Carter.

Wright (1968) with his BEF and Hsu (1995) with his EBEF provided a rational approach to box girder design. Box girders are thin walled beams with closed cross sections, and even that they are prone to suffer restrained warping; their governing differential equation is different from that of thin walled beams with open cross sections.

Boothby (1984) postulated the Bending Moment-Bimoment Analogy or Bending-Warping Analogy; that is, a bimoment distribution method for multi-span bars analogous to classical moment distribution methods of flexural analysis. He neither mentioned Vlasov's beam-column analogy or the book by Vlasov (1961).

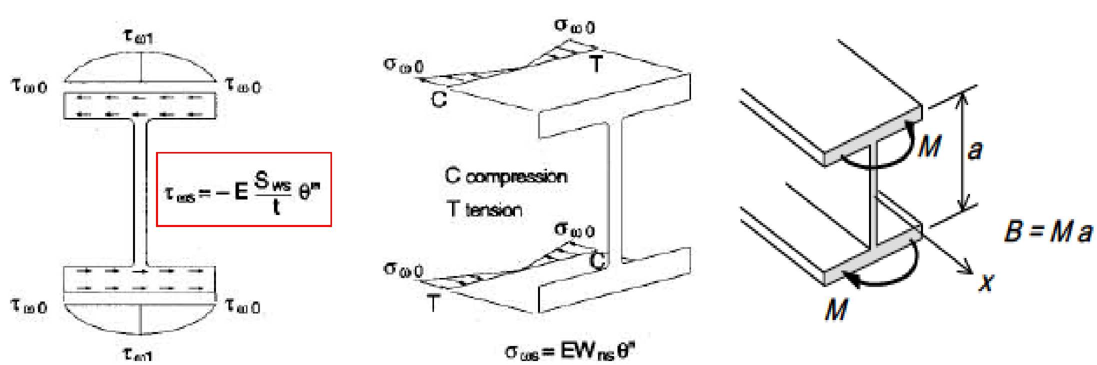



Figure 15. Bimoment and Related Stresses

Medwadowski (1985) states, quoting Bleich (1952), that "…the governing differential equation" of the beam-column "is identical to the equation governing the problem of beams in axial tension and bending." Medwadowski (1985) redistributed bimoments in multi-span beams by using beam-column formulations proper of classical moment distribution methods similar to the ones developed by Lightfoot (1961) and Gere (1963). In fact, Åkesson (1987) discussed on the paper by Medwadowski that the real beam in mixed torsion rather be represented by an analog beam-column in mixed bending and tension for the sake of clarity, accuracy and efficacy.

### 2.6 Some Features Introduced by the Restrained Warping Theory

It has been discussed that thin-walled members with open cross sections are prone to warp. A restrained I-beam under torsion undergoes opposite lateral curvatures with balanced opposite lateral shears and moments (Figure 10). A system of generalized forces statically equivalent to zero, coined by Vlasov as bimoment *B*, is developed. It is equal to the flexural moment around the minor axis in one flange times the separation between flanges. As a result, warping axial strains and stresses are developed along the member.

The Vlasov hypotheses of general torsion 1$^{st}$ order theory are: First, the cross section contour *s* does not bend or elongate; then, it does not suffer changes in curvature as expressed in Equation (11), or changes in length as expressed in Equation (12), thus, the middle line projection of the thin-walled beam in the cross section surface *xOy* does not change (See Figure 16(a)). And second, the middle surface of the thin-walled bar does not suffer shear deformations as denoted in Equation (13) and Figure 16(b).

$$\frac{1}{\rho_1} - \frac{1}{\rho} = 0, \tag{11}$$

$$\frac{u_r}{\rho} + \frac{\partial u_s}{\partial s} = 0; \tag{12}$$

$$\gamma_1 + \gamma_2 \approx 0. \tag{13}$$

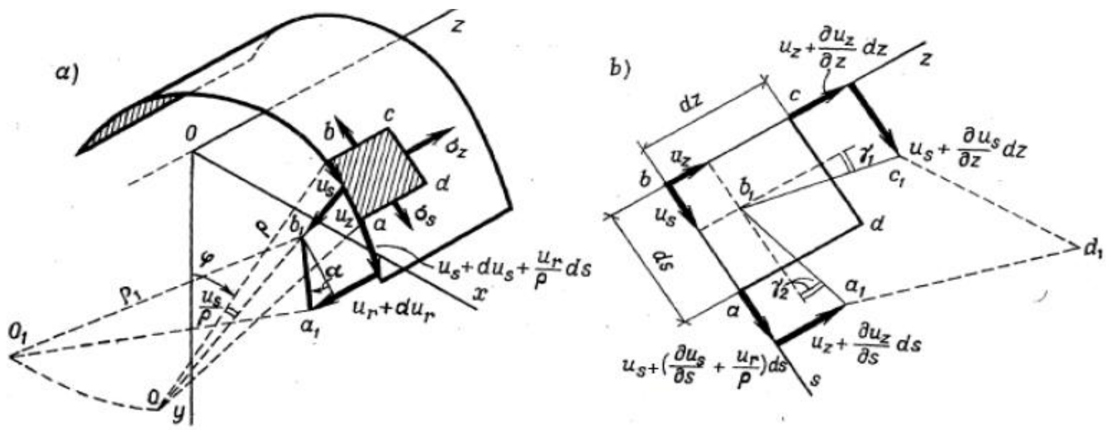
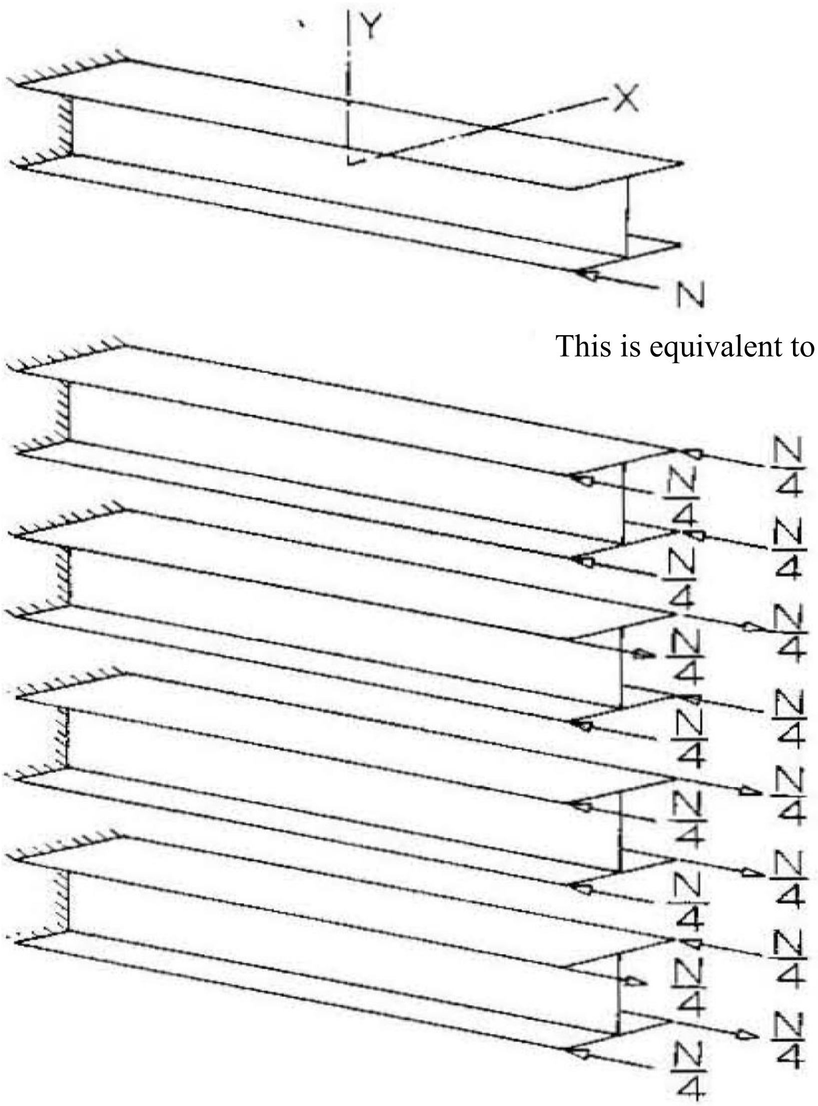


Figure 16. Differential Element in a Thin-Walled Beam

Equations (11) and (13) explain the cross section deformation out of its plane. The cross sections after the deformations will not remain plane but will obey the rule of sectorial areas according to Rekach (1978).

One of the consequences of this phenomenon is that the flanges of a thin-walled beam subjected to a centered axial tensile force *P* applied eccentrically as it is shown in Figure 17 would deform independently from one another. And each transverse lateral moment acting over each flange would spread over the entire length of the beam. The Saint Venant's principle does not hold true in this case (Feodosiev, 1972).

Figure 17 from Chaudhary (1982) illustrates a way in which the warping function could manifest; the example of an axial centered resultant force producing bimoments will be presented. If *b* and *d* are the section width and depth, respectively, Figure 17(e) shows equal and opposite lateral bending moments *Nb/4* acting at each flange producing opposite lateral curvatures in each flange, which are statically equivalent to zero. The product of each opposite bending moment times their distance is called bimoment *Nbd/4*.




(a)

This is equivalent to:-

(b)
AXIAL FORCE

(c)
B.M. ABOU X
AXIS

(d)
B.M. ABOUT Y
AXIS

(e)
BIMOMENT

Figure 17. Centered Axial Load Producing Restrained Warping

According to Vlasov (1961), the same result could have been obtained by multiplying the axial force by the warping function $(-N)\cdot(-bd/4) = Nbd/4$. The summation of forces is statically equivalent to a centered axial force *N* plus a bending moment *Nd/2* around mayor axis plus a bending moment *Nb/2* around minor axis, plus two opposite and equal moments producing opposite lateral curvatures in the flanges (Chaudbary, 1982).

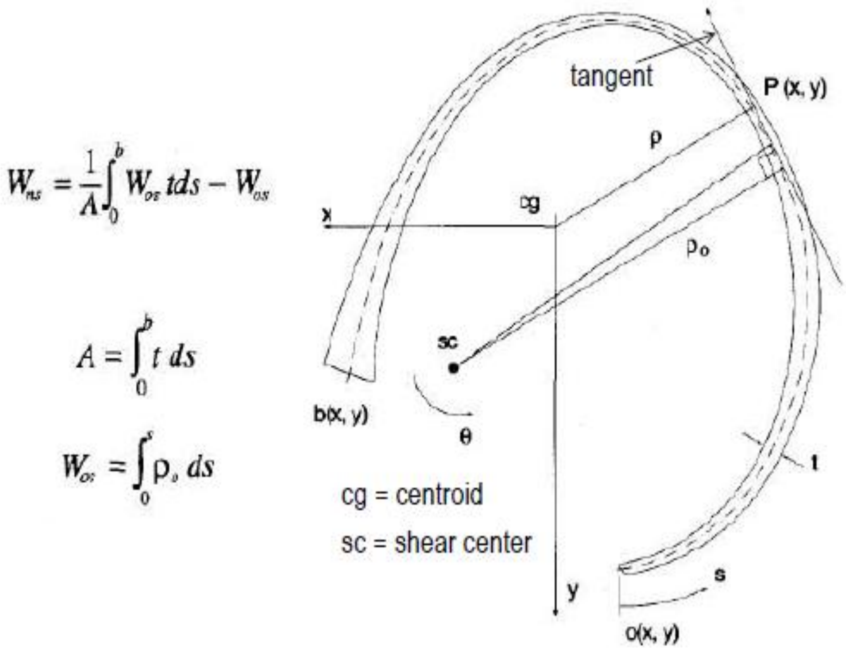



Figure 18. Definition of Warping Function

### 2.7 The Beam-Column Analogy

In Chapter II, Section 9 of his book, Vlasov (1961) restated the impeccable analogy between the three-term-differential-equation of $2^{nd}$ order analysis of transversely and axially loaded bar and the differential equation of $1^{st}$ order analysis of a beam under mixed St. Venant and Vlasov torsion.

Boothby (1984) produced a work on the application of the moment distribution method to torsional analysis of multi-span TWB with open cross sections. He renamed the bimoment as warping moment: The kind of warping that produces bending moment.

Medwadowski (1985) also resorted to the moment distribution method for the torsional analysis of multi-span TWB with open cross sections. His equations are presented in matrix form, and his transfer matrix helps to express the deformed configuration of each span of the beam once the MDM has converged.

Both Boothby (1984) and Medwadowski (1985) prepared separately formulas and tables with stiffness, carry over factors, fixed-end bimoments for various loading and boundary conditions based on the formulas of mixed torsion. Medwadowski (1985) also presented the analytic stiffness matrix for the beam under mixed torsion that will be used in this paper to recommend a size of the finite element to ne used in the software.

Boothby (1984) only made references to works on combined bending and torsion without even including Vlasov's. Conversely, Medwadowski (1985) made a number of references to seminal works on beam-column and on moment distribution methods. Boothby (1984), for example, stated that he had to compute the fixed moments on a beam under a concentrated load by dividing the beam at the point of load application into two parts, as a result he solves simultaneously 4 equations with 4 unknowns for each span; while both Ligthfoot (1961) and Gere (1963) had already solved the problem trough more straightforward methods for a beam-column under mixed tension and transverse load.

Åkesson (1987) stated that, works on the moment distribution method to torsional analysis of multi-span TWB with open cross sections were advanced by others including Pettersson (1955). He proposes to model and solve an analogue beam-column in mixed bending and tension. And Deschapelles (1987) acknowledged Medwadowski's effort to approach the statical redundancy in thin-walled bars under restrained warping and advised to better follow the philosophy of the finite element method.

### 2.7.1 The Beam-Column Differential Equation

If an infinitely small element bounded by two cross sections a distance *dz* apart is cut out of the BC, the equilibrium of moments leads to the Equation (14) from Hetényi (1961) illustrated in Figure 19; where $Q_v$ is the vertical shearing force, as shown in

Figure 19. The normal shear $Q_n$ acting in the plane of the section normal to the deflection line can also be obtained using Figure 19b with the results of Equations (15) and (16), where it could be noticed that the rate of change of curvature is almost exclusively responsible for the normal shear.

$$(M+dM)-M+Ndy-Q_v dz=0 \Rightarrow \frac{dM}{dz}+N\frac{dy}{dz}-Q_v=0; \tag{14}$$

$$Q_n = Q_v \cos\theta - N\sin\theta, \tag{15}$$

$$\begin{aligned} Q_n &= \cos\theta(Q_v - N\frac{\sin\theta}{\cos\theta}) = \cos\theta(Q_v - N\tan\theta) \Rightarrow \\ Q_n &= \cos\theta(Q_v - N\frac{dy}{dz}) = \cos\theta\frac{dM}{dz} \approx \frac{dM}{dz}, \forall\theta \ll 1 \end{aligned} \tag{16}$$

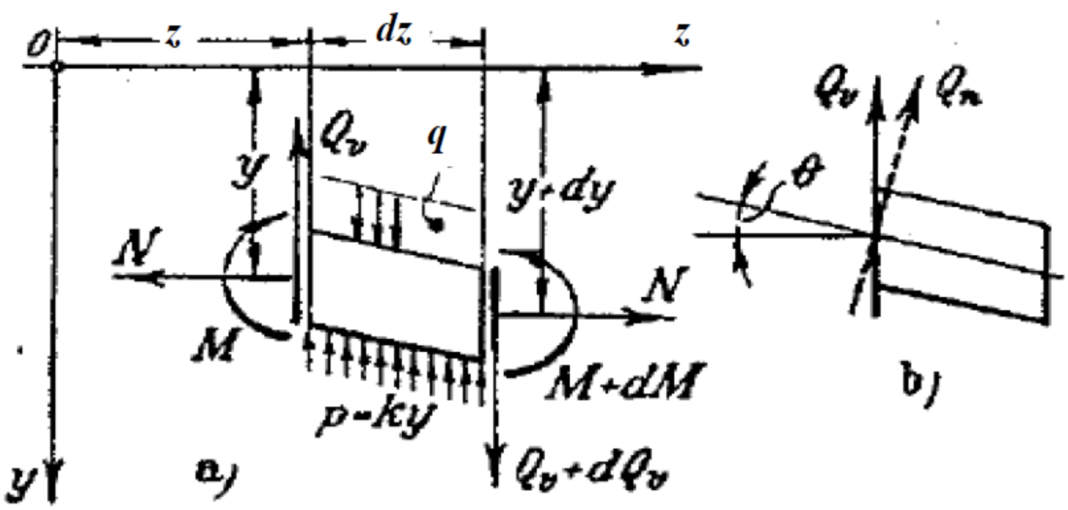



Figure 19. Beam Column under Mixed Axial and Transverse Loading

In the following derivations $Q_v$ from Equation (16) will be used. Replacing $M=-E\cdot I\cdot y''$ (bottom fibers are in positive location and under a tensile positive stress $\sigma_y$ = $\varepsilon{\cdot}y$ combined with an upwardly negative curvature). Then he differentiates with respect to the axial coordinate, making $Q_v'=ky$, thus obtaining the differential equation of the elastic line of a straight bar under mixed axial and transverse loading:

$$
\begin{aligned}
&+EI\frac{d^4y}{dz^4}-N\cdot\frac{d^2y}{dz^2}=-ky+q\Rightarrow \\
&EI\frac{d^3y}{dz^3}-N\frac{dy}{dz}=Q_v\Leftrightarrow ky\wedge q=0,
\end{aligned}
\qquad (17)
$$

where $E$ and $I$ are the modulus of elasticity and the cross section inertia of the member, and $k$ is the soil modulus. The 2nd of Equations (17) corresponds to the integration of the first one with both the modulus of the foundation $k$ and the uniformly distributed load $q$ equal to 0. Figure 20 illustrates a different convention for a BC with longitudinal axis along axis z, and where $ky$ and $q(z)$ are also replaced by a unique k(y,z).

$$
Q_v=N\frac{dy}{dz}-EI\frac{d^3y}{dz^3}. \qquad (18)
$$

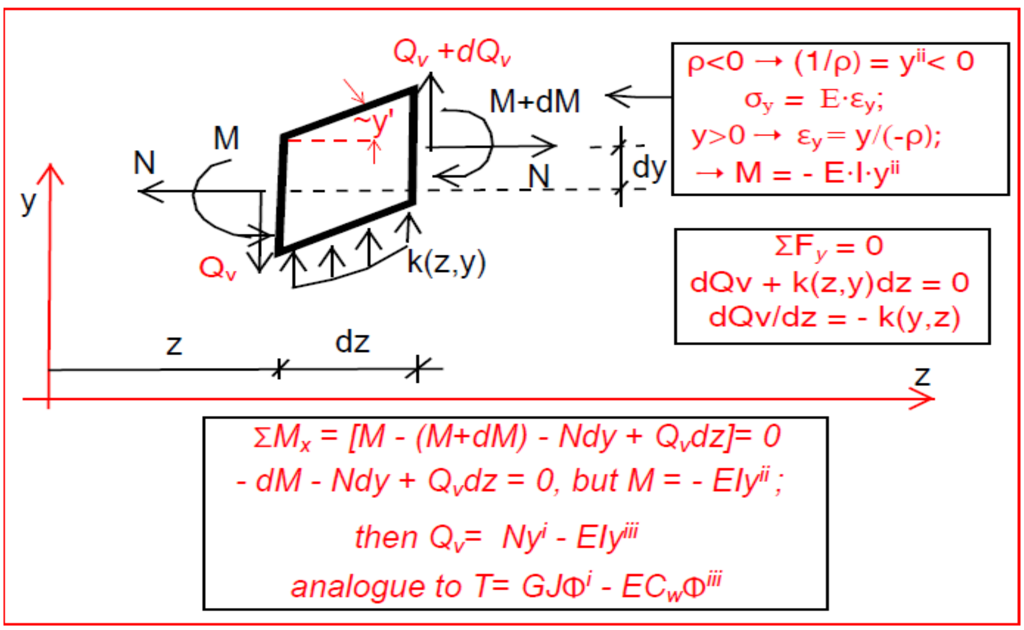



Figure 20. Another BC Sign Convention with Similar Results

### 2.7.2 Differential Equation of a Torsional TWB

The governing differential equation of a thin-walled bar under restrained warping has any of the forms shown in Equations (19), where $z$ is considered the longitudinal axis, $E$ is the modulus of elasticity of the material, $G$ is the shear modulus of elasticity, $Cw$ is the cross section warping constant, $J$ is the Saint Venant torsional constant, $\theta$ is the cross section twist angle, $t$ is a uniformly distributed torque, $t_j$ is the highest applied torque at right support[8] in the case of a linearly varying torque, and $T$ is the applied torque in the cross section under study. See Ugural (1987) Heins (1963) and Figure 26 from the AISC-Design Guide 9, 1997.

$$
\begin{gathered}
T = G \cdot J \frac{d\phi}{dz} - EC_W \cdot \frac{d^3\phi}{dz^3}, \\
E \cdot C_W \frac{d^4\phi}{dz^4} - G \cdot J \frac{d^2\phi}{dz^2} = t \Leftrightarrow \frac{dT}{dz} = -t \\
EC_W \frac{d^4\phi}{dz^4} - G \cdot J \frac{d^2\phi}{dz^2} = t_j \frac{x}{l} \Leftrightarrow \frac{dT}{dz} = t_j \frac{x}{l}
\end{gathered}
\tag{19}
$$

#### 2.7.3 The Analogue Beam-Column

The analogue beam-column is a deformed straight bar vertically loaded and pulled by a horizontal force *GJ* as illustrated in Figure 21, Figure 22, Figure 23, which obeys Equations (19). Its longitudinal and transverse loadings are not tangent and normal to the elastic line respectively: they are measured in their original directions before the bar is deformed.

That is why the BC in Figure 22 remains in equilibrium. When the beam-column Equation (16) is formulated with forces acting normally and along the deformed elastic line, the analogy with the TWB is lost as it could be seen by comparing Equations (19) On the other hand, Equations (19) governing open TWB, are perfectly analogue to Equations (17) governing BC. In this sense, when using the small deflection theory in the BC straight bar, care should be taken so that the axial and transverse loads remain in their original directions before and after bending.

[8] The AISC uses $t$ without subscript.

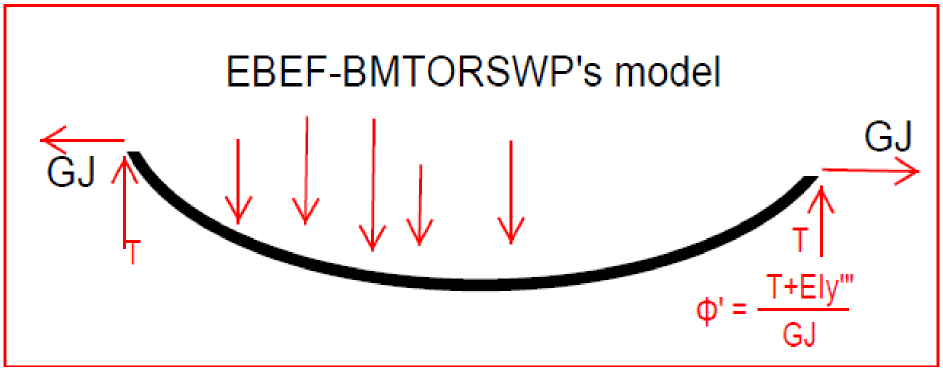



Figure 21. Analogue Model in Deformed Bar

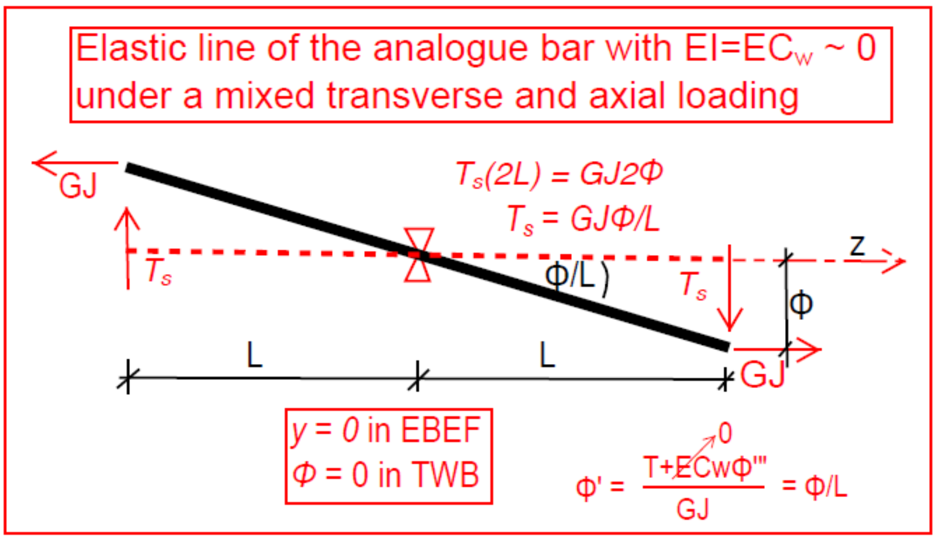



Figure 22. BC Model in Equilibrium

It is important to clarify statements by Åkesson (1987): "For a real beam in pure St. Venant torsion ($E \cdot Cw = 0$), the analogue beam-column degenerates into a transversely loaded cable stretched by a force *GJ*" (symbols have been adapted to those in AISC-DG-9). This statement should be understood under the light of Åkesson's model shown in

Figure 23 and never interpreted too literally as in Figure 24. Where it could be noticed that the concurring longitudinal forces $G \cdot J$ are incapable to provide opposition to the couple of forces formed by the transverse loads *Ts* and their arm *2L*. Because this transverse loads happen to be applied at the end nodes of the beam, and do not deform according to a commonly transversely loaded cable stretched by a tensional force.

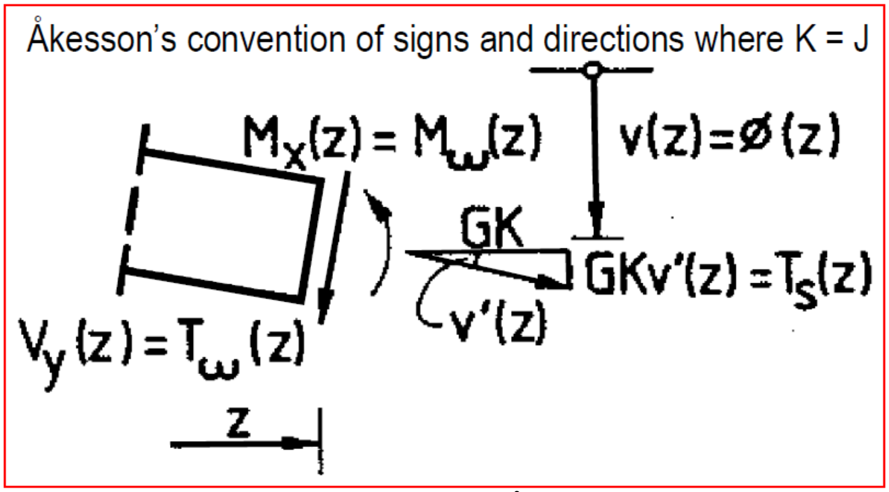



Figure 23. Equilibrium in Åkesson's Model

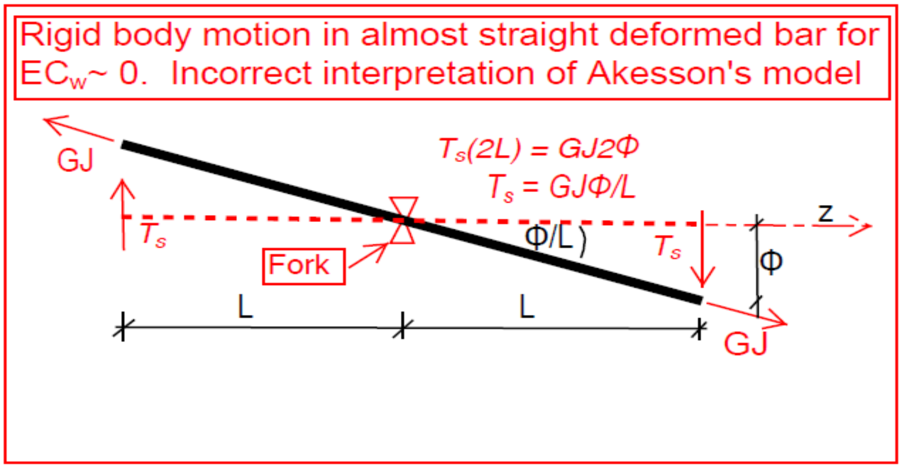



Figure 24. Incorrect Overly Literal Interpretation of Akesson's Model

### 2.7.4 Boundary Conditions and Equivalence of Symbols

The following are equivalences between the real TWB undergoing a twist angle $\theta$ and the BC undergoing a transverse displacement $y$ (Table 3 and Table 4):

Table 3. Analogue Components between a TWB and a BC

| **Beam in mixed uniform and non-uniform torsion, TWB** | **Beam in mixed transverse and axial loading, BC** |
|---|---|
| Angle of twist or torque angle = $\theta$ | Transverse displacement = y |
| $\theta \neq 0$, some torsion rotation<br>$\theta' \neq 0$, no warping restraint<br>$\theta'' \neq 0$, some bimoment | $y \neq 0$, some transverse displacement<br>$y' \neq 0$, slope, no fixed end<br>$y'' \neq 0$, some bending |

(continue)

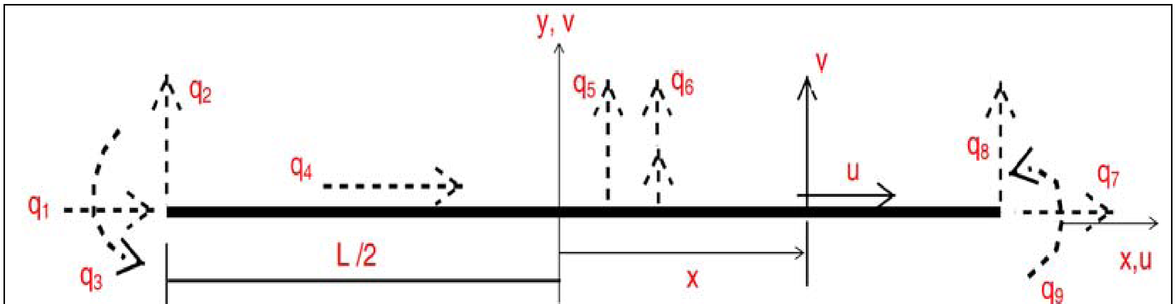
| | |
|---|---|
| *Cw* = *Iww* = *Γ*. Warping constant Principal 2nd order moment of area using sectorial area coordinates = $\int w^2 \cdot dA$, in$^6$ | I = moment of inertia $\int y^2 \cdot dA$, in$^4$ |
| $E \cdot Cw$ = Warping stiffness, kip- in$^4$<br>G·J = St. Venant torsion stiffness, kip-in² | E·I = flexural stiffness, kip- in²<br>N = tensile axial load in original coordinates, kip |
| θ = Torsion twist at a generic section<br>θ" = 2nd derivative → Bimoment<br>$B = -E \cdot Cw \cdot \theta''$ | y = Transverse displacement at a section<br>y" = 2nd derivative → bending moment<br>$M = -E \cdot I \cdot y''$, or $y'' = -M/ E \cdot I$ |
| θ = 0, no torsional rotation (clamped end or fork)<br>θ' = 0, no slope, warping restraint (clamped end)<br>θ" = 0, no torsional curvature, B = 0, (no end moment, pinned or free) | y = 0, no transverse displacement (pinned end or roller, fixed end)<br>y' = 0, no slope, flexural restraint (fixed end, symmetry)<br>y" = 0, no flexural curvature, M= 0 (no end moment, pinned end, roller) |
| Warping (lateral bending) axial stresses:<br>$\sigma_w = - E \cdot w\, \theta''$ | Vertical bending axial stresses:<br>$\sigma_f = - E \cdot y \cdot y''$ |

Table 4. Torsional Conditions in TWB from AISC-DG 9

| **Physical** | **End** | **Mathematical** |
|---|---|---|
| No rotation | Fixed (clamped)or pinned (fork) | θ = 0 |
| Cross-section cannot warp | Fixed (clamped) end | θ' = 0 |
| Cross section warps freely | Pinned or free | θ” = 0 |

### 2.8 The High Degree 9-DOF-Bar Element

The 9-DOF-BE is a beam-column element composed by a 3DOF rod element plus a 6-DOF beam-column element. The 3DOFs of the rod element operate in the field of axial displacements uncoupled with the transverse displacement and its derivatives without taking care of the 2nd order effect of the axial load. The 3DOF rod element has a quadratic expansion with one axial DOF at each end node plus a dimensionless DOF, the average of axial displacements. The 6DOF beam-column element operates in the uncoupled field of transverse displacements and will be described with more detail.



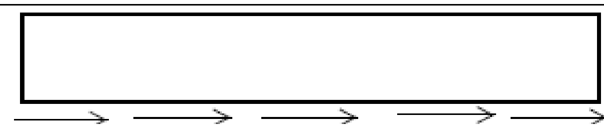
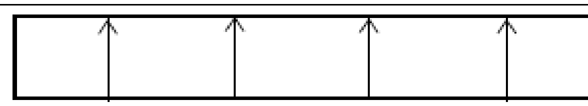
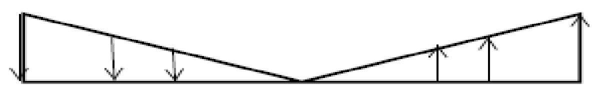

| Non Nodal DOF | Definition | Meaning | Corresponding Distributed Load |
|---|---|---|---|
| $q_4$ | $\int u \cdot dx / L$ | Average longitudinal displacement |  **Longitudinal Uniform** |
| $q_5$ | $\int v \cdot dx / L$ | Average transverse displacement |  **Transverse Uniform** |
| $q_6$ | $\int v \cdot x \cdot dx / L$ | First moment of transverse displacement s |  **Transverse Lineal** |

Figure 25. The 9DOF Frame Element with its Non-Nodal DOF

In order to avoid the presence of the element length $L$ in the expressions of the entire matrix entries, normalized-dimensionless degrees of freedom divided by powers of the length were used as it could be seen in Table 5 and Figure 25.

Local axes are located at member midspan, $A$ and $I$ are the cross section area and inertia respectively, and $Ce$ is a common dimensionless factor stated in Equation (20). To avoid matrix singularity, a value different from zero, i.e. 1, must be assigned to the cross section area to make possible the matrix inversion when running BMTORSWP. Table 6 shows one of the 9DOF components, the 6DOF stiffness matrix related to transverse displacement and derivatives.

Table 5. All the DOFs' Definitions in the 9DOF-BE

| DOFs | Definitions |
|---|---|
| 1 and 7 | Axial displacements at each end divided by length, L |
| 4 | Average, non nodal, axial displacements divided by length, L |
| 2 and 8 | Transverse displacements at each end divided by L |
| 3 and 7 | Rotational displacement at each end |
| 5 | Average, non nodal, transverse displacement, divided by L |
| 6 | Average, non nodal, first moment of transverse displacement, divided by $L^2$ |

Table 6. The 9-DOF-BE Elastic Stiffness Matrix

$$K_{.e9s} := \frac{4 \cdot E \cdot I}{L^3} \cdot \begin{pmatrix} 2 \cdot C_{.e} & 0 & 0 & -3 \cdot C_{.e} & 0 & 0 & C_{.e} & 0 & 0 \\ 0 & 300 & 30 & 0 & -90 & 2520 & 0 & -210 & 15 \\ 0 & 30 & 4 & 0 & -15 & 210 & 0 & -15 & 1 \\ -3 \cdot C_{.e} & 0 & 0 & 6 \cdot C_{.e} & 0 & 0 & -3 \cdot C_{.e} & 0 & 0 \\ 0 & -90 & -15 & 0 & 180 & 0 & 0 & -90 & 15 \\ 0 & 2520 & 210 & 0 & 0 & 25200 & 0 & -2520 & 210 \\ C_{.e} & 0 & 0 & -3 \cdot C_{.e} & 0 & 0 & 2 \cdot C_{.e} & 0 & 0 \\ 0 & -210 & -15 & 0 & -90 & -2520 & 0 & 300 & -30 \\ 0 & 15 & 1 & 0 & 15 & 210 & 0 & -30 & 4 \end{pmatrix}$$

$$C_e = \frac{A}{2I} L^2 \therefore if + A = 1 \Rightarrow C_e = \frac{1}{2I} L^2 \qquad (20)$$

### 2.8.1 Expediency and Importance of the Non Nodal DOF

Nodal degrees of freedom are precise points or directions at, along or around which only applied concentrated generalized loads could be accommodated. Conversely, non nodal DOFs could accommodate actions operating on continuously distributed domains, i.e. complex geometric parameters other than localized directions.

Since a distributed load acts over a disperse domain in the element, it can be related to a dispersed displacement measured over the same domain. Non nodal degrees of freedom are "distributed directions" that could be defined by integrations or averages of geometric parameters (consistently and carefully chosen to avoid matrix singularity) over the element domain. Non nodal degrees of freedom cover the action of distributed loads, just as "concentrated directions or coordinates" are required to ensure continuity of the deformed configuration at the nodes, providing accommodation for complex loading cases without entailing to find equivalent nodal loads.

Deschapelles (2002) formulated a detailed description of a refined beam element with five degrees of freedom, one of them was the first non-nodal degree of freedom explicitly described in a finite element publication. Nevertheless, this type of degree of freedom had been used by him for quite a while. Deschapelles (1987) had shown before an explicit stiffness matrix containing non nodal DOF and making a reference to the development of his 12-DOF frame-shear element in Deschapelles (1978, 1984).

In the case of the 6DOF element used by BMTORSWP, this ground-breaking and handy non nodal DOF was able to capture entirely the uniformly distributed load in its corresponding consistent load vector. Thus, no loads components were projected to the other DOF. Moreover, the stiffness, and geometric matrixes were successfully developed, explicitly formulated and applied to a successful 2$^{nd}$ order analysis, and a soil contribution matrix was also provided for the solution of a wide range of EBEF problems. In fact, after a research spanning almost 2 years without the knowledge, involvement or input of Deschapelles, a categorical statement can be made that no other reference to non nodal DOFs have been advanced in the literature of finite elements.

### 2.8.2 Adequacy of the 6-DOFs for the Transverse Displacement

The 6-DOF-BE (which is a component of the 9-DOF-BE) corresponds to a $5^{th}$ order polynomial acting in the field of transverse displacement *v*. The geometric and soil matrix are developed entirely in the field of transverse displacements, as well as the flexural elastic matrix. This element has a $5^{th}$ order expansion with two nodal degrees of freedom at each end node plus two dimensionless degrees of freedom that symbolically are situated at midspan of each finite element.

The transverse displacement *v* of the beam-column is analogous to the angle of twist of a TWB with open cross section. The real open TWB under a linearly varying torque (Figure 26) undergoes a deformed rotational configuration governed by a $5^{th}$ degree polynomial. Therefore, the beam-column should have a field of transverse displacements governed by an equal or higher-than a $5^{th}$ degree polynomial.

The beam-column field of transverse displacements is ruled by a $5^{th}$ degree polynomial. Thus, the 6DOF beam-column elastic line, which is a component of the 9DOF-BE can perfectly represent both a uniform and a linearly varying transverse load whose bending moments will be described at least by a $2^{nd}$ and a $3^{rd}$ degree polynomials respectively. On the other hand, the elastic lines will be described at least by a $4^{th}$ and a $5^{th}$ degree polynomials, respectively.

In fact, fields of transverse displacement containing powers lower than five cannot form the basis of a BC that provides a solution for general torsion problems. In addition, three derivatives are required to be produced in the output for design purposes.

Because, restrained warping shearing stresses depend on the angle of twist third derivative with respect to the axial coordinate, and warping normal stresses depend on the second derivative. The first derivative defines the free warping shearing stresses.

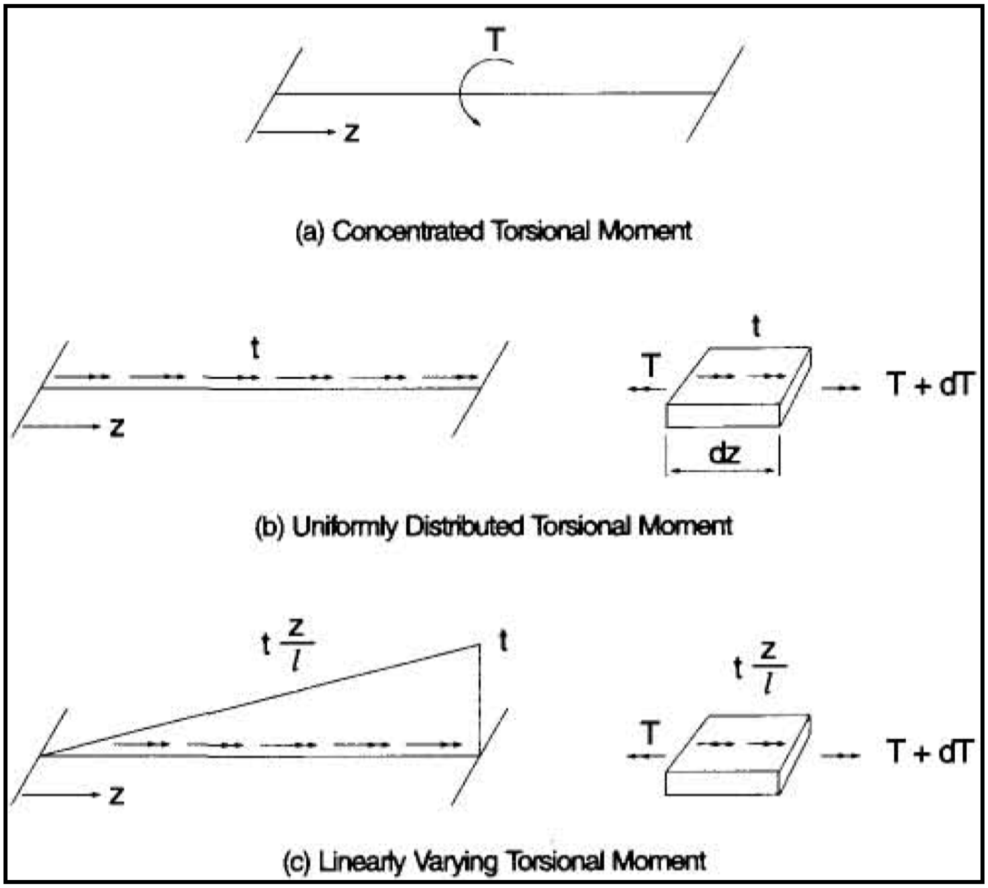



Figure 26. Torsional Loading Systems from AISC-DG9

It is pertinent to remark that this high order finite element can also accommodate in the corresponding non nodal load consistent vector shown in Equation (27) and Equation (28) all the torsional loading cases requested by the code just by using one algorithm. This additional feature makes the software application even more trustful.

### 2.8.3 Field of Transverse Displacements of the 6-DOF BE

The generic transversal field of displacements {g} at an arbitrary point along the 6-DOF BE is expressed with a 5$^{th}$ order polynomial in Equation (21). Where its variable of position $\xi$ has been normalized, and it's $\alpha$ coefficients are also called generalized displacements. The generic displacement could also be expressed as a function of the Shape Function [N] shown in Equation (22) in a one column matrix version, as explained below. Identifying x instead of *z* as the longitudinal axis, the corresponding DOF are: First, the $v(x(\xi))/L$ value for $\xi = -1$. Second, the $v(x(\xi)),x$ value for $\xi = -1$. Third, the non-nodal integral $(\int v(x(\xi))dx)/L^2$. Fourth, the non-nodal integral $(\int v(x(\xi))\cdot x\cdot dx)/L^3$. Fifth, the $v(x(\xi))/L$ value for $\xi = +1$. Sixth, the $v(x(\xi)),x$ value for $\xi = +1$.

$$v(\xi) \rightarrow \alpha_5\cdot\xi^5 + \alpha_4\cdot\xi^4 + \alpha_3\cdot\xi^3 + \alpha_2\cdot\xi^2 + \alpha_1\cdot\xi + \alpha_0 \qquad (21)$$

### 2.8.4 Shape Function Matrix [N] of the 6-DOF BE

The generic displacement at an arbitrary point in the element {g} is numerically defined by the shape function matrix [N] pre-multiplied by the vector of specific (reference) displacements {q} along the DOFs. That is, {g} = {q} [N]. The shape function matrix [N] contains each of the interpolation polynomials of the position coordinate. [N] is shown in Equation (22) as a column vector for convenience. Where $x = d\xi$, $dx = ad\xi$, $a = L/2$, and $L$ is the member length, and x is the longitudinal coordinate.

### 2.8.5 Geometric Interpretation of the Shape Function [N]

The $i^{th}$ shape function is the equation that describes the deformed configuration of the finite element elastic line when a unit displacement in the domain of the $i^{th}$ degree of freedom is imposed while any other displacement along or around the rest of DOFs is prevented.

The $i^{th}$ shape function is the equation describing the deformed configuration of the element elastic line when a unit displacement in the domain of the $i^{th}$ degree of freedom is imposed when any other displacement along the rest of DOFs is prevented.

$$N := L\cdot\begin{pmatrix} \frac{21\cdot\xi^5}{8} - \frac{15\cdot\xi^4}{16} - 5\cdot\xi^3 + \frac{15\cdot\xi^2}{8} + \frac{15\cdot\xi}{8} - \frac{7}{16} \\ \frac{7\cdot\xi^5}{32} - \frac{5\cdot\xi^4}{32} - \frac{5\cdot\xi^3}{16} + \frac{3\cdot\xi^2}{16} + \frac{3\cdot\xi}{32} - \frac{1}{32} \\ \frac{15\cdot\xi^4}{8} - \frac{15\cdot\xi^2}{4} + \frac{15}{8} \\ \frac{105\cdot\xi^5}{4} - \frac{105\cdot\xi^3}{2} + \frac{105\cdot\xi}{4} \\ 5\cdot\xi^3 - \frac{15\cdot\xi^4}{16} - \frac{21\cdot\xi^5}{8} + \frac{15\cdot\xi^2}{8} - \frac{15\cdot\xi}{8} - \frac{7}{16} \\ \frac{7\cdot\xi^5}{32} + \frac{5\cdot\xi^4}{32} - \frac{5\cdot\xi^3}{16} - \frac{3\cdot\xi^2}{16} + \frac{3\cdot\xi}{32} + \frac{1}{32} \end{pmatrix} \qquad (22)$$

### 2.8.6 The 6-DOF BE Elastic Matrix [B]

The elastic or strain matrix [B] is the second derivative of the shape function matrix expressed here with one subscript for each derivative [$N_{xx}$]. Therefore, it is related to the generic curvature of the member elastic line. The elastic matrix is obtained by integrating the projections of all the infinitesimal stiffness contributions $E\cdot I\cdot a d\xi$ along the member towards the geometric parameters chosen as the member DOFs. Where $x = a\xi$, $dx = ad\xi$, $a = L/2$, $L$ is the member length, and $x$ is the longitudinal coordinate. Equation (23) states the analytical formulation of the elastic matrix, while its explicit numeric results appear in Equation (24) whose coordinate system has origin at midspan.

$$\int_{-1}^{1} N_{xx}\cdot E\cdot I\cdot N_{xx}^{T}\cdot a\, d\xi \qquad (23)$$

$$\left(\frac{4\cdot E\cdot I}{L}\right)\cdot\begin{pmatrix} 300 & 30 & -90 & 2520 & -210 & 15 \\ 30 & 4 & -15 & 210 & -15 & 1 \\ -90 & -15 & 180 & 0 & -90 & 15 \\ 2520 & 210 & 0 & 25200 & -2520 & 210 \\ -210 & -15 & -90 & -2520 & 300 & -30 \\ 15 & 1 & 15 & 210 & -30 & 4 \end{pmatrix} \qquad (24)$$

### 2.8.7 The 6-DOF BE Soil Normal Stiffness Contribution Matrix

The normal modulus of the foundation or soil modulus at interface is $k_0$. The 6-DOF-BE soil stiffness contribution matrix is expressed numerically in Equation (26) and was obtained by integrating Equation (25). Due to the Boussinesq effect, it is not an uncommon engineering practice to assign a unit value to the effective convex interface width $b$ according to Terzaghi (1973); thus, $k_o$ and $k$ could be numerically equal except for their units:

$$\int_{-1}^{1} N \cdot k_o \cdot b \cdot N^T \cdot a \, d\xi \tag{25}$$

$$\frac{k_o \cdot b \cdot L^3}{13860} \cdot \begin{pmatrix} 3000 & 140 & -2970 & 16800 & -30 & -25 \\ 140 & 8 & -165 & 630 & 25 & -3 \\ -2970 & -165 & 19800 & 0 & -2970 & 165 \\ 16800 & 630 & 0 & 352800 & -16800 & 630 \\ -30 & 25 & -2970 & -16800 & 3000 & -140 \\ -25 & -3 & 165 & 630 & -140 & 8 \end{pmatrix} \tag{26}$$

### 2.8.8 Load Consistent Vectors for the 6-DOF BE

The effect of infinitesimal loads applied along or around the element domain can be projected over the DOFs directions by means of the interpolation polynomials contained in the shape functions. The former operation provides a load consistent vector for any kind of distributed load. Particular distributed loads can be exclusively projected over distributed degrees of freedom. Different distributed normalized loads from a constant uniform load to one with a $5^{th}$ order polynomial variation, are shown in Equation (27), where $w$ is a distributed constant load expressed in lb/in (force/length), and $W$ is a vector with powers of varying distributed loads.

In the case of a 6-DOF-BE, two load consistent vectors are shown in the first and second columns of the Equation (28) in matrix form one for a constant and other for a linearly varying distributed loads. It could be noticed that these 2 loadings do not project to DOFs other than the 3rd and 4th non nodal DOFs previously discussed.

This outstanding feature could very well be exploited in a number of finite element formulations. As it was already said, this fact was addressed in Deschapelles (2002). Unfortunately, in earlier papers, Deschapelles (1984, 1985, and 1987) restrained himself to just introduce finite element formulations of his own containing these suitable non nodal geometric parameters without a major discussion of their remarkable properties.

A great advantage of these non-nodal DOFs is that, in this particular problem, they could abridge in only one algorithm solutions for the 3rd and 4th order differential equations of members subjected to two different distributed loads. Concentrated loads in different locations could be handled by assigning each one node. Thus, it is not longer necessary to develop one algorithm for each version of Equation (17) for the BEF behavior or their corresponding two analog Equations (19) for the TWB behavior.

As a result of what has been previously stated, the $i^{th}$ column of any stiffness matrix expresses the system of forces in the coordinates of all the element degrees of freedom compatible with a unit displacement along the $i^{th}$ degree of freedom without any displacement in the coordinates of all other DOFs.

$$W^T \rightarrow \begin{pmatrix} w \\ \xi \cdot w \\ \xi^2 \cdot w \\ \xi^3 \cdot w \\ \xi^4 \cdot w \\ \xi^5 \cdot w \end{pmatrix} \tag{27}$$

$$\frac{1}{\left(L^2 \cdot w\right)} \cdot \left( \int_{-1}^{1} N \cdot W \cdot a \, d\xi \right) \rightarrow \begin{pmatrix} 0 & 0 & \frac{2}{21} & -\frac{1}{21} & \frac{8}{105} & -\frac{34}{693} \\ 0 & 0 & \frac{1}{210} & -\frac{1}{630} & \frac{1}{315} & -\frac{1}{693} \\ 1 & 0 & \frac{1}{7} & 0 & \frac{1}{21} & 0 \\ 0 & 2 & 0 & \frac{2}{3} & 0 & \frac{10}{33} \\ 0 & 0 & \frac{2}{21} & \frac{1}{21} & \frac{8}{105} & \frac{34}{693} \\ 0 & 0 & -\frac{1}{210} & -\frac{1}{630} & -\frac{1}{315} & -\frac{1}{693} \end{pmatrix} \tag{28}$$

### 2.9 Software Application

BMCOLD or BMCOLDGP, under the name of BMTORSW or BMTORSWSP is the principal software application that will be used to analyze multi-span beams under mixed torsion. BMCOLDGP will be eventually used to analyze the contributions of bending shear and normal stresses.

BMTORSWSP provides the opportunity to bypass the complexities in the restrained warping analysis. And it predicts the restrained warping behavior better than that of the beam-column on elastic foundation (due to the absence of the soil hard-to-predict-behavior) and does not demand much of a computer memory or time.

BMCOLD (by Deschapelles) is a double precision adaptation of the program ZAPEL (by Deschapelles). It is a rather small (92KB) software application for Windows (32 bits) able to undertake the analysis of a bar in mixed bending and axial loading. It can also be used to solve problems of the general torsion theory due to the BC analogy previously discussed. BMCOLD is based upon a nine degree-of freedom-beam-column element (9-DOF-BE) including 3 non nodal smeared DOFs.

As it has been seen before, the solution of restrained warping problems under distributed non uniform torsional loads requires a field of displacements expressed in a polynomial of 5$^{th}$ or higher order. That is to accommodate uniform and linearly varying torsional loads prescribed in manuals including Seaburg (1997) or Heins (1963.)

BMCOLD considers a continuous soil modulus that may vary according to 3 parameters between the element nodes, and a geometric parameter from an axial force considered positive in compression. The program works with or without the elastic foundation. However, the elastic soil input data is not necessary in open TWB torsion analysis but it is necessary in box girders distortion analysis.

Input forms show *x* as the longitudinal axis. BMCOLD processes applied concentrated forces parallel to the longitudinal axis *x*, parallel to transverse axis *y* and applied concentrated moments around axis *z*. End nodes of each element are numbered NI and NJ, where number NI is smaller than number NJ; and the local *x* axis is defined by the direction going from node NI to node NJ (Deschapelles, 2008-2011).

Information on nodes with any type of displacement restraint must be provided for any node N, three integers are given to cover conditions along the 3 global axes. Integer +1 denotes full restraint, zero indicates no restraint at all, and –1 implies the existence of a spring; thus, the number of springs increases each time the integer –1 is entered.

The data of spring constants must be given if an input on some negative integers have been written to indicate the existence of elastic restraints; otherwise the information of prescribed non zero displacements is omitted.

For each node *n*, an integer is given to define the direction along which the restraint operates or the displacement is specified. Integers 1, 2 and 3 correspond to directions *x* (longitudinal), *y* (vertical transverse) and *z* (lateral transverse) respectively. Values of spring constants or specified displacements are also specified in the input (Deschapelles, 2008-2011).

BMCOLDG is the version of BMCOLD able to export graphic data for applications like excel. BMTORSW is a 32-bit-software application adapted from the 32-bit-software application BMCOLDG, and named after beam-torsion-warping. While, BMTORSWP is a 64-bit-software application adapted from the 64-bit-software application BMCOLDGP, and named after beam-torsion-warping Portland.

# CHAPTER 3

# OBJECTIVES AND METHOD

To recommend a finite element size and to undertake single-span and multi-span problems including a single-span performance experiment by using the recommended size. Some user friendly software features and conclusions and recommendations will be also provided. The process will be undertaken in an orderly, logical, systematic fashion in accordance with the AISC-DG-9. Obrébski (2005) states that the application of the finite element method to single span bar analysis and torsion analysis can produce errors up to 394% and 270% respectively.

## 3.1 Scope

This study is concerned with the analysis of thin-walled beams with open cross-sections subjected to first order mixed torsion. A software application based on the EBEF analogy and able to handle 3 parameter soils with axial loadings will be used.

This work is not concerned with the analysis of box girders, which are thin walled beams with closed sections whose governing differential equation is analogous to that of an EBEF as pointed by Vlasov (1961), Wright and Abdel-Samad (1968) and Hsu (1995) and confirmed by tests according to Heins (1981). However, it should be also pointed that the proposed software application is able to handle box girder distortional problems.

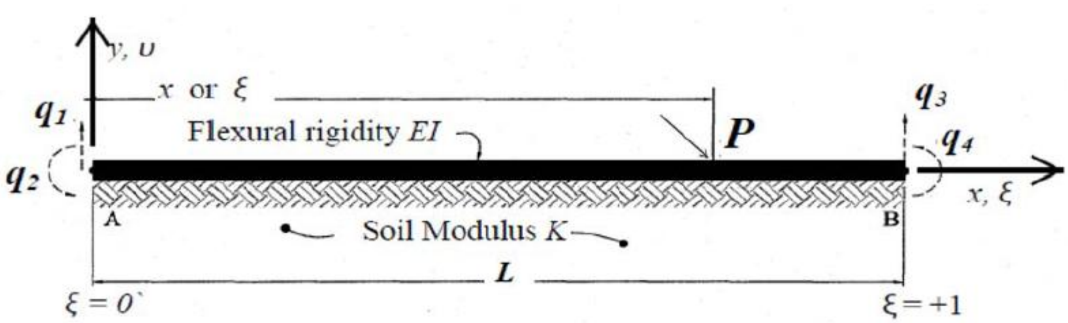



Figure 27. Four-DOF-BEF with Origin at Left Node

### 3.2 Methodology

The process will be undertaken in an orderly, logical, systematic manner. First, a finite element size calibration will be performed by means of a convergence study regarding the tensile geometric parameter of the EBC—analogous to $G{\cdot}J$ in the TWB. Then, maximum and minimum dimensions will be recommended for the finite element.

A challenging performance experiment with an open TWB single supported at midspan under a torsional load at far end will be performed and analyzed with the software.

In this experiment, two conditions prone to errors pointed by Obrébski (2005) will be simultaneously fulfilled: This will be challenging, due to the fact that the application of the finite element method to single-span bars and torsion analysis can produce errors up to 394% and 270% respectively according to Obrébski (2005).

Examples 5.1, 5.4, 5.5 from the AISC DG 9, Boothby (1984), and Medwadowski (1985) will be reanalyzed and compared. Computations, graphics error comparisons, adjustments, printing, drawings, tables, and standards will be provided. No corrections will be imposed to the problems original data.

Comparisons will be made and conclusions and recommendations will also be provided. Finally, input forms, the Software Application Manual, and conclusions and recommendations will be provided

#### 3.2.1 Convergence Study

The finite element used by the software is composed by two uncouple fields represented in a 3DOF rod element and a 6 DOF beam element. The 6 DOF-BC described before is the tool that provides the algorithm to solve the 1st order general torsion problem analogous to the 2nd order beam column problem.

On the other hand, to study the convergence, one modus operandi could be to develop or select the exact stiffness matrix for the first order general torsion problem and compare its entries with those of the finite 6DOF-BE. For that purpose, the local axis and the DOFs must be similar in both matrixes.

Unfortunately to develop an exact 6x6-matrix with the DOFs prescribed in the 6DOF-BE will be a very difficult task due to the more complex nature of the non-nodal degrees of freedom.

Instead, the elastic and geometric finite element 6x6-matrixes will be transformed into local axis with origin at the left node of the member and then condensed to a 4x4-matrix. The transformation to local axis will be made independently for both the elastic and geometric matrixes.

Therefore, Equations (24) and (26) whose local axis coordinate systems have origin at midspan have been transformed into Equations (29) and (31) whose coordinate systems have origin at left node according to Figure 27. In Equation (30) appears a common factor.

$$K_{e6} := \frac{E \cdot I}{L} \cdot \begin{pmatrix} 1200 & 120 & -5400 & 10080 & -840 & 60 \\ 120 & 16 & -480 & 840 & -60 & 4 \\ -5400 & -480 & 25920 & -50400 & 4680 & -360 \\ 10080 & 840 & -50400 & 100800 & -10080 & 840 \\ -840 & -60 & 4680 & -10080 & 1200 & -120 \\ 60 & 4 & -360 & 840 & -120 & 16 \end{pmatrix} \tag{29}$$

$$\lambda := L \cdot \sqrt{\frac{N}{E \cdot I}} \tag{30}$$

$$K_{g6} := \lambda^2 \cdot \begin{pmatrix} \frac{100}{7} & \frac{23}{42} & -\frac{410}{7} & 100 & -\frac{40}{7} & \frac{5}{42} \\ \frac{23}{42} & \frac{4}{63} & -\frac{44}{21} & \frac{10}{3} & -\frac{5}{42} & -\frac{1}{126} \\ -\frac{410}{7} & -\frac{44}{21} & \frac{2080}{7} & -560 & \frac{290}{7} & -\frac{26}{21} \\ 100 & \frac{10}{3} & -560 & 1120 & -100 & \frac{10}{3} \\ -\frac{40}{7} & -\frac{5}{42} & \frac{290}{7} & -100 & \frac{100}{7} & -\frac{23}{42} \\ \frac{5}{42} & -\frac{1}{126} & -\frac{26}{21} & \frac{10}{3} & -\frac{23}{42} & \frac{4}{63} \end{pmatrix} \tag{31}$$

$$K_{red} \rightarrow \begin{pmatrix} 12 & 6 & -12 & 6 \\ 6 & 4 & -6 & 2 \\ -12 & -6 & 12 & -6 \\ 6 & 2 & -6 & 4 \end{pmatrix} \tag{32}$$

$$K_{gred} \rightarrow \begin{pmatrix} \frac{15\cdot\lambda^2}{14} & \frac{\lambda^2}{28} & -\frac{15\cdot\lambda^2}{14} & \frac{\lambda^2}{28} \\ \frac{\lambda^2}{28} & \frac{3\cdot\lambda^2}{70} & -\frac{\lambda^2}{28} & -\frac{\lambda^2}{140} \\ -\frac{15\cdot\lambda^2}{14} & -\frac{\lambda^2}{28} & \frac{15\cdot\lambda^2}{14} & -\frac{\lambda^2}{28} \\ \frac{\lambda^2}{28} & -\frac{\lambda^2}{140} & -\frac{\lambda^2}{28} & \frac{3\cdot\lambda^2}{70} \end{pmatrix} \tag{33}$$

Equations (32) and (33) present the finite element 4x4 matrixes with origin at left node were condensed independently from each other. It is important to make clear that the condensation must not be made independently for each matrix. This procedure is prone to error. In this work, instead, both the elastic and geometric 6x6 matrixes will be combined and subsequently condensed. See Equations (39) and (40).

Ligthfoot (1961) and Gere (1963) developed generalized equations for 2$^{nd}$ order beam-columns under mixed bending and axial loading. Pettersson (1955), Chaudbary

(1982), Boothby (1984), Dvorkin (1988), and others developed generalized stiffness matrix for thin walled bars or just elastic line formulas like those in Roark (1982).

Nevertheless, the explicit version of the exact stiffness matrix corresponding to the solution of the governing differential equation of a prismatic beam under restraining warping developed by Medwadowski (1985) has been chosen in this work and it is reproduced in Equation (34). The complete entries of Medwadowski matrix are defined in Equations (35), (36), (37), and (38). Medwadowski matrix entries have been rearranged and normalized into dimensionless expressions according to the coordinate and DOF systems used for the development of the 6DOF finite element built into the software application BMTORSWP.

$$K_{DE} := \begin{pmatrix} -\delta(\lambda) & \gamma(\lambda) & \delta(\lambda) & \gamma(\lambda) \\ \gamma(\lambda) & \alpha(\lambda) & -\gamma(\lambda) & \beta(\lambda) \\ \delta(\lambda) & -\gamma(\lambda) & -\delta(\lambda) & -\gamma(\lambda) \\ \gamma(\lambda) & \beta(\lambda) & -\gamma(\lambda) & \alpha(\lambda) \end{pmatrix} \tag{34}$$

$$\alpha(\lambda) := \frac{\lambda \cdot sinh(\lambda) - \lambda^2 \cdot cosh(\lambda)}{2(cosh(\lambda) - 1) - \lambda \cdot sinh(\lambda)} \tag{35}$$

$$\beta(\lambda) := \frac{\lambda^2 - \lambda \cdot sinh(\lambda)}{2(cosh(\lambda) - 1) - \lambda \cdot sinh(\lambda)} \tag{36}$$

$$\gamma(\lambda) := \frac{\lambda^2 - \lambda^2 \cdot cosh(\lambda)}{2(cosh(\lambda) - 1) - \lambda \cdot sinh(\lambda)} \tag{37}$$

$$\delta(\lambda) := \frac{\lambda^3 \cdot sinh(\lambda)}{2(cosh(\lambda) - 1) - \lambda \cdot sinh(\lambda)} \tag{38}$$

Equation (39) shows the first two columns and Equation (40) shows the last two columns of the condensed finite element stiffness matrix. Each entry combines both the elastic and geometric effect in a 4x4 matrix with origin at left node condensed from a 6x6 matrix with non nodal DOFs. Ten figures, each containing graphics of both the exact stiffness matrix for the thin-walled beam under mixed torsion and the finite element stiffness matrix for the analogue beam-column under mixed bending and axial load will be provided. Each chart corresponds to one of the 10 symmetric entries presented as functions of λ as defined in Equation (30). The corresponding form of λ for the analogue thin-walled beam is λ = Le/a = Le√(G J/ E Cw) as per the AISC definition. See Figure 28 to Figure 37 .

$$\begin{pmatrix} \dfrac{15\cdot\lambda^4 + 1680\lambda^2 + 15120}{14\cdot\lambda^2 + 1260} & \dfrac{\lambda^4 + 420\lambda^2 + 15120}{28\cdot\lambda^2 + 2520} & \cdots \\[2ex] \dfrac{\lambda^4 + 420\lambda^2 + 15120}{28\cdot\lambda^2 + 2520} & \dfrac{3\cdot\lambda^6 + 1050\lambda^4 + 72240\lambda^2 + 1058400}{70\cdot\lambda^4 + 9240\lambda^2 + 264600} & \cdots \\[2ex] -\dfrac{15\cdot\lambda^4 + 1680\lambda^2 + 15120}{14\cdot\lambda^2 + 1260} & -\dfrac{\lambda^4 + 420\lambda^2 + 15120}{28\cdot\lambda^2 + 2520} & \cdots \\[2ex] \dfrac{\lambda^4 + 420\lambda^2 + 15120}{28\cdot\lambda^2 + 2520} & \dfrac{210\lambda^4 - \lambda^6 + 19320\lambda^2 + 1058400}{140\lambda^4 + 18480\lambda^2 + 529200} & \cdots \end{pmatrix} \tag{39}$$

$$\begin{pmatrix} \cdots & -\dfrac{15\cdot\lambda^4 + 1680\lambda^2 + 15120}{14\cdot\lambda^2 + 1260} & \dfrac{\lambda^4 + 420\lambda^2 + 15120}{28\cdot\lambda^2 + 2520} \\[2ex] \cdots & -\dfrac{\lambda^4 + 420\lambda^2 + 15120}{28\cdot\lambda^2 + 2520} & \dfrac{210\lambda^4 - \lambda^6 + 19320\lambda^2 + 1058400}{140\lambda^4 + 18480\lambda^2 + 529200} \\[2ex] \cdots & \dfrac{15\cdot\lambda^4 + 1680\lambda^2 + 15120}{14\cdot\lambda^2 + 1260} & -\dfrac{\lambda^4 + 420\lambda^2 + 15120}{28\cdot\lambda^2 + 2520} \\[2ex] \cdots & -\dfrac{\lambda^4 + 420\lambda^2 + 15120}{28\cdot\lambda^2 + 2520} & \dfrac{3\cdot\lambda^6 + 1050\lambda^4 + 72240\lambda^2 + 1058400}{70\cdot\lambda^4 + 9240\lambda^2 + 264600} \end{pmatrix} \tag{40}$$

A more accurate convergence study will be made with the condensed matrix shown in Equations (39) and (40), which show a matrix very different from that of Equation (33). Current recommendations on the element size are for analyses of EBEF with BMCOLDG, are based on a convergence study related to the soil parameter made with the 4x4 matrix developed exclusively from nodal DOFs as an upper limit for accuracy purposes.

Figure 28, Figure 29, Figure 30, Figure 31, Figure 32, Figure 32, Figure 33, Figure 34, Figure 35, Figure, 36, and figure 37 each contain the curves showing one entry of the finite element stiffness matrix (continuous curve) and its corresponding entry of the exact stiffness matrix in the 4DOF-TWB (dotted curve). More entries are not needed due to the symmetry of the matrixes.

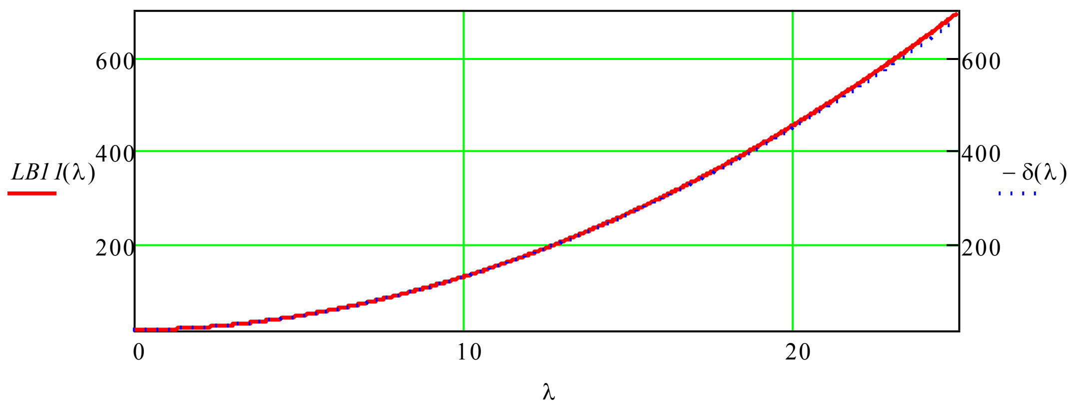


Figure 28. Entry 1-1 in the FE and Exact Stiffness Matrixes in the 4DOF-TWB

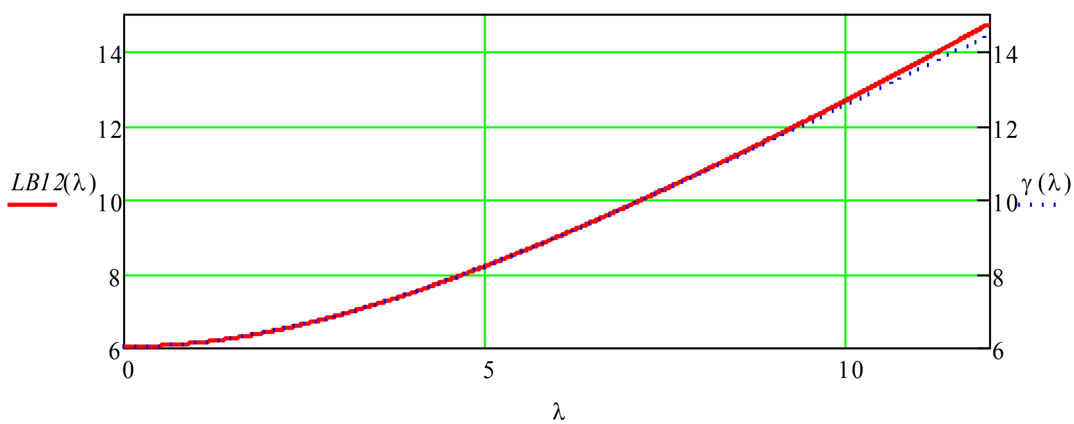


Figure 29. Entry 1-2 in the FE and Exact Stiffness Matrixes in the 4DOF-TWB

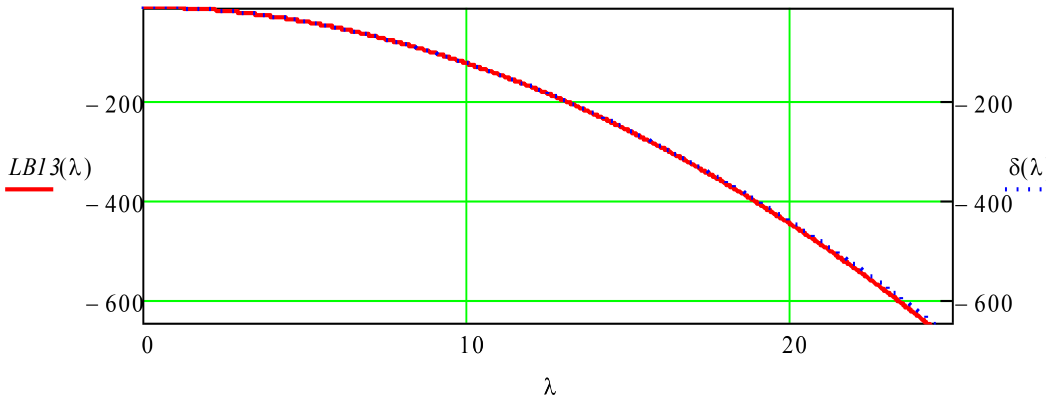



Figure 30. Entry 1-3 in the FE and Exact Stiffness Matrixes in the 4DOF-TWB

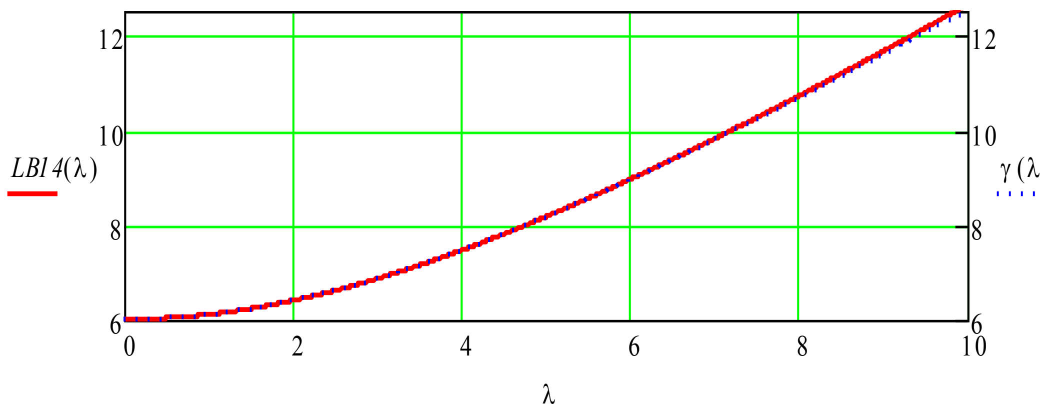



Figure 31. Entry 1-4 in the FE and Exact Stiffness Matrixes in the 4DOF-TWB

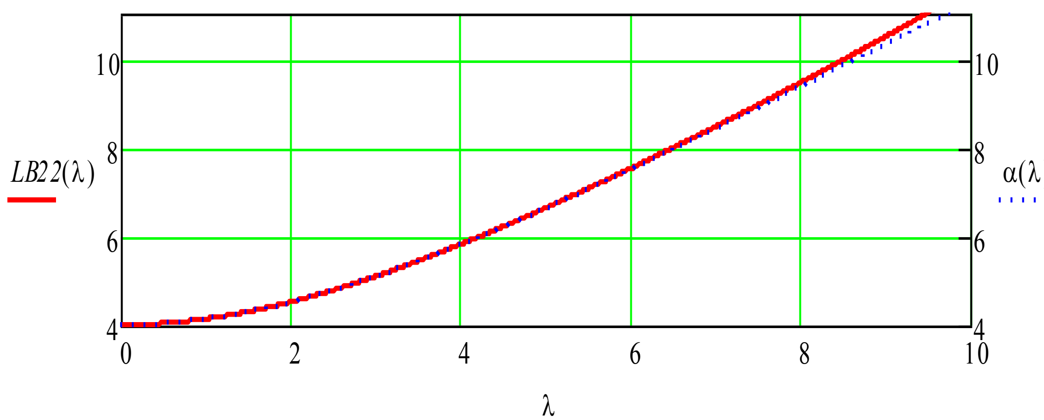



Figure 32. Entry 2-2 in the FE and Exact Stiffness Matrixes in the 4DOF-TWB

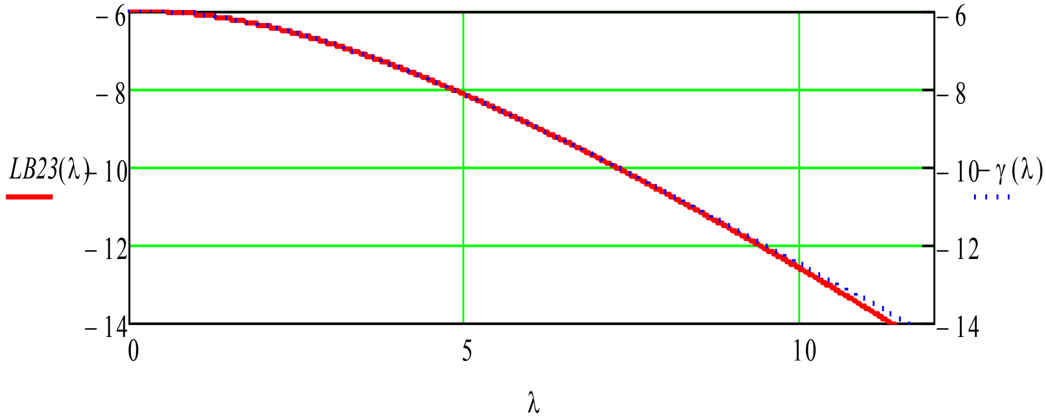



Figure 33. Entry 2-3 in the FE and Exact Stiffness Matrixes in the 4DOF-TWB

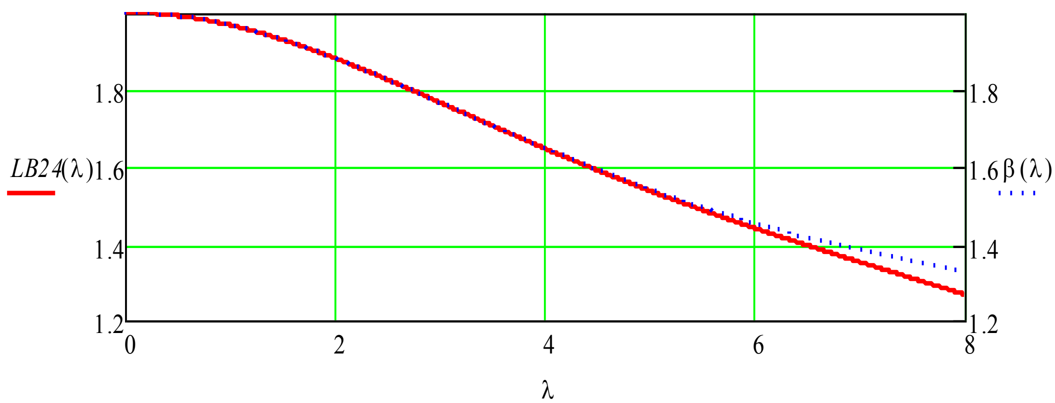



Figure 34. Entry 2-4 in the FE and Exact Stiffness Matrixes in the 4DOF-TWB

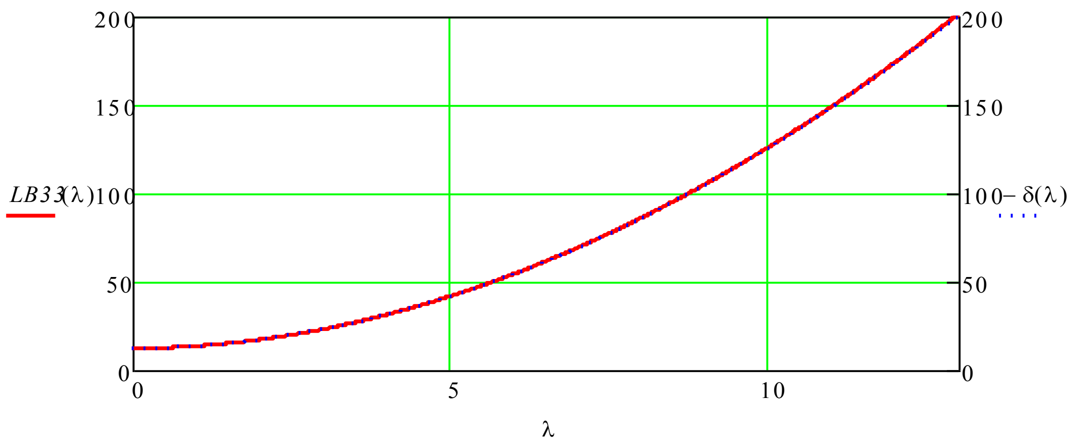



Figure 35. Entry 3-3 in the FE and Exact Stiffness Matrixes in the 4DOF-TWB

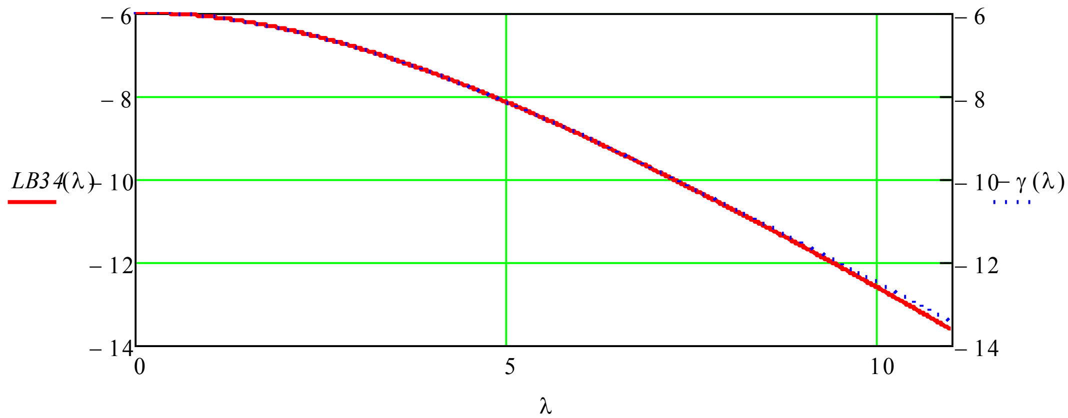


Figure 36. Entry 3-4 in the FE and Exact Stiffness Matrixes in the 4DOF-TWB

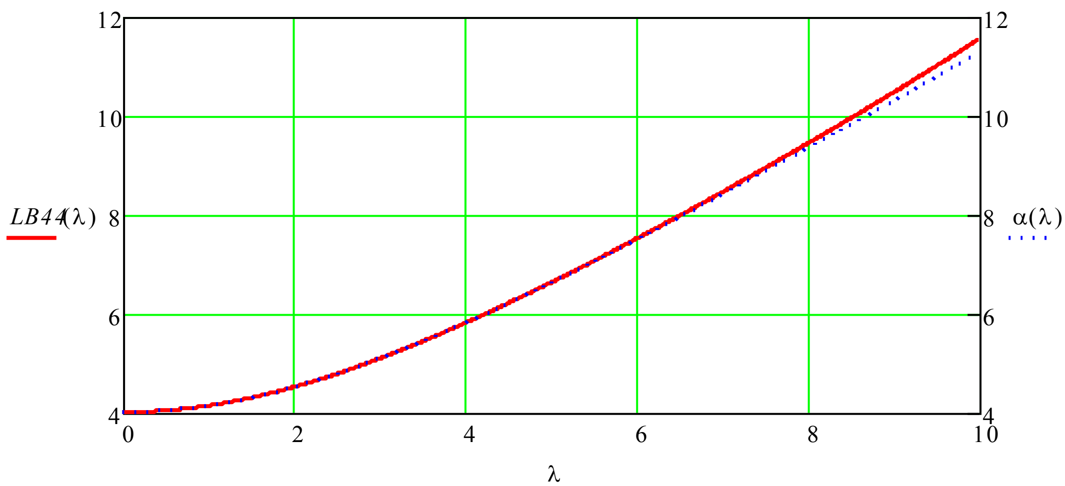


Figure 37. Entry 4-4 in the FE and Exact Stiffness Matrixes in the 4DOF-TWB

Entry Relative Errors of FE Condensed Stiffness Matrix at specific $\lambda$ values will be discussed. For $\lambda = 10$, an unacceptable maximum error of 13% emerges in the stiffness entries *LB24* related to the DOFs 2 and 4 (slopes or minor angles in the BC elastic line). For a $\lambda = 6$, accuracy improves 15 times with a maximum error of 0.9%. For a λ = 5, accuracy improves 45 times with a maximum error of 0.29%. For a λ = 4, accuracy improves 200 times with a maximum error of 0.065%. For a λ = 2, the maximum error is 0.0004% and accuracy is not expected to keep improving considering other factors influencing errors. Even though, convergence can improve in theory, errors also depend on factor including but not limited to the final size of the global matrix assembled (Wilkinson, 1963), as well as the precision of the algorithm built in the software. On the other hand, it is not a best practice to multiply the number of FEs ad infinitum. A discussion on this topic is not the theme of this work.

A preliminary advice would be to use λ (Le/a) values between 2 and 4. Thus, $2 \leq \lambda = Le/a \leq 4$ for the TWB. Thus, $2a \leq Le \leq 4a$, as it is shown in Equation (41). The minimum number of elements in a given span should be an integer $Ne > (L/Le)$.

Table 7. Relative % Entry Error in Condensed Matrix for Different λ Values

| **1/a = λ** | **Matrix of % Entry Errors** |
|---|---|
| 10 | $\begin{pmatrix} 0.1894 & 0.9482 & 0.1894 & 0.9482 \\ 0.9482 & 2.5332 & 0.9482 & -13.3456 \\ 0.1894 & 0.9482 & 0.1894 & 0.9482 \\ 0.9482 & -13.3456 & 0.9482 & 2.5332 \end{pmatrix}$ |
| 6 | $\begin{pmatrix} 0.0200 & 0.0602 & 0.0200 & 0.0602 \\ 0.0602 & 0.2481 & 0.0602 & -0.9083 \\ 0.0200 & 0.0602 & 0.0200 & 0.0602 \\ 0.0602 & -0.9083 & 0.0602 & 0.2481 \end{pmatrix}$ |
| 5 | $\begin{pmatrix} 0.0075 & 0.0191 & 0.0075 & 0.0191 \\ 0.0191 & 0.0915 & 0.0191 & -0.2914 \\ 0.0075 & 0.0191 & 0.0075 & 0.0191 \\ 0.0191 & -0.2914 & 0.0191 & 0.0915 \end{pmatrix}$ |
| 4 | $\begin{pmatrix} 0.0021 & 0.0043 & 0.0021 & 0.0043 \\ 0.0043 & 0.0240 & 0.0043 & -0.0653 \\ 0.0021 & 0.0043 & 0.0021 & 0.0043 \\ 0.0043 & -0.0653 & 0.0043 & 0.0240 \end{pmatrix}$ |
| 3 | $\begin{pmatrix} 0.00033 & 0.00055 & 0.00033 & 0.00055 \\ 0.00055 & 0.00363 & 0.00055 & -0.00833 \\ 0.00033 & 0.00055 & 0.00033 & 0.00055 \\ 0.00055 & -0.00833 & 0.00055 & 0.00363 \end{pmatrix}$ |
| 2 | $\begin{pmatrix} 0.00002 & 0.00003 & 0.00002 & 0.00003 \\ 0.00003 & 0.0002 & 0.00003 & -0.00039 \\ 0.00002 & 0.00003 & 0.00002 & 0.00003 \\ 0.00003 & -0.00039 & 0.00003 & 0.0002 \end{pmatrix}$ |
| 1 | $\begin{pmatrix} 0.000000 & 0.000000 & 0.000000 & 0.000000 \\ 0.000000 & 0.000001 & 0.000000 & -0.000002 \\ 0.000000 & 0.000000 & 0.000000 & 0.000000 \\ 0.000000 & -0.000002 & 0.000000 & 0.000001 \end{pmatrix}$ |

$$Le \leq 2\sqrt{(ECw/GJ)} = 2a < L/3 \qquad (41)$$

$$Ne > 0.5L\sqrt{(GJ/ECw)} = 0.5L/a > 3 \qquad (42)$$

A similar study of the stiffness coefficients for a 4DOF-BC was made, and the comparison of the respective stiffness matrixes error-wise is shown and commented in Figure 38 and Figure 39. From the point of view of computer memory, it could be said that the 6DOF-BE increases accuracy and saves computer memory spaces.

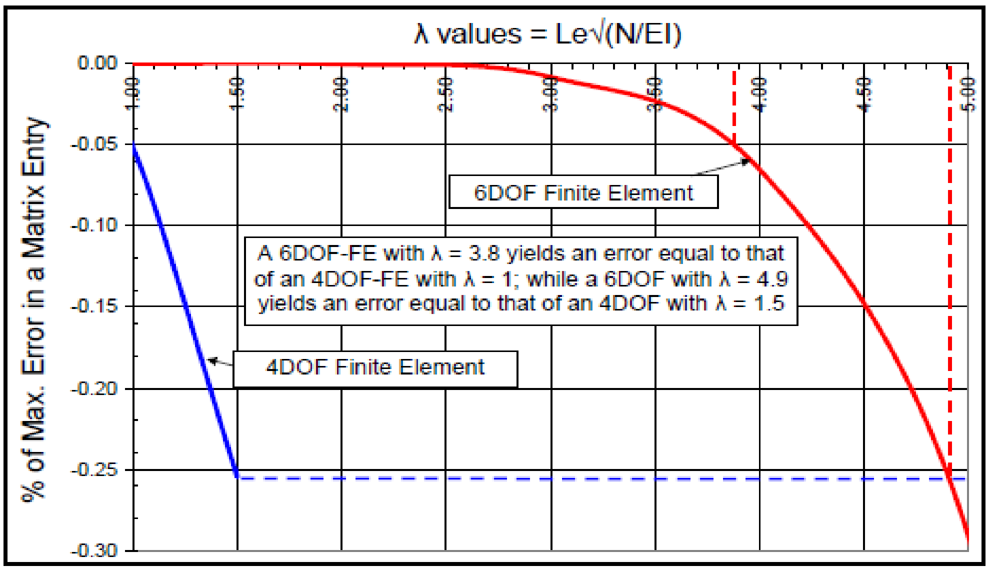



Figure 38. Four and 6DOF FE Stiffness Matrix Convergence for BC

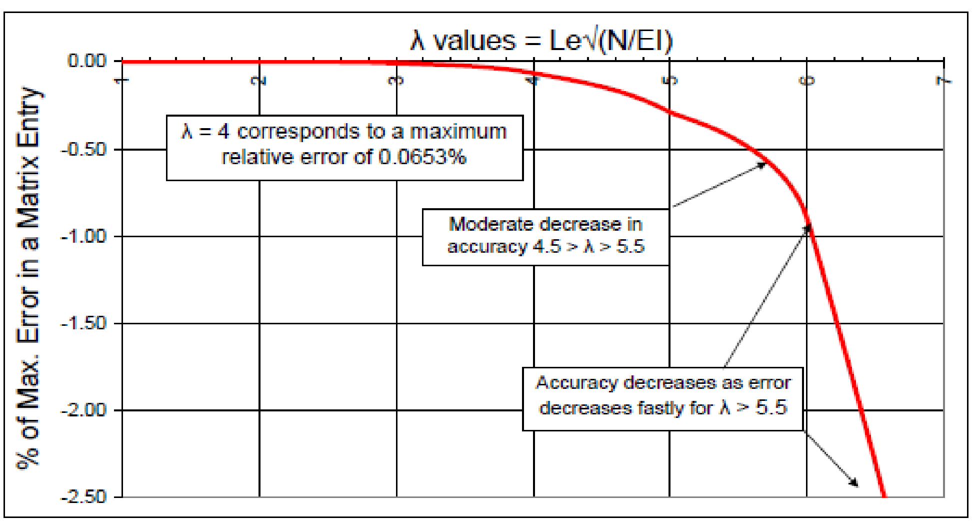



Figure 39. Six-DOF FE Stiffness Matrix Convergence for BC

As a sweeping statement, it could be said that the accuracy of the 6DOF beam column is 3.8 to 4 times that of the 4 DOF beam column. Only one 6DOF-BE is required to achieve the accuracy of four-4DOF-BE or five 6DOF-BE are required to achieve the accuracy of nineteen 4DOF-BE.

Therefore, one 6x6 matrix requires 18x1+3 = 21 entries in a global matrix upper triangle and four 4x4 matrixes require 7x4+3 = 31 entries in a global matrix upper triangle respectively. Similarly, five 6x6 matrixes require 18x5+3 = 93 entries in a global matrix upper triangle and nineteen 4x4 matrixes require 7x19+3 = 135 entries in a global matrix upper triangle respectively.

This difference will increase linearly as the number of required elements increases. In general the global matrix assembled with a higher degree finite element requires using less matrix entries in the memory to achieve a given accuracy; thus, the 6DOF is more convenient.

This advantage is in addition to the ones already discussed regarding the two non nodal DOFs properties, and the fact that for a solution of the mixed torsion problem a higher degree polynomial is mandatory.

### 3.2.2 List of Case Studies

The examples to be solved will be the following: Two parametric experiments of a bar restrained under two different restraints at midspan and subjected to a torque at right end, and example analytically solved by Ugural (1987), examples 5.1, 5.4 and 5.5 from the AISC Design Guide 9, an example from Boothby (1984), an example from Medwadowski (1985). Trials with the finite elements size recommended in the convergence study will be undertaken and mesh adjustments will be made if any.

### 3.2.3 Case Study One

The first case study is a parametric experiment where a TWB is cantilevered at midspan and subjected to a 1k-in torque at far end. The data is as follows $L$ = 15 in at each side, of the support. $E$ = 29000 ksi, $C_w$ = 0.01 $in^6$, $GJ$ = 10 kip-$in^2$. The minimum element size is $2\sqrt{ECw/GJ}$ = $2\sqrt{290}/10 \sim 11$ from Equation (41). The number of elements per span must be an integer larger than 15/11. As two elements did not work out, 3 elements were used from then on. Obrébski (2005) has commented on the difficulty presented by finite element formulations in the case of TWB and one span bars.

The cross section shown in Figure 40 is not that of the real bar in the parametric experiment. It just provides an order of magnitude for the bogus cross section torsional properties in order to make verisimilar the data handling.

Figure 40 shows the analog EBC subdivided in finite elements, Figure 41 shows an exaggerated EBC elastic line and Figure 42 shows the torque graph. More information can be found in Appendix A.

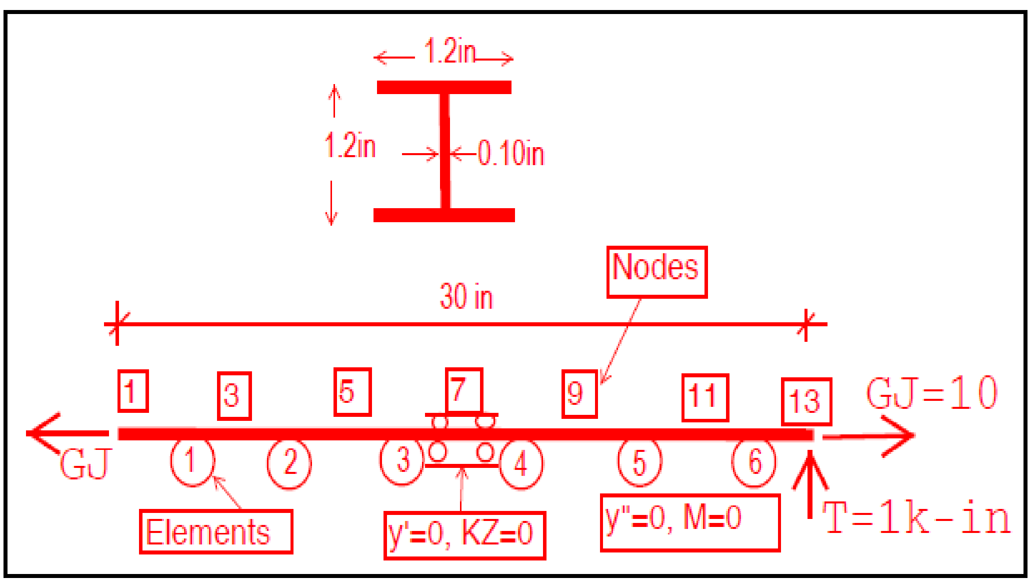



Figure 40. Model of Parametric Experiment to Test the Software

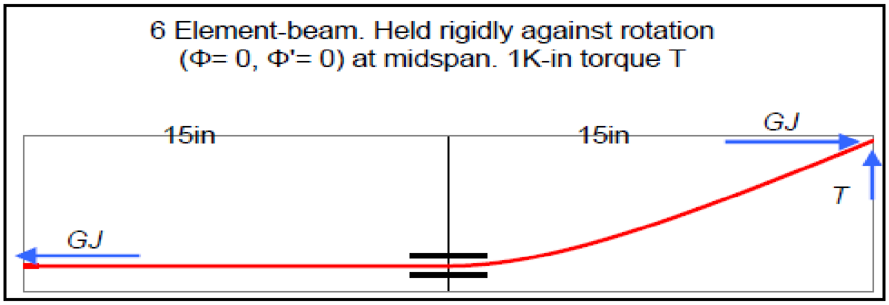


Figure 41. Elastic Line of the Beam Column Analogous to the TWB

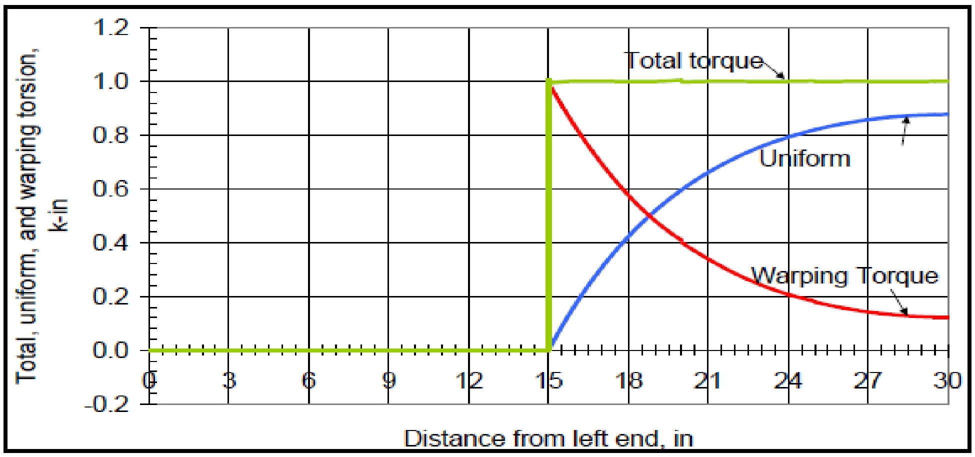


Figure 42. Asymptotic Behavior near Restraints from BMTORSWP

### 3.2.4 Case Study Two

This second case is a parametric experiment where the TWB is not cantilevered but just restrained against rotation at midspan. The data is the same as the first experiment. The results are shown in APPENDIX B including pictures; the input model, data, and forms; output data; and charts.

Three effects detected in the parametric experiment have been corroborated in the analysis: First, there is twist at the free unloaded near end; second, the twist at far end is the largest; and third, there could be a zero external torque with restrained warping along the full first span as seen in Figure 43 and Figure 44 and Figure 47. In addition to the contents in the respective APPENDIX B, more materials illustrating this problem could be found from Error! Reference source not found. to Error! Reference source not found.

These results are crucial to reformulate any previous advice on element size, particularly in the case of single span bars. Obrébski (2005) has commented on the inconvenience of both TWB and single span bars in the case of finite element formulations.

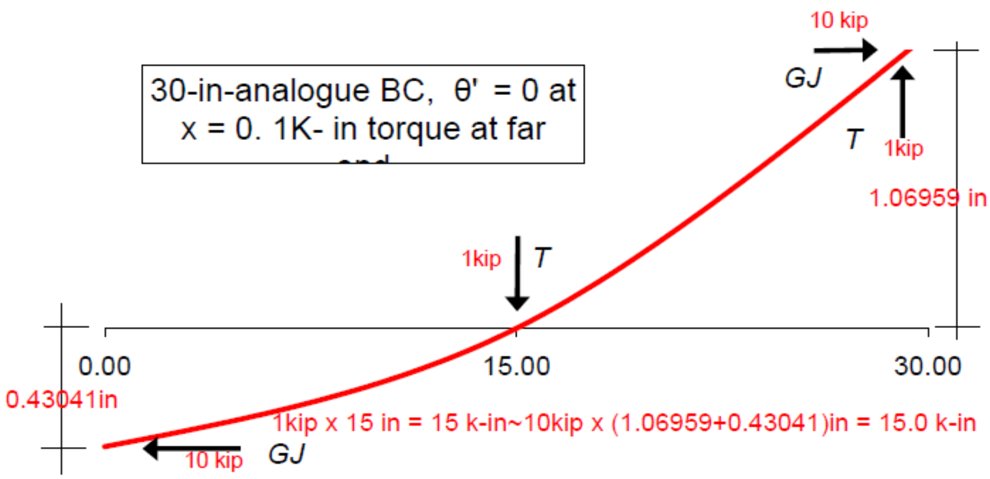



Figure 43. Analogue EBC in 2$^{nd}$ Order Equilibrium

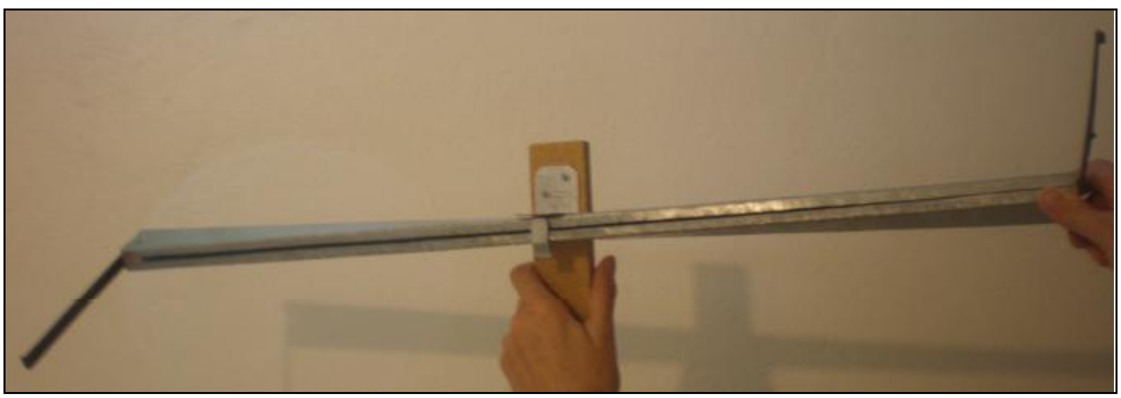


Figure 44. Zero External Torque with Restrained Warping at First Span

```
                                6EL
6 Elements' beam. Torque angle restrained ad midspan. 1K-in torque at far end
  6 13  1     9  0  1  0  29000.
  1  1  2  3 0.03    1.  5.                             -10.
  2  3  4  5 0.03    1.  5.                             -10.
  3  5  6  7 0.03    1.  5.                      - GJ   -10.
  4  7  8  9 0.03    1.  5.                             -10.
  5  9 10 11 0.03    1.  5.                             -10.
  6 11 12 13 0.03    1.  5.                             -10.
 1 7 0 1 0 <------------------ KX=KZ=0, KY=1 in Node 7
113    1. <-- Total Torque
```

Figure 45. Input Data

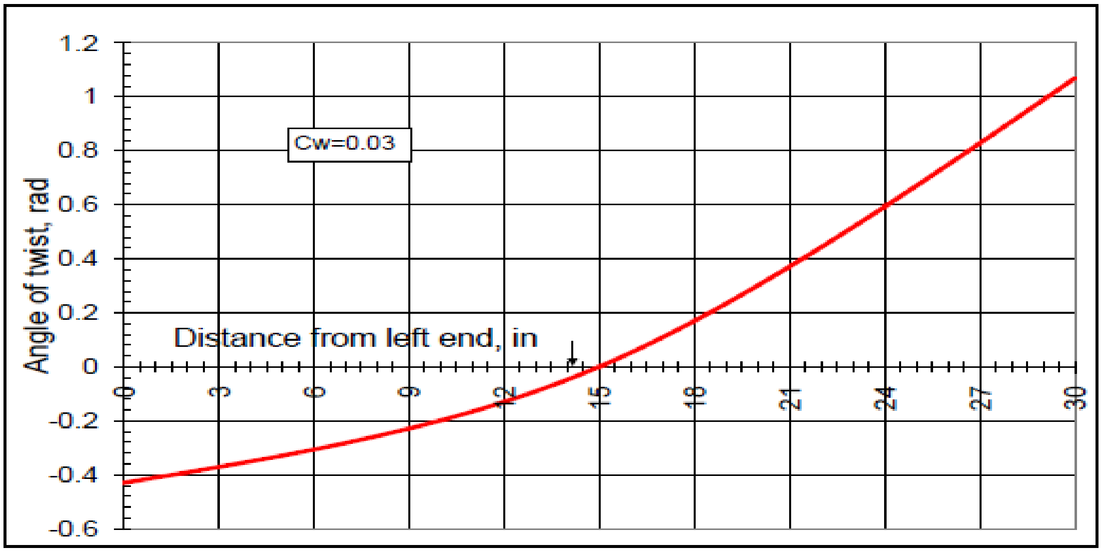



Figure 46. EBC with 6 Elements, θ=0 at Midspan, 1K-in Torque at Far End

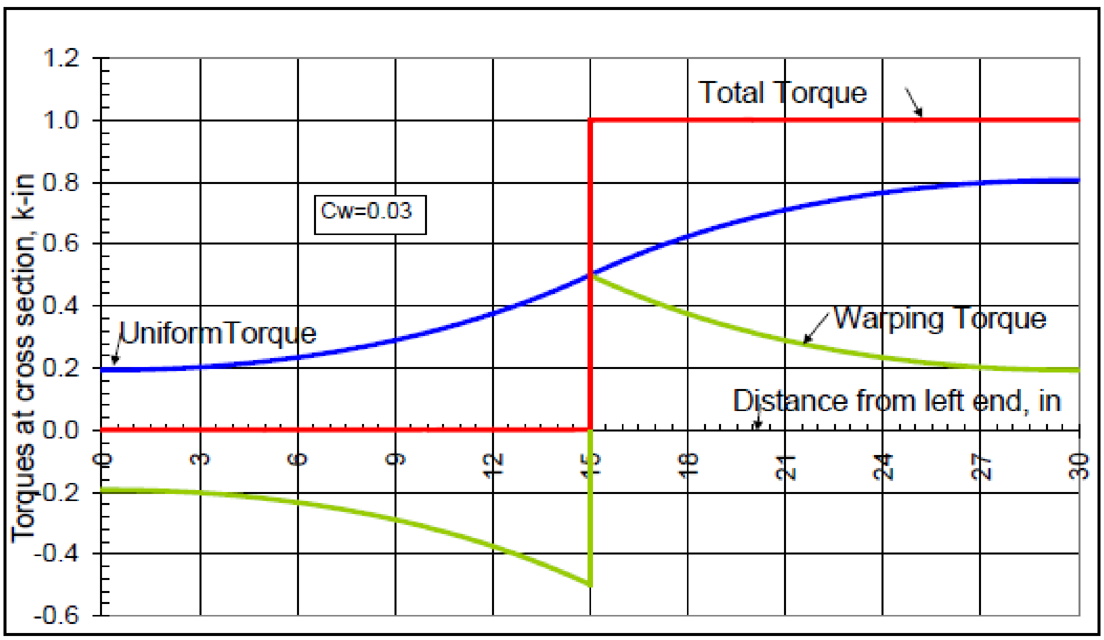



Figure 47. TWB with 6 Elements, θ=0 at Midspan, 1K-in Torque at Far End

### 3.2.5 Case Study Three

The case study three is a single span cantilevered TWB, $L$ = 30 in, under torsion held rigidly ($\theta'(0) = 0, \theta''(L) = 0$) in left end taken from pages 206-209 of Ugural (1987.) The TWB has been assigned the same cross section properties as in Case Study # 1 to compare BMTORSWP outputs with the analytic solution. The minimum element size is $2\sqrt{ECw/GJ} = 2\sqrt{290/10} \sim 11$ from Equation (41), and the number of elements should be and integer larger than 30/11 ~ 3, say 6 to edit the data from the former input form. Figure 48 illustrates the analogous EBC, while Figure 49 and Figure 50 show the similarity of the finite element solution and the analytic solution of the torque diagram.

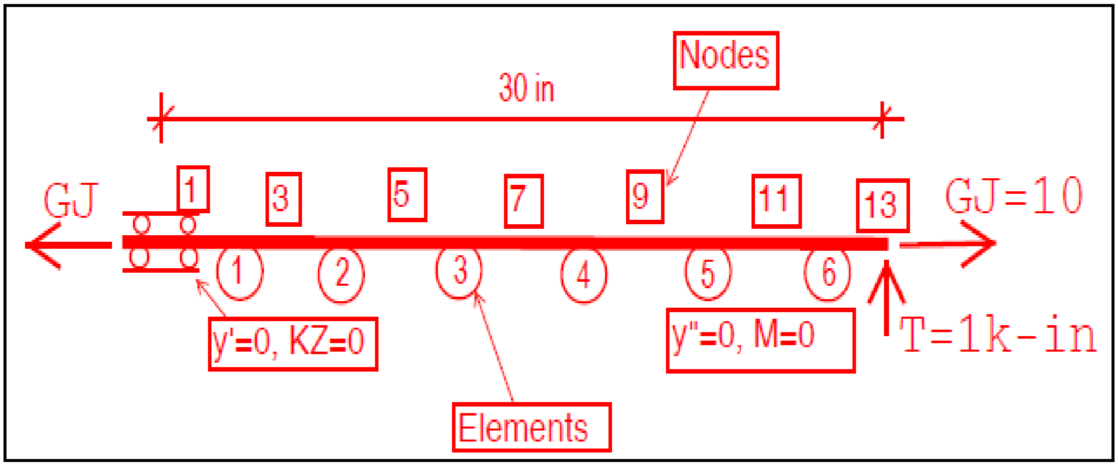



Figure 48. Model of Cantilevered Long Beam

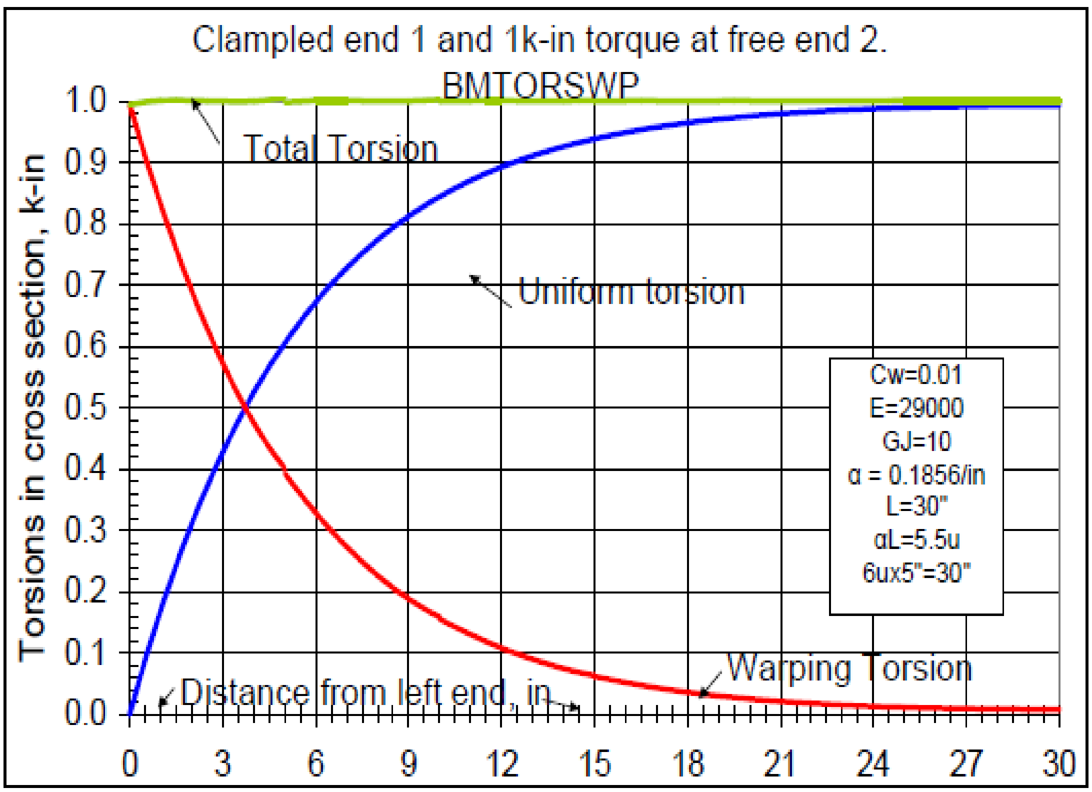



Figure 49. L=30”, TWB Clamped at Near End, Torque at Far End, BMTORSWP

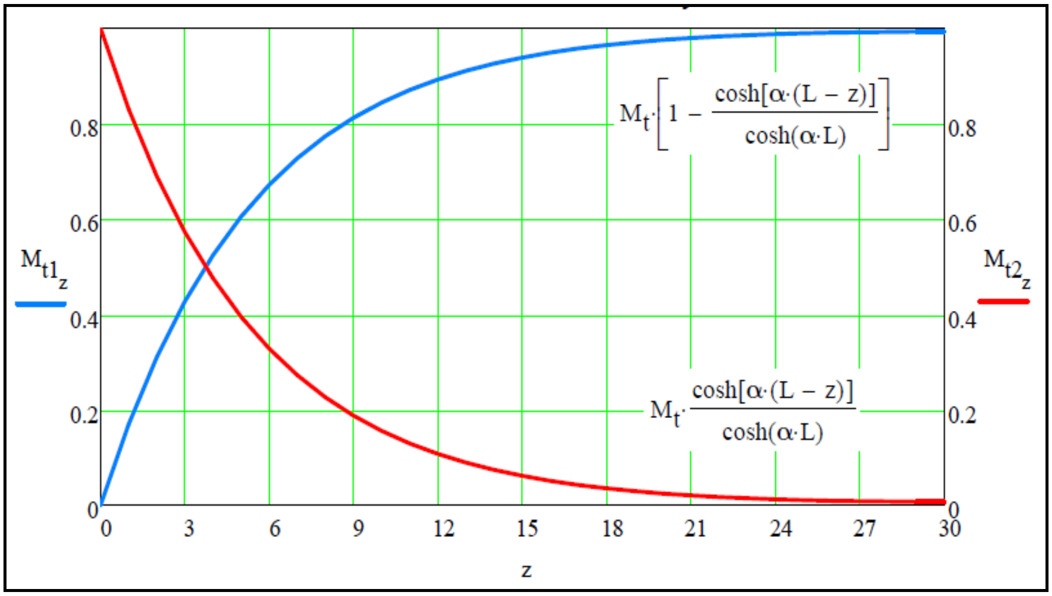


Figure 50. L=30", Cantilevered Beam under Torsion, Exact Solution

More information can be found in APPENDIX C, whose charts show the asymptotic behavior of the twist angle derivatives increases as the order of derivative increases as seen in Figure 51. This is a very suitable feature of the particular finite element upon which the software is based.

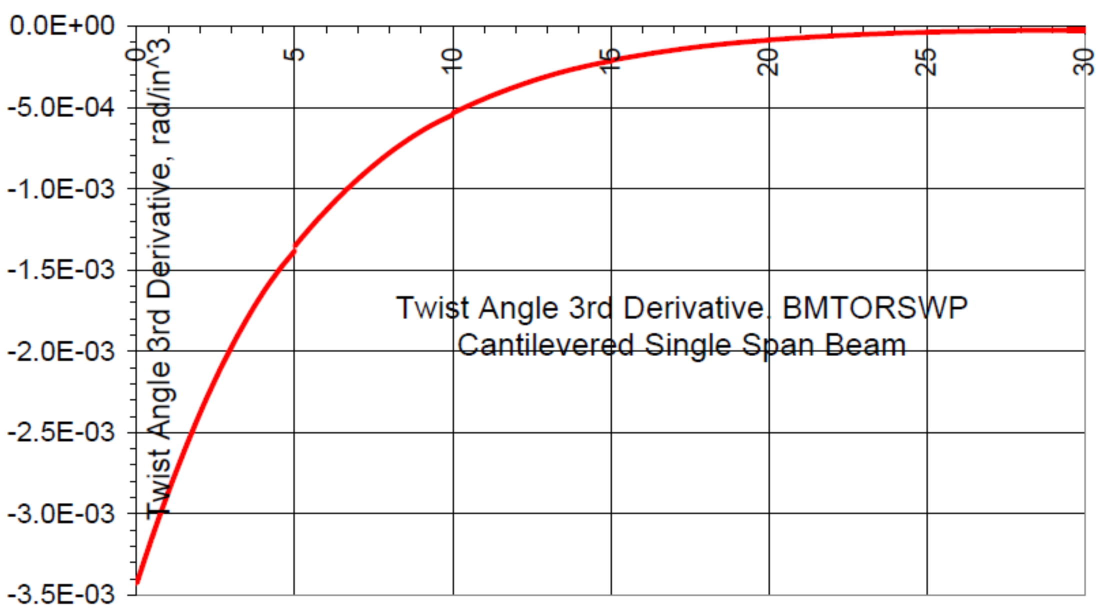


Figure 51. Twist Angle Third Derivative

### 3.2.6 Case Study Four

Examples 5.1 from the AISC DG-9 illustrated in Figure 52.a is solved, in which a W10x49 spans 15 ft (180 in) and supports a 15-kip factored load (10-kip service load) at midspan that acts at a 6 in eccentricity with respect to the shear center.

The stresses on the cross-section and the torsional rotation should be computed and compared with those provided by the AISC-DG9.

The ends are flexurally and torsionally pinned. The eccentric load can be resolved into a torsional moment and a load applied through the shear center as shown in Figure 52(b).

The resulting flexural and torsional loadings are illustrated in the same Figure 52. More detailed information can be found in APPENDIX D and a summary of results can be found from Figure 53 to Figure 58.

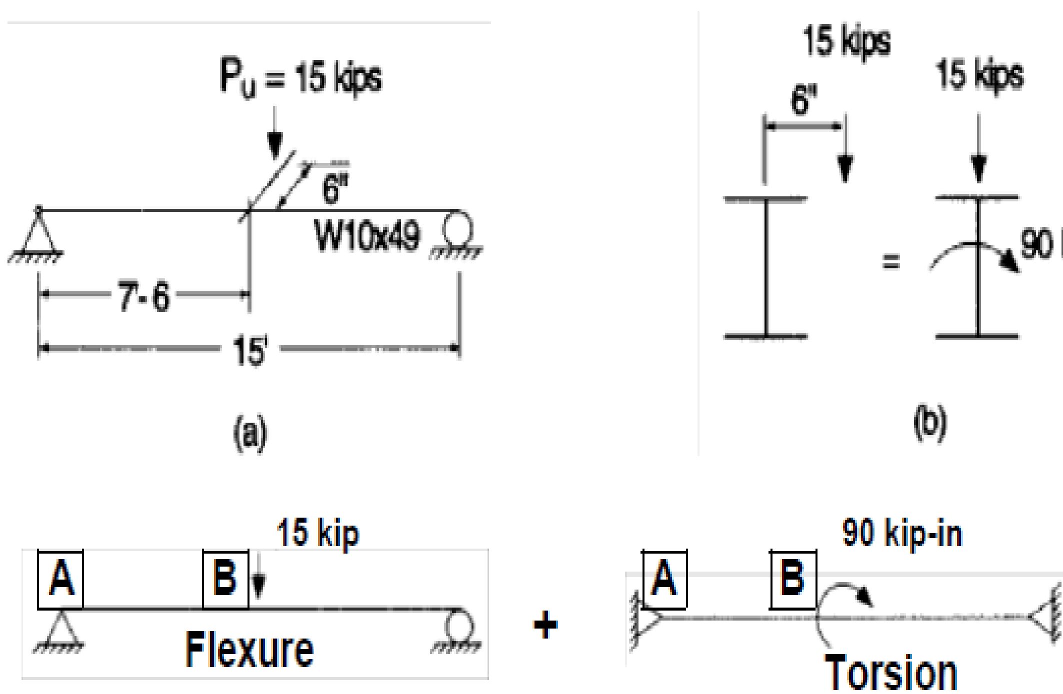



Figure 52. AISC-Design Guide 9, Example 5.1

Shear stress in web (w) at the TWB support:

$$\tau_{t\_wsBmtrs} := G \cdot t_w \cdot \left( \theta_{sBmtrs_1} \cdot \frac{1}{in} \right) = -6.11 \cdot ksi$$ From BMTORSW

$$\tau_{t\_wsAisc} := G \cdot t_w \cdot \left( \theta_{sAisc_1} \cdot \frac{1}{in} \right) = -6.16 \cdot ksi$$ From AISC-DG9

Shear stress in flange (f) at the support (s):

$$\tau_{t\_fsBmtrs} := G \cdot t_f \cdot \left( \theta_{sBmtrs_1} \cdot \frac{1}{in} \right) = -10.06 \cdot ksi$$ From BMTORSW

$$\tau_{t\_fsAisc} := G \cdot t_f \cdot \left( \theta_{sAisc_1} \cdot \frac{1}{in} \right) = -10.15 \cdot ksi$$ From AISC-DG9

Figure 53. Shear Stress in Web and Flange at Midspan

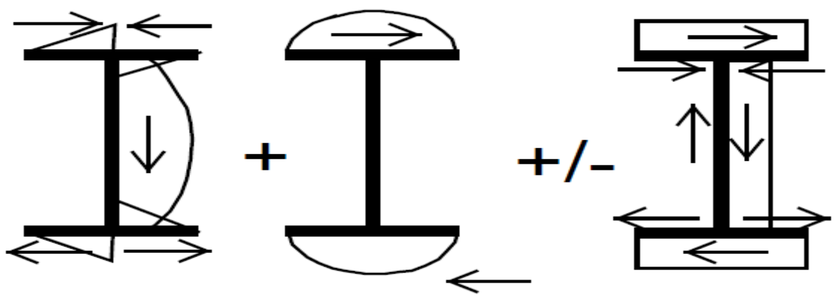
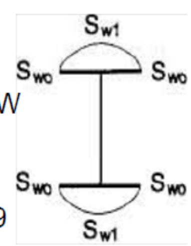



Figure 54. Two Solutions for Total Shear Stresses

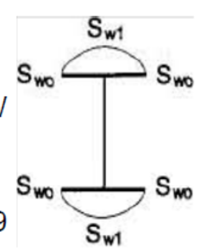
Warping Shear Stress in flange at midspan (fm):




$$\tau_{w\_fmBmtrs} := -E \cdot S_{wl} \cdot \left( \theta_{mBmtrs_3} \cdot \frac{1}{in^3} \right) \cdot \frac{1}{t_f} = -1.28 \cdot ksi$$ BMTORSW

$$\tau_{w\_fmAisc} := -E \cdot S_{wl} \cdot \left( \theta_{mAisc_3} \cdot \frac{1}{in^3} \right) \cdot \frac{1}{t_f} = -1.28 \cdot ksi$$ AISC-DG9

Warping Shear Stress in flange at the support (fs):




$$\tau_{w\_fsBmtrs} := -E \cdot S_{wl} \cdot \left( \theta_{sBmtrs_3} \cdot \frac{1}{in^3} \right) \cdot \frac{1}{t_f} = -0.57 \cdot ksi$$ BMTORSW

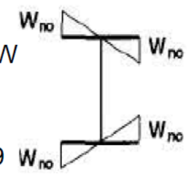
$$\tau_{w\_fsAisc} := -E \cdot S_{wl} \cdot \left( \theta_{sAisc_3} \cdot \frac{1}{in^3} \right) \cdot \frac{1}{t_f} = -0.56 \cdot ksi$$ AISC-DG9

Figure 55. Shear Stress from Warping

Warping Normal Stresses at midspan due to warping

$$\sigma_{w\_mBmtrs} := E \cdot W_{no} \cdot \left(\theta_{mBmtrs_2} \cdot \frac{1}{in^2}\right) = 28.53 \cdot ksi$$ BMTORSW

$$\sigma_{w\_mAisc} := E \cdot W_{no} \cdot \left(\theta_{mAisc_2} \cdot \frac{1}{in^2}\right) = 28.04 \cdot ksi$$ AISC-DG9




Warping Normal Stresses at the support $\sigma_w = 0$, since θ"=0

$$\sigma_{w\_sBmtrs} := E \cdot W_{no} \cdot \left(\theta_{sBmtrs_2} \cdot \frac{1}{in^2}\right) = -0.003 \cdot psi$$ BMTORSW

$$\sigma_{w\_sAisc} := E \cdot W_{no} \cdot \left(\theta_{sAisc_2} \cdot \frac{1}{in^2}\right) = 0\,psi$$ AISC-DG9

Figure 56. Normal Stresses due to Warping

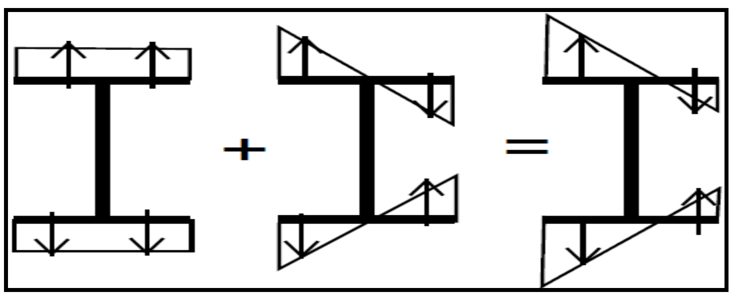


Figure 57. Superposition of Normal Stresses

| Location | Normal Stresses | | | Shear Stresses | | | |
|---|---|---|---|---|---|---|---|
| | $\sigma_{uw}$ | $\sigma_{ub}$ | $f_{un}$ | $\tau_{ut}$ | $\tau_{uw}$ | $\tau_{ub}$ | $f_{uv}$ |
| Midspan | 28.53 | 12.36 | 40.89 | | | | |
| Flange | ±28.1 | ±12.4 | ±40.4 | 0 | -1.28 | ±0.640 | -1.92 |
| Web | ---- | ---- | ---- | 0 | ---- | ±2.45 | -2.45 |
| Support | | | | 10.06 | 0.57 | 0.64 | 11.27 |
| Flange | 0 | 0 | 0 | -10.2 | -0.564 | ±0.640 | -11.4 |
| Web | ---- | ---- | ---- | -6.16 | ---- | ±2.45 | -8.61 |
| Maximum | | 40. 89 | ±40.4 | | | 11.27 | -11.4 |

BMTORSWSP figures without sign, next to AISC DG9 figures

Figure 58. Comparisons Total of Flexure and Torsion Stresses

### 3.2.7 Case Study Five

Examples 5.4 from the AISC DG-9 illustrated in The welded plate-girder shown in Figure 5.4a spans 25 ft (300 in.) and supports 310-kip and 420-kip factored loads (210-kip and 285-kip service loads). As illustrated in Figure 59, these concentrated loads are acting at a 3" eccentricity with respect to the shear center. Determine the stresses on the cross-section and the torsional rotation if the end conditions are assumed to be flexurally and torsionally pinned. The results and comparisons are found in Figure 60, Figure 61, and Figure 62. More detailed information can be found in APPENDIX E.

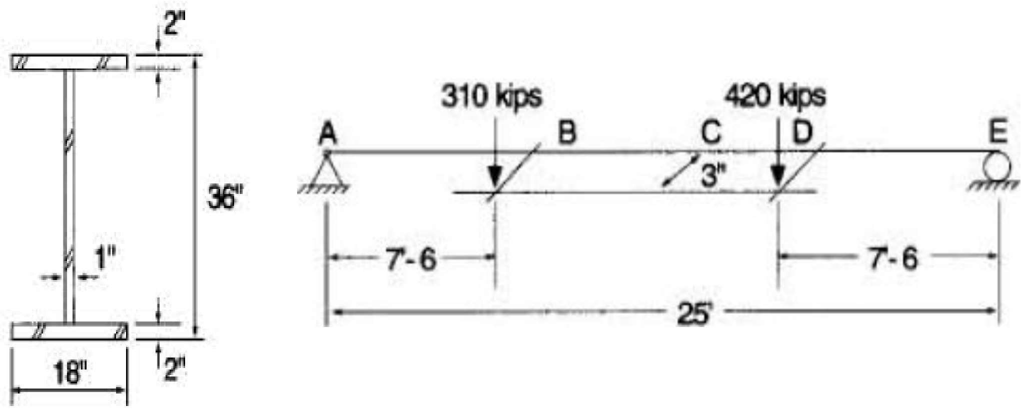



Figure 59. TWB Cross Section and Interest Points along Beam

| Location | | $\sigma_w$ | $\sigma_b$ | $f_{un}$ |
|---|---|---|---|---|
| Point D | flange | - 18.63<br>–18.4 | ±26.6 | -45.63<br>–45.0 |
| | web | — | — | — |
| Point E | flange | 0 | 0 | 0 |
| | web | — | — | — |
| Maximum | | | | -45.63<br>–45.0 |

Figure 60. Discrepancy in Total Maximum Normal Stresses in Interest Points

| Location | | $\tau_t$ | $\tau_w$ | $\tau_b$ | $f_{uv}$ |
|---|---|---|---|---|---|
| | | - 4.73 | - 1.11 | | - 8.21 |
| Point D | flange | −4.47 | −1.11 | ± 2.37 | − 7.95 |
| | web | −2.24 | — | ±12.1 | −14.3 |
| | | - 8.75 | -0.84 | | -11.9 |
| Point E | flange | −8.75 | −0.87 | ± 2.37 | −12.0 |
| | web | −4.37 | — | ±12.1 | −16.5 |
| | | - 4.38 | | | -16.51 |
| Maximum | | | | | −16.5 |

Figure 61. No Discrepancy in Total Maximum Shear Stresses in Interest Points

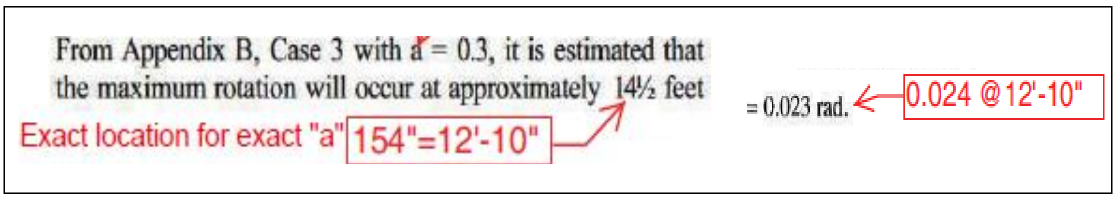



Figure 62. Discrepancies on Maximum Twist Angle and Location

### 3.2.8 Case Study Six

The MC18x42.7 channel illustrated in Figure 5.5(a) spans 12ft (144 in.) and supports a uniformly distributed factored load of 16 kips/ft (2.4 kips/ft service load) acting through the centroid of the channel. Determine the stresses on the cross section and the torsional rotation.

The beam ends are flexurally and torsionally fixed. The eccentric load can be resolved into a torsional moment and a load applied through the shear center according to the AISC-DG9.

The resulting flexural and torsional loadings are taken from AISC-DG9 and illustrated in Figure 64; while Figure 63 shows the input model to be used in the software.

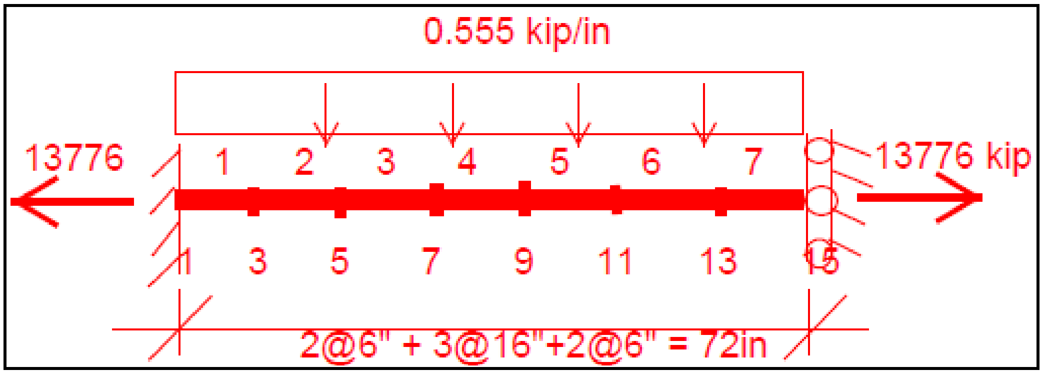



Figure 63. EBC Model for BMTORSWP

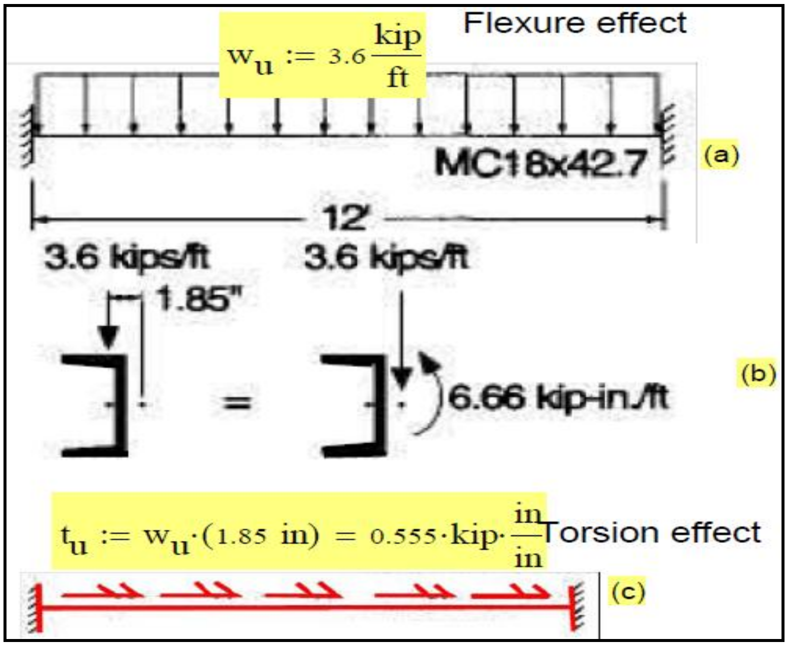



Figure 64. Flexure and Torsion Effect

$$\tau_{tBmt} = \begin{pmatrix} 0 & 0 & 0 & 0 \\ 2.923 & 2.923 & 2.923 & 2.105 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix} \cdot ksi$$

BMTORSW at
support
0.2L
0.5L
L

$$\tau_{t} = \begin{pmatrix} 0 & 0 & 0 & 0 \\ 2.843 & 2.843 & 2.843 & 2.047 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix} \cdot ksi$$

AISC-DG9 at
support
0.2L
0.5L
L

Figure 65. Shear Stress due to Pure Torsion along the Beam and Profile

$$\tau_{wBmt} = \begin{pmatrix} 0 & 1.306 & -1.013 & 0.703 \\ 0 & 0.595 & -0.462 & 0.321 \\ 0 & 0 & -0 & 0 \\ 0 & 1.306 & -1.013 & 0.703 \end{pmatrix} \cdot ksi$$

BMTORSW at
support
0.2L
0.5L
L

$$\tau_{w} = \begin{pmatrix} 0 & 1.303 & -1.011 & 0.702 \\ 0 & 0.599 & -0.465 & 0.323 \\ 0 & 0 & 0 & 0 \\ 0 & 1.303 & -1.011 & 0.702 \end{pmatrix} \cdot ksi$$

AISC-DG9 at
support
0.2L
0.5L
L

Figure 66. Shear Stress due to Warping Torsion along the Beam and Profile

$$\sigma_{wBmt} = \begin{pmatrix} 21.009 & 0 & -9.932 & 0 \\ 0.18 & 0 & -0.085 & 0 \\ -9.187 & 0 & 4.343 & 0 \\ 21.009 & 0 & -9.932 & 0 \end{pmatrix} \cdot ksi$$

BMTORSW at
support
0.2L
0.5L
L

$$\sigma_{w} = \begin{pmatrix} 20.078 & 0 & -9.491 & 0 \\ 0 & 0 & 0 & 0 \\ -8.729 & 0 & 4.127 & 0 \\ 20.078 & 0 & -9.491 & 0 \end{pmatrix} \cdot ksi$$

AISC-DG9 at
support
0.2L
0.5L
L

Figure 67. Warping Normal Stresses along the Beam and Profile

The real point where maximum angle of twist occur is not at 0.2L but at 0.204L. It can be seen in the curve made from BMTORSWP data. However, the results from the AISC DG9 are very close. The maximum service load rotation θ, at midspan with the service-load torque is 0.0123 for the AISC and 0.0130 for BMTORSWP.

$$f_{unBmt} = \begin{pmatrix} 12.594 & -8.416 & -18.347 & 0 \\ -0.156 & -0.337 & -0.422 & 0 \\ -4.979 & 4.208 & 8.551 & 0 \\ 12.594 & -8.416 & -18.347 & 0 \end{pmatrix} \cdot ksi$$

BMTORSW at
support
0.2L
0.5L
L

$$f_{un} = \begin{pmatrix} 11.662 & -8.416 & -17.907 & 0 \\ -0.337 & -0.337 & -0.337 & 0 \\ -4.522 & 4.208 & 8.334 & 0 \\ 11.662 & -8.416 & -17.907 & 0 \end{pmatrix} \cdot ksi$$

AISC-DG9 at
support
0.2L
0.5L
L

Figure 68. Combined Normal Stresses along the Beam and Profile

| Location | Point | $\sigma_w$ | $\sigma_b$ | $f_{un}$ | |
|---|---|---|---|---|---|
| Support | 0 | 20.1(C) | 8.41(T) | 11.7(C) | 12.594 |
| | 1 | 0 | 8.41(T) | 8.41(T) | -8.416 |
| | 2 | 9.49(T) | 8.41(T) | 17.9(T) | -18.347 |
| | 3 | 0 | 0 | 0 | 0. |
| Midspan | 0 | 8.73(T) | 4.20(C) | 4.53(T) | |
| | 1 | 0 | 4.20(C) | 4.20(C) | |
| | 2 | 4.13(C) | 4.20(C) | 8.33(C) | |
| | 3 | 0 | 0 | 0 | |
| $z/l = 0.20$ | 0 | 0 | — | — | |
| | 1 | 0 | — | — | |
| | 2 | 0 | — | — | |
| | 3 | 0 | — | — | |
| Maximum | | | | 17.9(T) | -18.347 |

Figure 69. Total Normal Stresses along the Beam and Profile

The maximum normal stress (tension) occurs at the support at point 2 in the flange. A discrepancy of 2.5% is noticed: 17.907 ksi vs. 18.347 ksi from DG9 and the software respectively (Figure 68). The AISC DG9 data have been recalculated to the third decimal place to be compared with BMTORSWP output calculated to the third decimal place.

$$f_{uv} = \begin{pmatrix} 0 & 1.303 & 0.218 & 3.986 \\ 2.843 & 3.442 & 3.115 & 4.34 \\ 0 & 0 & 0 & 0 \\ 0 & 1.303 & -2.24 & -2.582 \end{pmatrix} \cdot \text{ksi}$$

BMTORSW at support
0.2L
0.5L
L

$$f_{uvBmt} = \begin{pmatrix} 0 & 1.306 & 0.216 & 3.987 \\ 2.923 & 3.518 & 3.199 & 4.396 \\ 0 & 0 & -0 & 0 \\ 0 & 1.306 & -2.242 & -2.58 \end{pmatrix} \cdot \text{ksi}$$

AISC-DG9 at support
0.2L
0.5L
L

Figure 70. Combined Normal Stresses along the Beam and Profile

| Location | Point | $\tau_t$ | $\tau_w$ | $\tau_b$ | $f_{uv}$ | |
|---|---|---|---|---|---|---|
| Support | 0 | 0 | 0 | 0 | 0 | |
| | 1 | 0 | 1.30← | — | 1.30← | |
| | 2 | 0 | 1.01← | 1.23→ | 0.22→ | |
| | 3 | 0 | 0.702↓ | 3.28↓ | 3.98↓ | |
| Midspan | 0 | 0 | 0 | 0 | 0 | |
| | 1 | 0 | 0 | 0 | 0 | |
| | 2 | 0 | 0 | 0 | 0 | |
| | 3 | 0 | 0 | 0 | 0 | |
| $z/l = 0.20$ | 0 | 2.84⇆ | 0 | 0 | 2.84⇆ | 2.923 |
| | 1 | 2.84⇆ | 0.599← | — | 3.44← | 3.158 |
| | 2 | 2.84⇆ | 0.465← | 0.740→ | 3.12→ | 3.199 |
| | 3 | 2.05⇅ | 0.323↓ | 1.98↓ | 4.35↓ | 4.396 |
| Maximum | | | | | 4.35↓ | 4.396 |

Figure 71. Summary of Total Shear along the Beam and Profile

Figure 65, Figure 65, Figure 66, Figure 67, Figure 68, Figure 69, Figure 70, and Figure 71 contain the results of isolated and combined stresses and maximum angle of twist were presented. More comments, data, graphs and charts are provided in APPENDIX F.

#### 3.2.9 Case Study Seven

Boothby (1984) proposed the use of the moment distribution method to analyze multi-span bars under restrained warping. A three-span continuous C12x30 beam with J = 0.864 in$^4$, Cw = 151 in$^6$, maximum Wn = 11.7 in2, G/E = 0.4, E=29000ksi must be solved. The beam with its support conditions are shown in Figure 72. The beam was divided in half due to the physical and loading symmetries. See APPENDIX G.

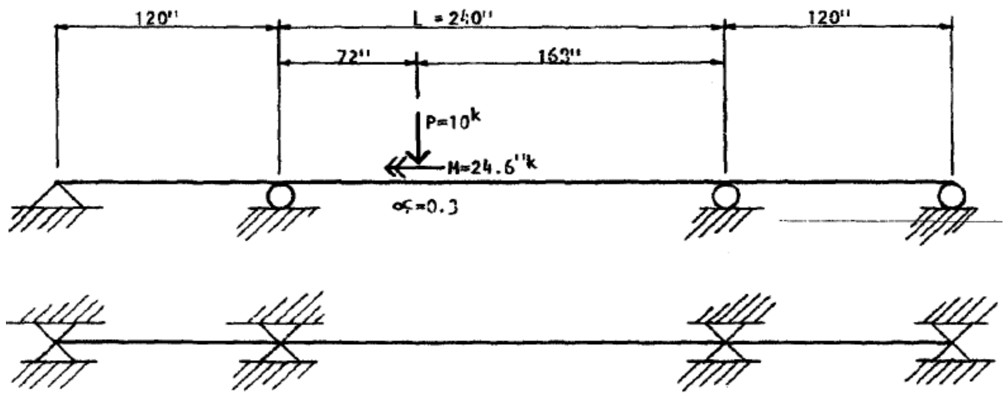



Figure 72. Boothby's Bending and Torsional Support Conditions

The properties of $1/a$ = 0.04787/in or $a$ = 20.9 in; thus, each finite element could span 2a to 4a. Nevertheless, the elements will be chosen smaller as shown in Figure 73. The number each of the 23elements is shown below the beam, and the node numbering is shown on top of the beam.

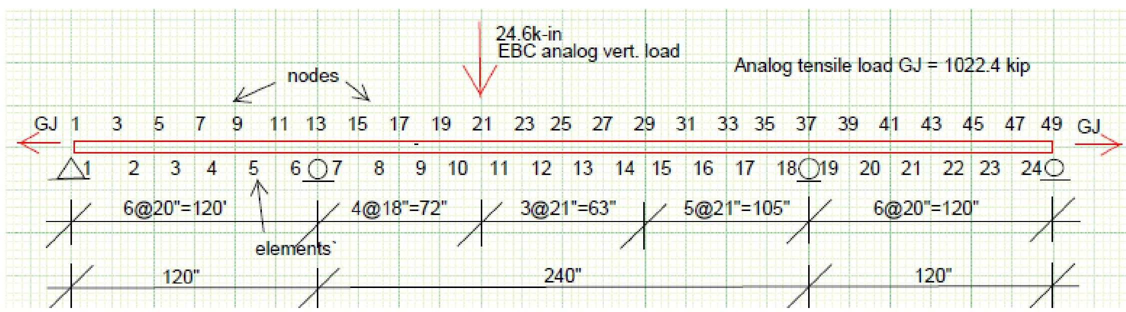
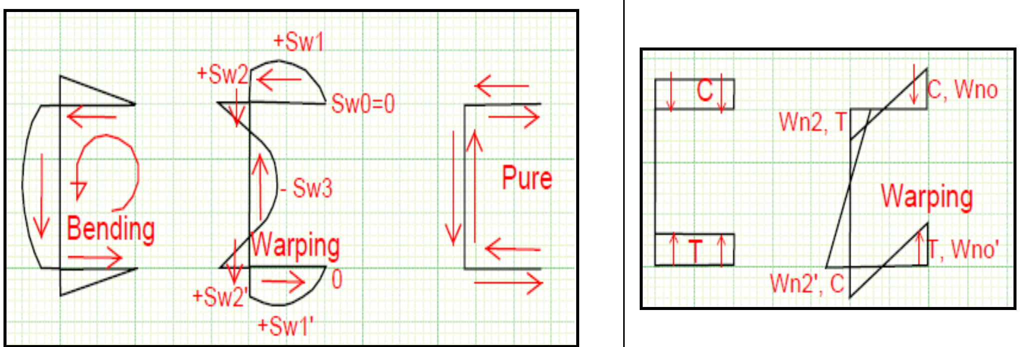



Figure 73. Model of EBC with Finite Elements

This is due to the fact that the program is based on a 9-DOF-beam-element. Whose three non nodal DOF are assigned to a fictitious middle span node. Therefore a structure with one element will carry 2 real nodes and one fictitious node; a structure with 2 elements will carry 3 real nodes and 2 fictitious nodes and so on and so forth. Therefore, a structure with 23 elements will carry 24 real nodes and 23 fictitious nodes.

The tensile analog force is -GJ = -0.4*0.864in^4*29000ksi= -10022 kip-in². *JBW* is calculated by the formula 3(NJ-NI+1), being I and J the real nodes of any element with the maximum difference for J − I, such that I < J. That is, in our case JBW= 3(3-1+1) = 9

(See APPENDIX F). The positive direction of forces and stresses along the cross section profile are those assigned to the beam under bending shear that appears in Figure 74.




Bending, Warping and Pure Shear

Normal Stresses

Figure 74. Stresses and Forces in Cross Section Profile

While there is a match between Boothby's and BMTORSWP charts (Figure 75 and Figure 76), it does not happen in the warping moment charts. Boothby's error on the warping moment is at the external sides of the first and last interior supports as it could be seen in Figure 77.

| | Joint 2 | | | Joint 3 | |
|---|---|---|---|---|---|
| | .523 | .477 | C.O.=0.095 | .477 | .523 |
| *FEB* | | –363.1 | | +134.0 | |
| Dist 1. | 189.9 | +173.2 | | –63.9 | –70.1 |
| C.O. 1 | | –6.1 | | +16.5 | |
| Dist. 2 | +2.9 | +3.2 | | –7.9 | –8.6 |
| C.O. 2 | | –0.8 | | +0.3 | |
| Dist. 3 | +0.4 | +0.4 | | –0.1 | –0.2 |
| | +193.2 | –193.2 | | +78.9 | –78.9 |
| | 193.62 | | | 78.93 | |

Figure 75. Bimoment Results. BMTORSWP Output Shown in Red

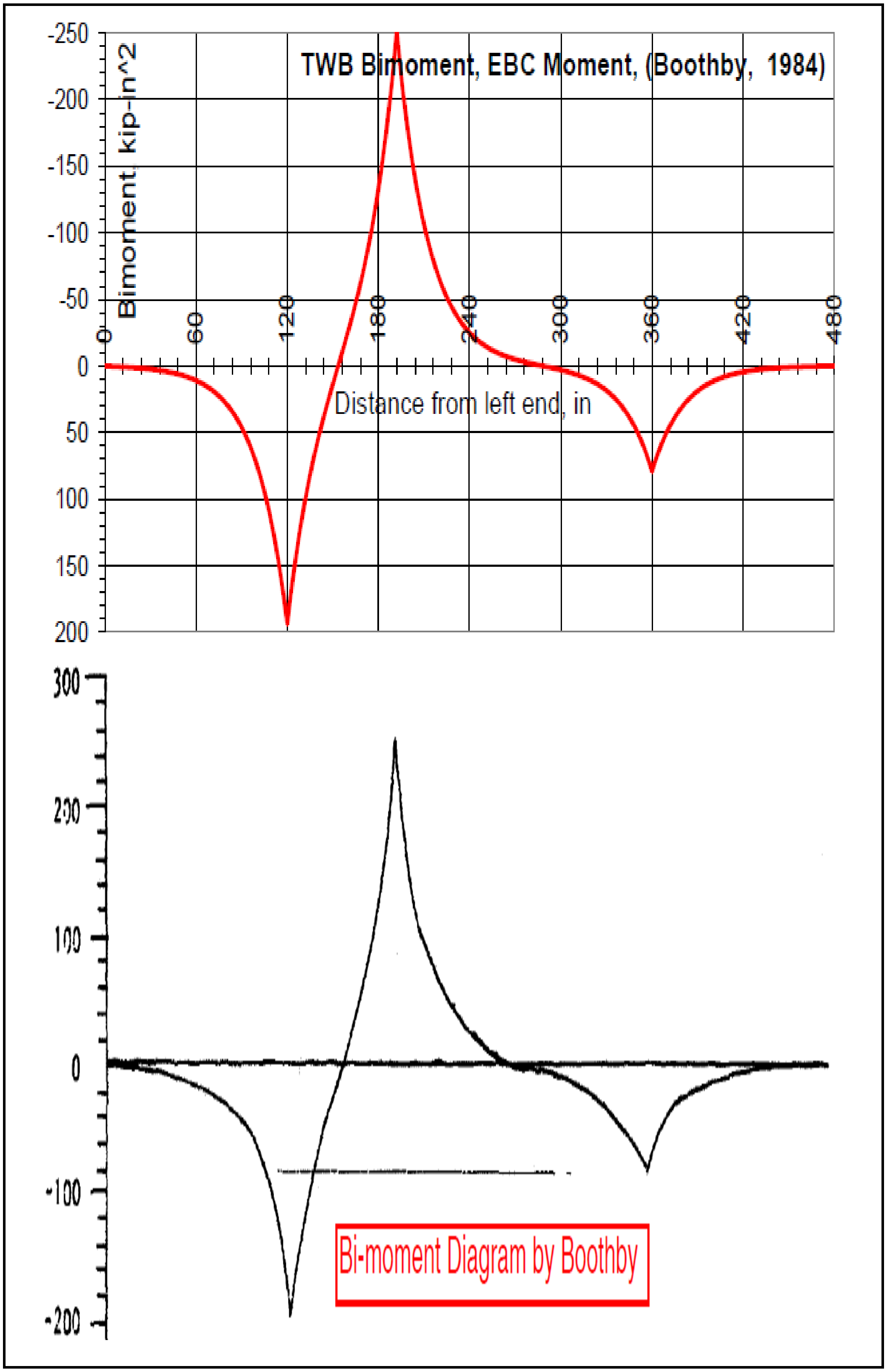



Figure 76. Bimoment Match between BMTORSWP and Boothby's

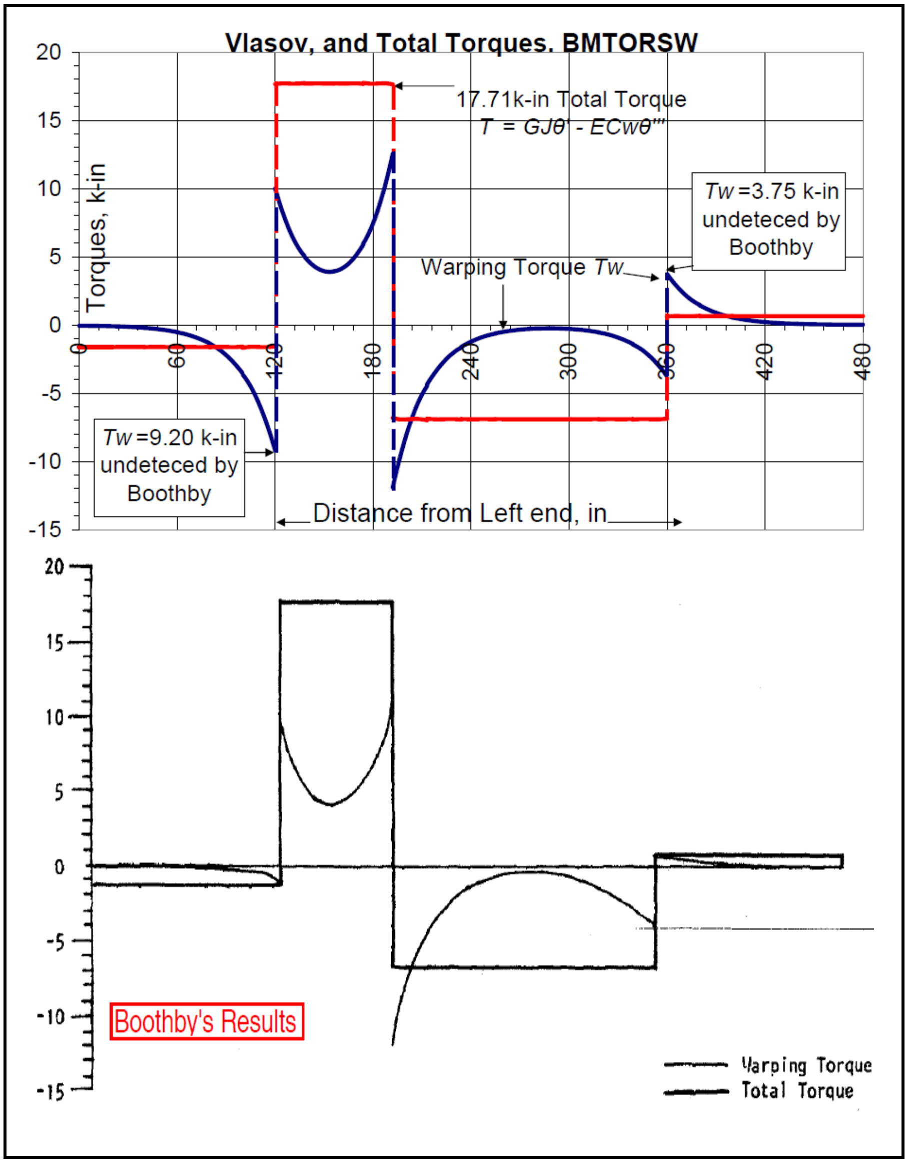



Figure 77. Total and Warping Torque from BMTORSWP and Boothby

Regarding stresses and strains Boothby made another mistake by selecting the wrong sectorial coordinate at the flange-web corner is illustrated in Figure 78 to evaluate the warping stresses. Nevertheless, it does not affect Boothby's Stresses at Right Side of 1st Interior Support (Figure 78). Regarding shear stresses, no discrepancies were found as shown in the same figure. More materials regarding this problem with a meticulous comprehensive study of stresses along the beam axial coordinate and along the cross sectional profile coordinate s may be found in APPENDIX G.

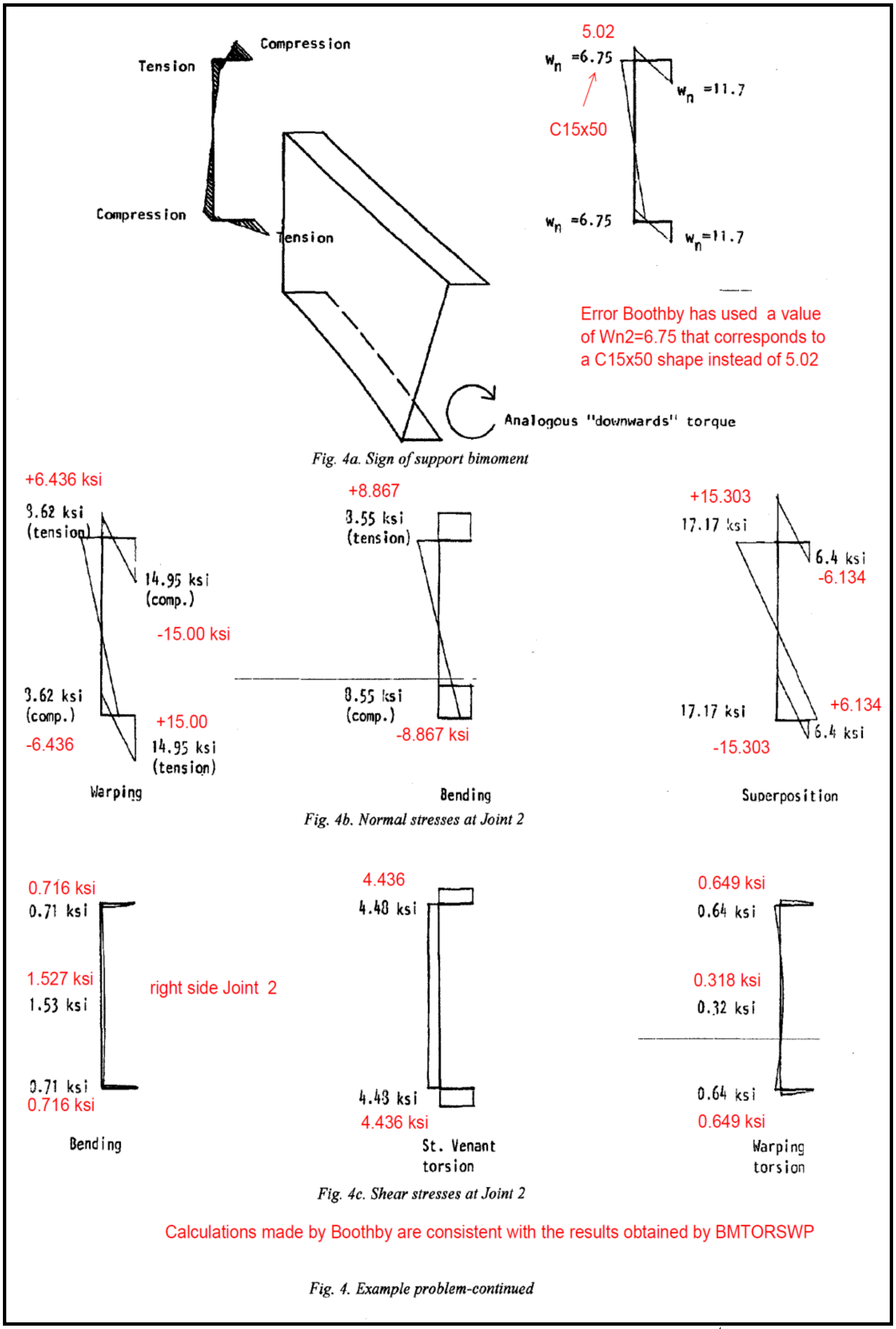



Figure 78. BMTORSWP and Boothby's Stresses at Right Side of 1st Interior Support

### 3.2.10 Case Study Eight

Consider the four-span continuous crane girder shown in Figure 79. The only torsional load in span AB and is due to the horizontal forces H = 6.7 kip exerted by the wheels of the bridge at the top of the crane rail. The wheels' location causes the largest warping moments. The lateral load and the cross section of the crane girder are shown in Figure 81. The distance from the lateral force to the shear center profile is 8.365 in. The total torque at each wheel is T = 56.0 kip-in. Figure 81 shows the values of the sectorial coordinate Wni and the values of the first moment of the sectorial coordinate Sw. The St. Vt. torsion constant J is 11.11 in^4, and the sectorial moment of inertia Cw is 81,989 in^6. E = 29,000 kip/in^2, and G = 11,154 kip/ in^2 for steel. And finally, Figure 80 shows the input model to be used in BMTORSWP.

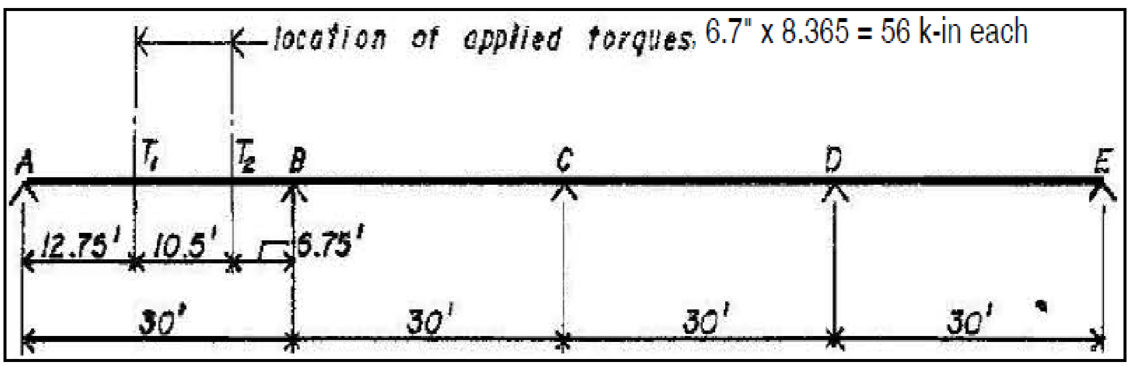



Figure 79. Continuous Crane Girder with Applied Torques

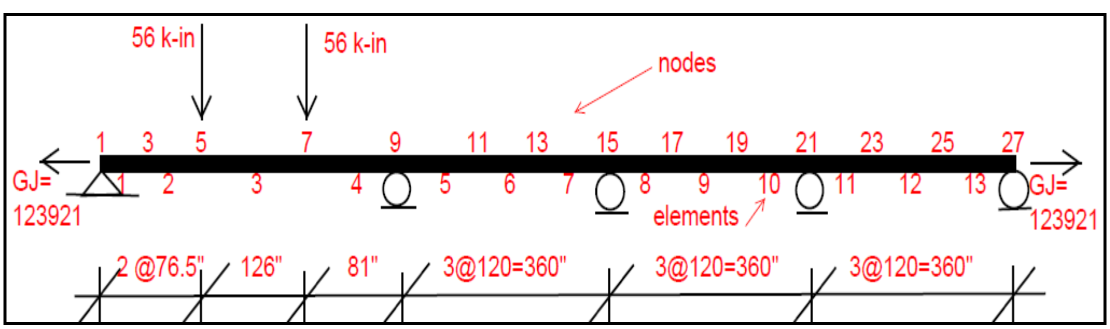



Figure 80. EBC Model for BMTORSWP

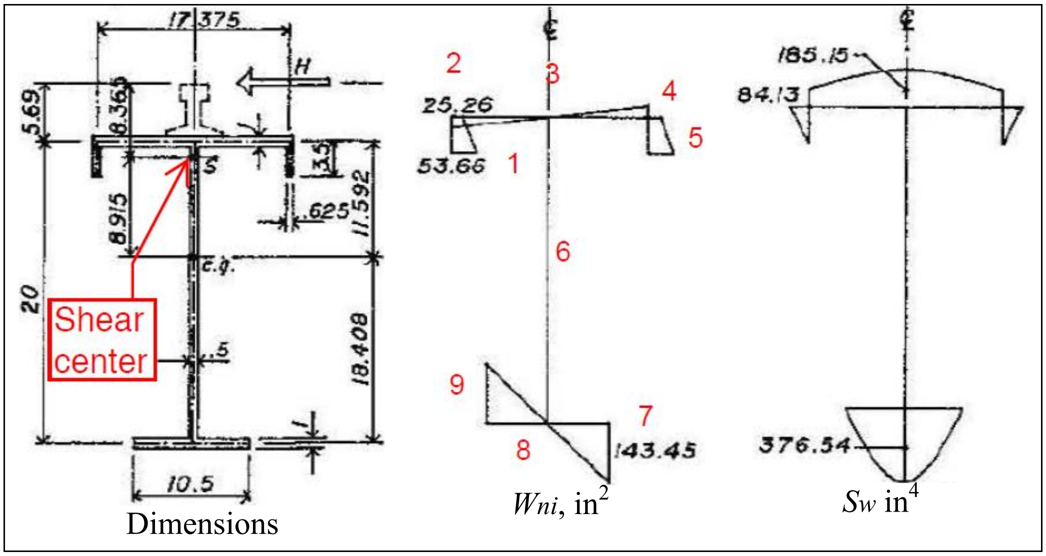


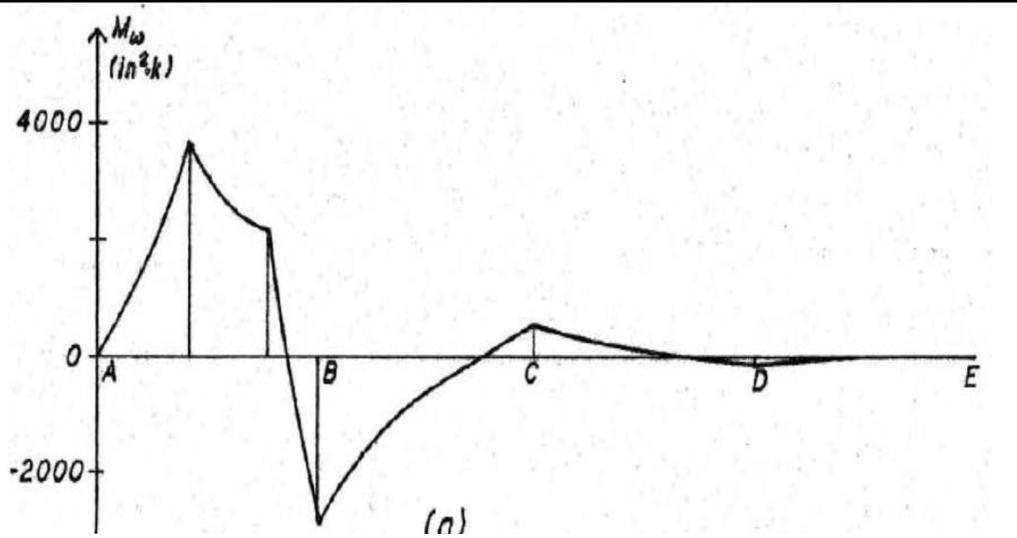

Figure 81. Section Properties and Profile Interest Points "s"

In APPENDIX H, material corresponding to the input model, input data, input forms, figures of output data with checks in notepad version, and excel processed output data and charts can be found. Charts are presented containing both partial and combined tresses along interest points of the beam and cross section profile.

Again, the asymptotic behavior of the thin-walled beam elastic line is successfully shown in the charts, which evidences the efficiency of the high order finite element used by BMTORSWP. Positive shear and axial stresses are assumed similar to those occurring at the cross section flanges and web when the beam undergoes bending.

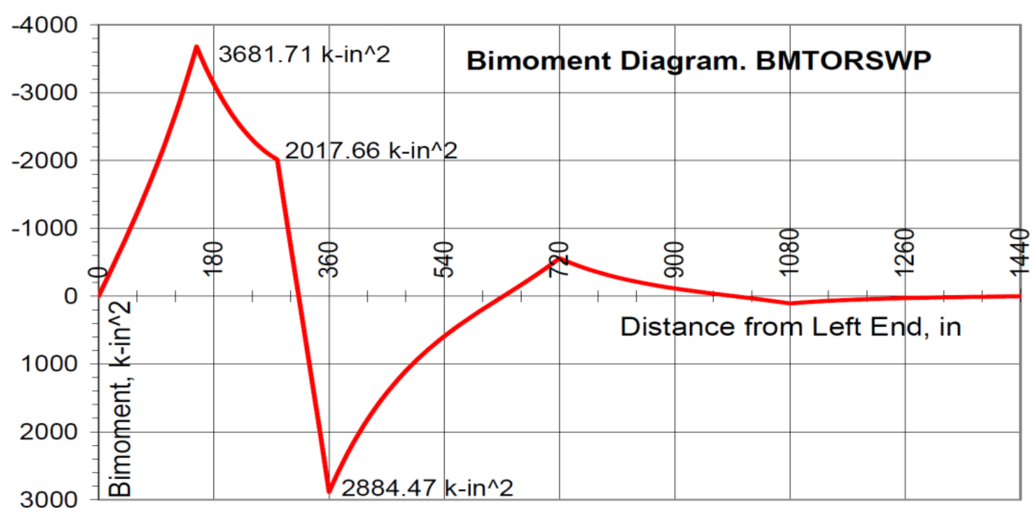
There is a match between the form of BMTORSWP and Medwadowski charts (Figure 82). Nevertheless, Medwadowski committed a huge error in the scale of torques. He found torques 100 times larger than expected, which in terms of stresses is not acceptable at all as in can be seen in Figure 83.

The reasoning to support this statement is as follows: Given the hyperbolic nature of the functions involve in the angle of twist θ(z), each of its derivatives decreases an order of magnitude equal to the characteristic length "a". On the other hand, the ratio between bimoment B (= – ECwθ”, ksi-in^2) and warping moment Tw (= – ECwθ’”, ksi-in) should be in the range of “a” (√ECw/ GJ =138.5 in). Thus, using the Medwadowski bimoment data, the warping moment Tw is expected to be around (4000+3000) ksi-in^2 /138.50in ~ 50 k-in; as shown in the BMTORSWP chart.




Bimoment Values Obtained by Medwadowski (1985)

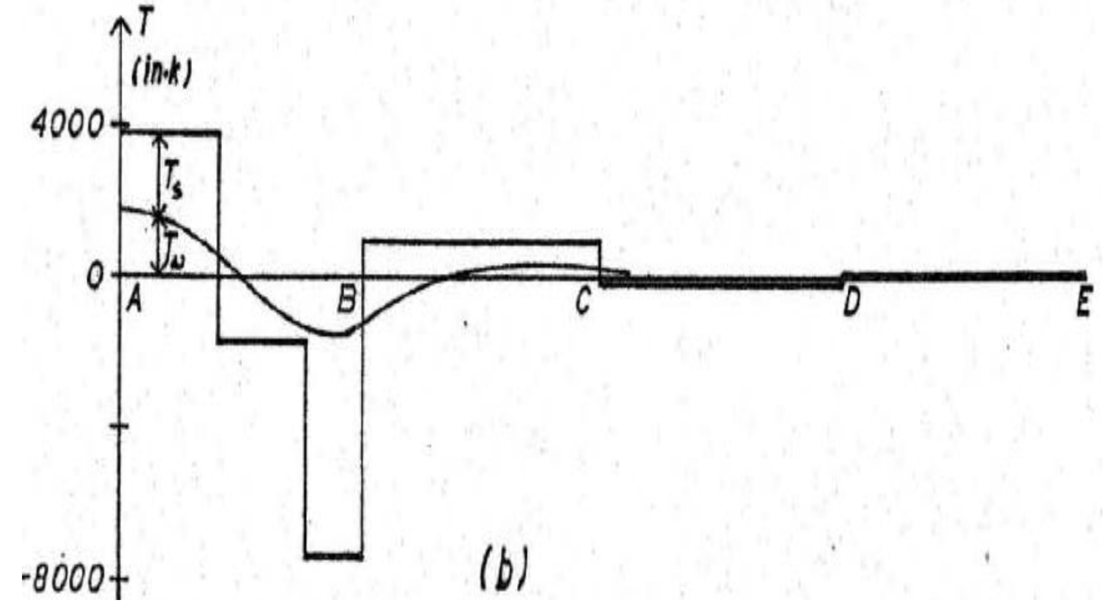
| | | | | | | | |
|---|---|---|---|---|---|---|---|
| 0 | -2885 | 2885 | 562 | -562 | -104 | 104 | 0 |



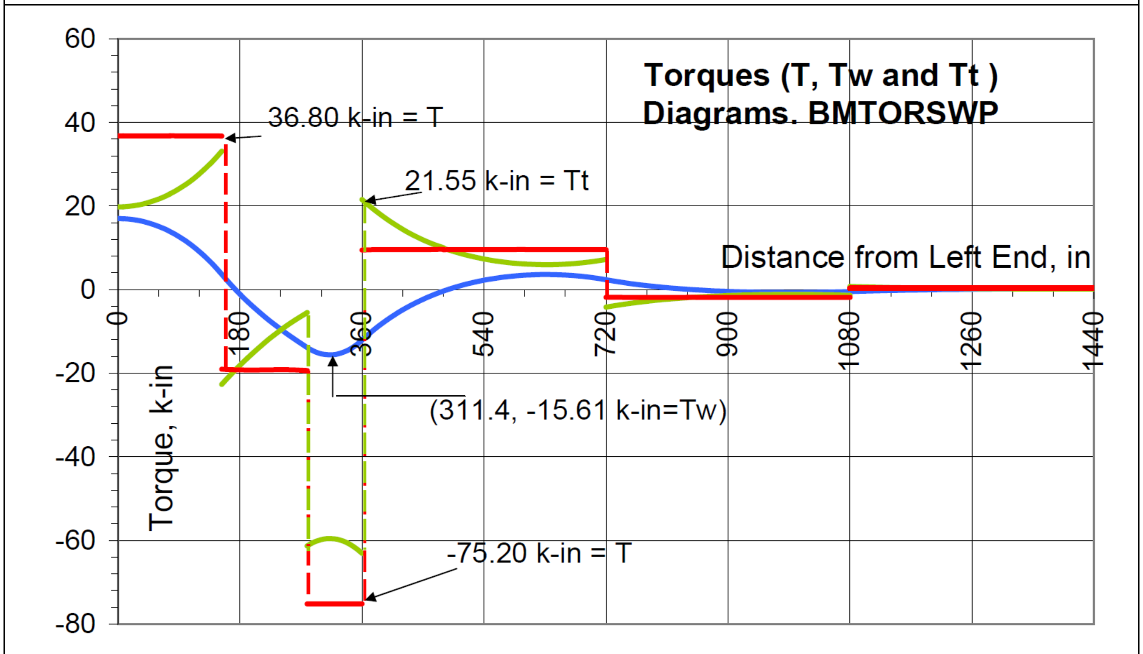

Obtained with BMTORSWP

| | | | | | | | |
|---|---|---|---|---|---|---|---|
| 0 | -2884.5 | 2884.5 | 561.5 | -561.5 | -105.4 | 105.4 | 0 |

Figure 82. Comparison of Bimoment Charts by Medwadowski and with BMTORSWP




Medwadowski's with Scale Error. Given the hyperbolic nature of the functions involve in $\theta(z)$, each of its derivatives decreases an order of magnitude equal to the characteristic length "a". Thus B= $-ECw\theta''$ should be around "a" ($\sqrt{ECw/GJ}$ =138.5) times Tw = $-ECw\theta'''$. From the bimoment chart, Tw could be reasonably expected to be around (4000+3000)/138.50 ~ 50; that is, a two-figure-number as shown below




Obtained with BMTORSWP

Figure 83. Comparison of Torque Charts by Medwadowski and with BMTORSWP

### 3.2.11 Results' Interpretation

According to Saadeé (2004-2005), polynomial interpolation functions are usually used for bending while the hyperbolic functions are often used for non uniform torsion. Nevertheless, the efficacy of BMTORSWP to represent hyperbolic functions has been proven despite the fact it is based upon a finite element developed by means of polynomial interpolation functions.

On the other hand, the solutions in the AISC-DG9 by Seaburg (1997) have been found correct. They were originated in the works by Heins (1963) and Seaburg, actualized by Seaburg (1997) and Carter. Nevertheless, in the case of C shape profiles, the AISC-DG9 omits the shear stress analysis in the profile interest point at s = 1. A computer solution allows the practitioner to analyze this shear using interpolation of the value for the sectorial coordinate.

Conversely, the errors found in the works by Boothby (1984) and Medwadowski (1985) provide evidence that the solution of restrained warping problem is prone to error. Therefore, it is suitable to bring in a tool as BMTORSWP able to simplify the problems involving first order general torsion theory. In addition to the complexity that structural analysis of TWB share with $2^{nd}$ order analysis of beam-columns, the calculation of TWB stresses along the cross section profile is tiresome. And BMTORSWP provides the suitable option to export the data to a spreadsheet for a more systematic handling of stress computations.

As a comment on the trustiness of the finite element upon which the software is based, it is important to state that the accuracy of the 6DOF beam column is 3.8 to 4 times that of the 4 DOF beam column. Therefore, just one 6DOF-BE is required to achieve the accuracy of four-4DOF-BE or five 6DOF-BE are required to achieve the accuracy of nineteen 4DOF-BE. On the other hand, one 6DOF-BE matrix occupies 18x1+3 = 21 entries in a global matrix upper triangle, while four 4DOF-BE matrixes occupy 7x4+3 = 31 entries in a global matrix upper triangle respectively. Similarly, five 6DOF-BE matrixes occupy 18x5+3 = 93 entries in a global matrix upper triangle and nineteen 4DOF-BE matrixes occupy 7x19+3 = 135 entries in a global matrix upper triangle respectively.

This difference will increase linearly as the number of required elements increases. In general the global matrix assembled with a higher degree finite element requires a lesser number of matrix entries in the program memory in order to achieve a given accuracy; thus, it is more convenient. This advantage of the 6DOF-BE is in addition to the ones related to its two non nodal DOFs, and adds to the fact that for a solution of the mixed torsion problem a higher degree polynomial is mandatory.

Regarding the aforementioned errors by Boothby and Medwadowski, it is pertinent to comment that perfect coincidence was found in the bimoment diagrams. However the chart of torques by Boothby contains 2 mistakes. At left side of first interior support and right side of last interior support, warping torques of 9.2 and 3.5 respectively were not detected by Boothby with corresponding errors of 700 and 300 per cent. See Chart Total and Warping Torque from BMTORSWP and Boothby.

Nevertheless, these errors would not be critical for design purposes. It was found that stresses calculations by Boothby correspond to the right side of the first interior support, and they are correct. Nevertheless, another mistake by Boothby was noticed regarding the cross section sectorial coordinate at the flange-web corner: A Wn2 = 6.75 $in^2$ corresponding to a C15x50 shape was used instead of 5.02 $in^2$.

Regarding the aforementioned errors by Medwadowski, a perfect coincidence was found in the bimoment diagrams. However the chart of torques by Medwadowski contains one error. He found torques 100 times larger than expected, which in terms of stresses is not acceptable at all.

The reasoning to sustain this statement is as follows: Given the hyperbolic nature of the functions involve in the angle of twist θ(z), each of its derivatives decreases an order of magnitude equal to the characteristic length “a”. Therefore, the ratio between bimoment B (= – ECwθ”, ksi-in^2) and warping moment Tw (= – ECwθ’”, ksi-in) should be in the range of “a” (√ECw/ GJ =138.5 in). Thus, using the Medwadowski bimoment data, the warping moment Tw is expected to be around (4000+3000) ksi-in^2 /138.50in ~ 50 k-in; as shown in Figure 83.

# CHAPTER 4

# CONCLUSIONS AND RECOMMENDATIONS

It is appropriate for designers of thin-walled open cross sections frequently used in steel structures to count on straightforward and affordable tools enabling them to bypass the common difficulties related to the design of elements under restrained warping. BMTORSW or BMTORSWP fulfill very well all these requirements.

## 4.1 Conclusions

Considering the asymptotic character of the governing hyperbolic functions, the use of smaller finite elements may be more appropriate near restraints. An additional consideration must be taken into account to check the equilibrium of forces when distributed loads are involved. Due to the fact that the location of the smeared DOF is unknown, a more refined mesh reduces the error when it comes to arbitrarily assigning a reasonable location for the smeared DOF action during the equilibrium checks.

The accuracy of the 6DOF beam column is 3.8 to 4 times that of the 4 DOF beam column as shown in Figure 38. Therefore, it could be said that "n" 6DOF-BE matrixes require 18n+3 entries in a global matrix upper triangle which, in terms of accuracy, are equivalent to 3.8·n 4DOF-BE matrixes that require about 27n +3 (7(3.8n) +3) entries in a global matrix upper triangle respectively.

Two effects detected in the parametric experiments have been corroborated in the analysis: First, there is twist and restrained warping along the unloaded span; second, the twist at the loaded end is larger than that at the unloaded end.

### 4.2 Recommendations

Definitely BMTORSWP is the recommended software application to handle problems of multi-span beams subjected to restrained warping. It is straightforward, versatile and powerful enough. And it has survived the test of handling single span bar under restrained warping (Obréski, 2005).

The recommended size of the finite element to be used in BMTORSWP is between 2a and 4a; where $a = \sqrt{E \cdot Cw / G \cdot J}$ according to the AISC. Nevertheless, an additional advice is to use a minimum of 3 finite elements per span to reproduce the asymptotic behavior (which accentuates as $C_w$ gets smaller Figure 50) of the angle of twist and its derivatives. Nodes are mandatory at restraints and points of application of concentrated loads. An additional recommendation is that smaller finite elements can be used neighboring the nodes.

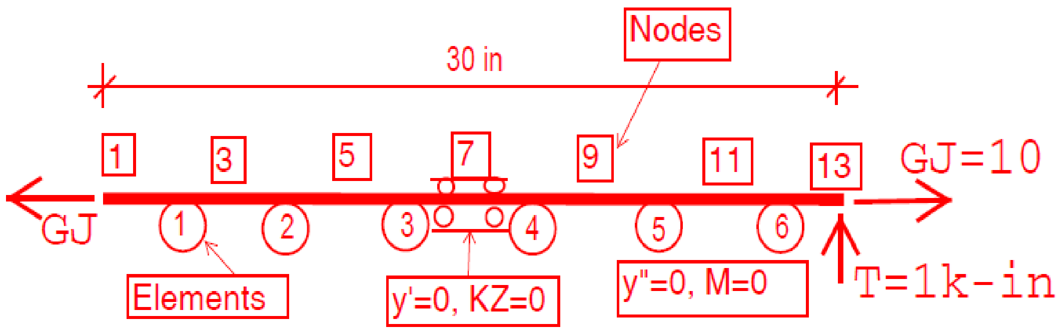
Inexpensive experiments like that of the bar restrained against torsion at midspan could be used to experimentally measure cross section torsional properties, and to verify the range of linear behavior of GJ. They could also be used to study large deformation behavior, and to explain behavior of moving large slim structures like trucks and catamarans when struck or impacted near midspan.

Finally, the software application is recommended to solve problems of distorted box girders. However, for a box girder solution, a study of convergence similar to the one presented in this work is recommended to be advanced. This more accurate convergence study must be made by combining the 6x6 elastic and soil matrixes related to the field of transverse displacements.

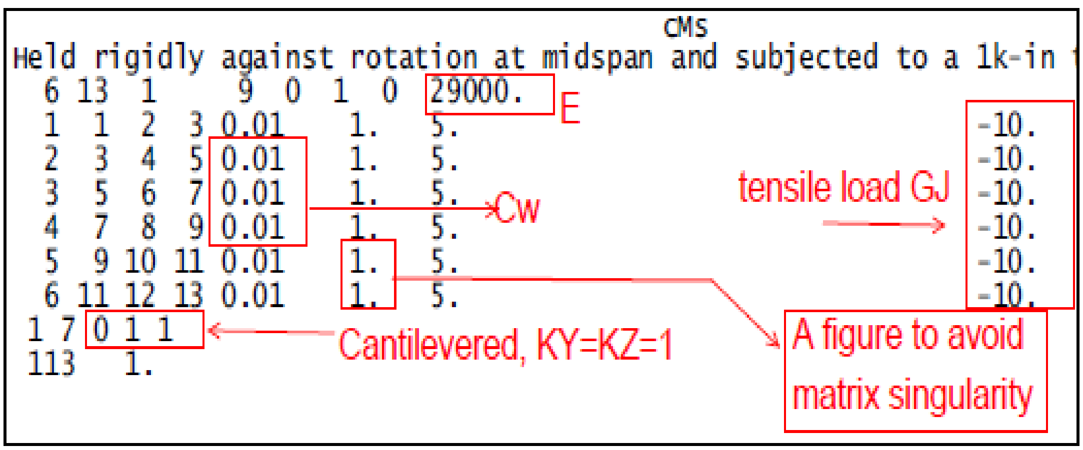
## APPENDIX A

## BAR CANTILEVERED AT MIDSPAN

The equivalent beam-column model, input forms, input and output notepads as well as the charts can be found in the following pages of this part of the document.



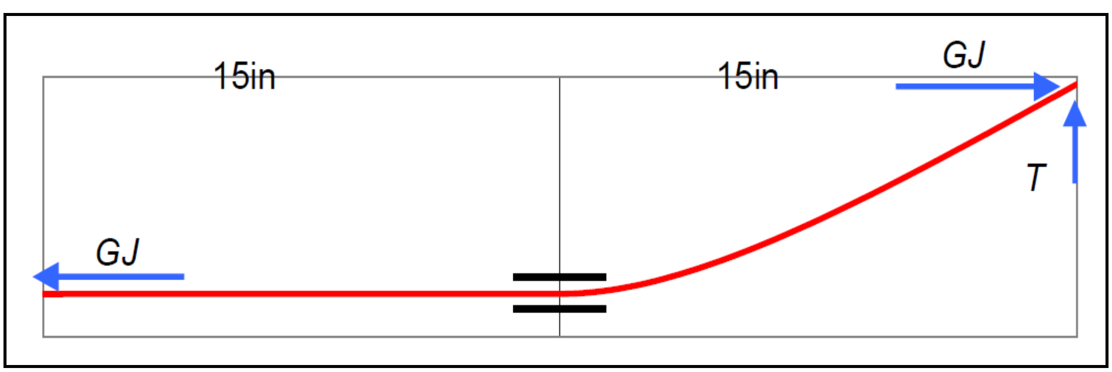

Input model for Bar Cantilevered at Midspan



```
                                                   CMs
Held rigidly against rotation at midspan and subjected to a 1k-in
  6 13  1      9  0  1  0  29000.
  1  1  2  3 0.01    1.   5.                            -10.
  2  3  4  5 0.01    1.   5.                            -10.
  3  5  6  7 0.01    1.   5.                            -10.
  4  7  8  9 0.01    1.   5.                            -10.
  5  9 10 11 0.01    1.   5.                            -10.
  6 11 12 13 0.01    1.   5.                            -10.
 1 7 0 1 1
 113    1.
```



Input Notepad




Bar Cantilevered (θ = 0, θ’’ = 0) at Midspan with 1K-in Torque at far End

```
                                         cMsOut
 ------------------------------------------------------------------------
 YOU ARE USING COMPUTER PROGRAM BMTORSW,DEVELOPED BY DR. BERNARDO DESCHAPELLES
0        INPUT DATA FILE NAME IS = cMs.txt
0                OUTPUT FILE NAME IS = cMsOut.txt
0          STORAGE FILE FOR POST-PROCESSING WITH EXCEL = cMsGrf.grf

0------------------------------------------------------------------------
 Held rigidly against rotation at midspan and subjected to a 1k-in torque at far
0modulus of elasticity of the material=   29000.  k/ft2
0ELEM nodes inertia length distrib. load  AXIAL  SOIL NORMAL MODULUS,Ksf  angle
      i   j  ft.4     ft     at i    at j   LOAD    1st END       2nd END   rad
  1   1   3 0.01000  5.00  0.000  0.000  -10.00      0.0           0.0   0.000 00
  2   3   5 0.01000  5.00  0.000  0.000  -10.00      0.0           0.0   0.000 00
  3   5   7 0.01000  5.00  0.000  0.000  -10.00      0.0           0.0   0.000 00
  4   7   9 0.01000  5.00  0.000  0.000  -10.00      0.0           0.0   0.000 00
  5   9  11 0.01000  5.00  0.000  0.000  -10.00      0.0           0.0   0.000 00
  6  11  13 0.01000  5.00  0.000  0.000  -10.00      0.0           0.0   0.000 00
0    INPUT DATA RELATED TO THE  1 SUPPORTS
  1 7 0 1 1
0    INPUT OF NODAL FORCES RELATED TO GLOBAL AXIS  2
         113   1.00
0      FINAL  SOLUTION   FOUND   AFTER    1  ITERATIONS
0     output of nodal displacements in reference to  global axes
0node  displ.       displ.       displ.     node  displ.       displ.       displ.
     along  x     along  y     around  z         along  x     along  y     around  z
     or nonn1     or nonn2     or nonn 3         or nonn1     or nonn2     or nonn 3

+  1 0.0000E+00  0.0000E+00  0.0000E+00
+                                          2 0.0000E+00  0.0000E+00  0.0000E+00

+  3 0.0000E+00  0.0000E+00  0.0000E+00
+                                          4 0.0000E+00  0.0000E+00  0.0000E+00

+  5 0.0000E+00  0.0000E+00  0.0000E+00
+                                          6 0.0000E+00  0.0000E+00  0.0000E+00

+  7 0.0000E+00  0.0000E+00  0.0000E+00
+                                          8 0.0000E+00  0.6168E-01  0.1475E-01

+  9 0.0000E+00  0.1724E+00  0.5967E-01
+                                         10 0.0000E+00  0.3451E+00  0.3048E-01

+ 11 0.0000E+00  0.5363E+00  0.8202E-01
+                                         12 0.0000E+00  0.7486E+00  0.3586E-01

+ 13 0.0000E+00  0.9656E+00  0.8771E-01
0-----------------------------------------------------------------------
      OUTPUT OF SOIL REACTIONS,STRESSES AND TRANSVERSE DISPLACEMENTS
0-----------------------------------------------------------------------
0      ELEMENT     1  DISPLACEMENTS IN INCIDENCES    1    2    3
         NODE      1         0.00000E+00         0.00000E+00        0.00000E+00
         NODE      2         0.00000E+00         0.00000E+00        0.00000E+00
         NODE      3         0.00000E+00         0.00000E+00        0.00000E+00
0      FORCES ACTING ALONG THE 9 DOF
         NODE      1         0.00000E+00         0.00000E+00        0.00000E+00
         NODE      2         0.00000E+00         0.00000E+00        0.00000E+00
         NODE      3         0.00000E+00         0.00000E+00        0.00000E+00
0ELEMENT   1, FROM NODE   1, TO NODE   3 - LENGTH =   5.00 ft
0 left half of span,at tenth points of length
                   span       span       span       span       span       span
+              0.0        0.1        0.2        0.3        0.4        0.5
 soil,k/ft       0.000      0.000      0.000      0.000      0.000      0.000
 shear,k         0.00       0.00       0.00       0.00       0.00       0.00
                                                  Page 1
```

Tensile

GJ=10; T = 1; L = 15;

(- GJ*0.9656 + TL) = Bimomento = 5.344

Output Checks, Page 1

```
                                      cMsOut
 bmom,kft      0.00      0.00      0.00      0.00      0.00      0.00
 tdisp,ft   0.00000   0.00000   0.00000   0.00000   0.00000   0.00000
 axial,k       0.00 AT 1st END and      0.00 AT 2nd END
Oright half of span,at tenth points of length
               span      span      span      span      span      span
+          0.5       0.6       0.7       0.8       0.9       1.0
 soil,k/ft    0.000     0.000     0.000     0.000     0.000     0.000
 shear,k       0.00      0.00      0.00      0.00      0.00      0.00
 bmom,kft      0.00      0.00      0.00      0.00      0.00      0.00
 tdisp,ft   0.00000   0.00000   0.00000   0.00000   0.00000   0.00000
 axial,k       0.00 AT 1st END and      0.00 AT 2nd END
0-----------------------------------------------------------------------
O     ELEMENT   2  DISPLACEMENTS IN INCIDENCES   3   4   5
         NODE   3          0.00000E+00          0.00000E+00          0.00000E+00
         NODE   4          0.00000E+00          0.00000E+00          0.00000E+00
         NODE   5          0.00000E+00          0.00000E+00          0.00000E+00
O     FORCES ACTING ALONG THE 9 DOF
         NODE   3          0.00000E+00          0.00000E+00          0.00000E+00
         NODE   4          0.00000E+00          0.00000E+00          0.00000E+00
         NODE   5          0.00000E+00          0.00000E+00          0.00000E+00
OELEMENT  2, FROM NODE  3, TO NODE  5 - LENGTH =  5.00 ft
0 left half of span,at tenth points of length
               span      span      span      span      span      span
+          0.0       0.1       0.2       0.3       0.4       0.5
 soil,k/ft    0.000     0.000     0.000     0.000     0.000     0.000
 shear,k       0.00      0.00      0.00      0.00      0.00      0.00
 bmom,kft      0.00      0.00      0.00      0.00      0.00      0.00
 tdisp,ft   0.00000   0.00000   0.00000   0.00000   0.00000   0.00000
 axial,k       0.00 AT 1st END and      0.00 AT 2nd END
Oright half of span,at tenth points of length
               span      span      span      span      span      span
+          0.5       0.6       0.7       0.8       0.9       1.0
 soil,k/ft    0.000     0.000     0.000     0.000     0.000     0.000
 shear,k       0.00      0.00      0.00      0.00      0.00      0.00
 bmom,kft      0.00      0.00      0.00      0.00      0.00      0.00
 tdisp,ft   0.00000   0.00000   0.00000   0.00000   0.00000   0.00000
 axial,k       0.00 AT 1st END and      0.00 AT 2nd END
0-----------------------------------------------------------------------
O     ELEMENT   3  DISPLACEMENTS IN INCIDENCES   5   6   7
         NODE   5          0.00000E+00          0.00000E+00          0.00000E+00
         NODE   6          0.00000E+00          0.00000E+00          0.00000E+00
         NODE   7          0.00000E+00          0.00000E+00          0.00000E+00
O     FORCES ACTING ALONG THE 9 DOF
         NODE   5          0.00000E+00          0.00000E+00          0.00000E+00
         NODE   6          0.00000E+00          0.00000E+00          0.00000E+00
         NODE   7          0.00000E+00          0.00000E+00          0.00000E+00
OELEMENT  3, FROM NODE  5, TO NODE  7 - LENGTH =  5.00 ft
0 left half of span,at tenth points of length
               span      span      span      span      span      span
+          0.0       0.1       0.2       0.3       0.4       0.5
 soil,k/ft    0.000     0.000     0.000     0.000     0.000     0.000
 shear,k       0.00      0.00      0.00      0.00      0.00      0.00
 bmom,kft      0.00      0.00      0.00      0.00      0.00      0.00
 tdisp,ft   0.00000   0.00000   0.00000   0.00000   0.00000   0.00000
 axial,k       0.00 AT 1st END and      0.00 AT 2nd END
Oright half of span,at tenth points of length
               span      span      span      span      span      span
+          0.5       0.6       0.7       0.8       0.9       1.0
 soil,k/ft    0.000     0.000     0.000     0.000     0.000     0.000
 shear,k       0.00      0.00      0.00      0.00      0.00      0.00
 bmom,kft      0.00      0.00      0.00      0.00      0.00      0.00
 tdisp,ft   0.00000   0.00000   0.00000   0.00000   0.00000   0.00000
 axial,k       0.00 AT 1st END and      0.00 AT 2nd END
                                       Page 2
```

(-Di + Dj)* GJ + Vj*Lij = Mj-Mi, where GJ=10, Lij=5

-0.0 +0.0)*10 -0*5 = 0

-0 + 0.0 = 0

Output Checks, Page 2

GJ=10; T = 1; L = 15;

(-GJ*0.9656 + TL) = Bimomento = 5.344

```
                                                     cMsOut
0-----------------------------------------------------------------------
O     ELEMENT   4  DISPLACEMENTS IN INCIDENCES   7   8   9
         NODE   7          0.00000E+00         0.00000E+00         0.00000E+00
         NODE   8          0.00000E+00         0.61681E-01         0.14750E-01
         NODE   9          0.00000E+00         0.17239E+00         0.59675E-01
O     FORCES ACTING ALONG THE 9 DOF
         NODE   7          0.00000E+00        -0.10000E+01        -0.53443E+01
         NODE   8          0.00000E+00        -0.21316E-13         0.11369E-12
         NODE   9          0.00000E+00         0.10000E+01         0.20682E+01
OELEMENT  4, FROM NODE  7, TO NODE  9 - LENGTH =  5.00 ft
0 left half of span,at tenth points of length
             span      span      span      span      span      span
+          0.0       0.1       0.2       0.3       0.4       0.5
 soil,k/ft    0.000     0.000     0.000     0.000     0.000     0.000
 shear,k     -1.00     -1.00     -1.00     -1.00     -1.00     -1.00
 bmom,kft     5.34      4.87      4.43      4.03      3.67      3.34
 tdisp,ft   0.00000   0.00223   0.00866   0.01892   0.03265   0.04955
 axial,k         0.00 AT 1st END and       0.00 AT 2nd END
0right half of span,at tenth points of length
             span      span      span      span      span      span
+          0.5       0.6       0.7       0.8       0.9       1.0
 soil,k/ft    0.000     0.000     0.000     0.000     0.000     0.000
 shear,k     -1.00     -1.00     -1.00     -1.00     -1.00     -1.00
 bmom,kft     3.34      3.04      2.76      2.51      2.28      2.07
 tdisp,ft   0.04955   0.06934   0.09174   0.11652   0.14347   0.17239
 axial,k         0.00 AT 1st END and       0.00 AT 2nd END
0-----------------------------------------------------------------------
O     ELEMENT   5  DISPLACEMENTS IN INCIDENCES   9  10  11
         NODE   9          0.00000E+00         0.17239E+00         0.59675E-01
         NODE  10          0.00000E+00         0.34515E+00         0.30482E-01
         NODE  11          0.00000E+00         0.53626E+00         0.82015E-01
O     FORCES ACTING ALONG THE 9 DOF
         NODE   9          0.00000E+00        -0.10000E+01        -0.20682E+01
         NODE  10          0.00000E+00         0.10658E-12        -0.11369E-11
         NODE  11          0.00000E+00         0.10000E+01         0.70689E+00
OELEMENT  5, FROM NODE  9, TO NODE 11 - LENGTH =  5.00 ft
0 left half of span,at tenth points of length
             span      span      span      span      span      span
+          0.0       0.1       0.2       0.3       0.4       0.5
 soil,k/ft    0.000     0.000     0.000     0.000     0.000     0.000
 shear,k     -1.00     -1.00     -1.00     -1.00     -1.00     -1.00
 bmom,kft     2.07      1.88      1.70      1.54      1.39      1.25
 tdisp,ft   0.17239   0.20309   0.23541   0.26919   0.30430   0.34061
 axial,k         0.00 AT 1st END and       0.00 AT 2nd END
0right half of span,at tenth points of length
             span      span      span      span      span      span
+          0.5       0.6       0.7       0.8       0.9       1.0
 soil,k/ft    0.000     0.000     0.000     0.000     0.000     0.000
 shear,k     -1.00     -1.00     -1.00     -1.00     -1.00     -1.00
 bmom,kft     1.25      1.12      1.01      0.90      0.80      0.71
 tdisp,ft   0.34061   0.37799   0.41634   0.45557   0.49557   0.53626
 axial,k         0.00 AT 1st END and       0.00 AT 2nd END
0-----------------------------------------------------------------------
O     ELEMENT   6  DISPLACEMENTS IN INCIDENCES  11  12  13
         NODE  11          0.00000E+00         0.53626E+00         0.82015E-01
         NODE  12          0.00000E+00         0.74858E+00         0.35859E-01
         NODE  13          0.00000E+00         0.96557E+00         0.87706E-01
O     FORCES ACTING ALONG THE 9 DOF
         NODE  11          0.00000E+00        -0.10000E+01        -0.70689E+00
         NODE  12          0.00000E+00        -0.63949E-13         0.56843E-12
         NODE  13          0.00000E+00         0.10000E+01        -0.15632E-12
OELEMENT  6, FROM NODE 11, TO NODE 13 - LENGTH =  5.00 ft
0 left half of span,at tenth points of length
```

(-Di + Dj)* GJ + Vj*Lij = Mj-Mi, where GJ=10; Lij=5

Page 3

-0.17239 +0.53626)*10 -1*5 = -1.3613

-2.07 + 0.71 = -1.36 OK

Output Checks, Page 3

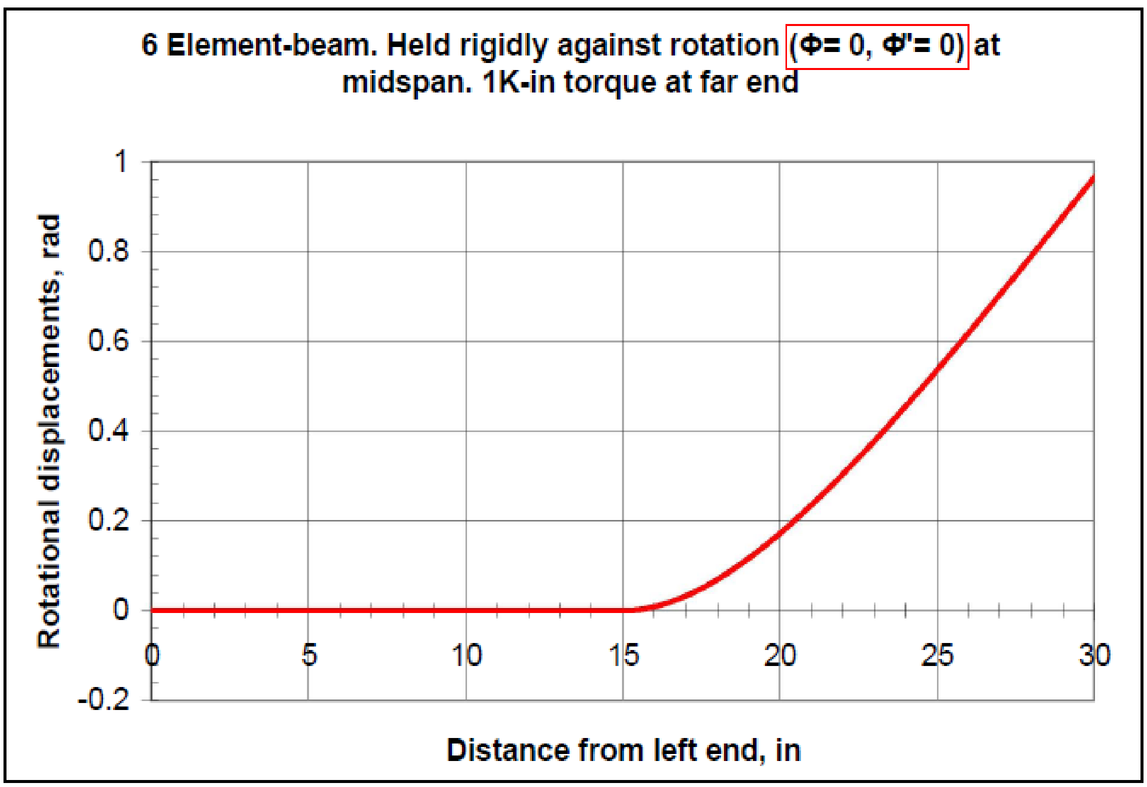
```
                                    cMsOut
               span       span       span       span       span       span
+           0.0        0.1        0.2        0.3        0.4        0.5
 soil,k/ft     0.000      0.000      0.000      0.000      0.000      0.000
 shear,k      -1.00      -1.00      -1.00      -1.00      -1.00      -1.00
 bmom,kft      0.71       0.62       0.54       0.46       0.39       0.32
 tdisp,ft    0.53626    0.57756    0.61939    0.66169    0.70439    0.74742
 axial,k       0.00 AT 1st END and      0.00 AT 2nd END
0right half of span,at tenth points of length
               span       span       span       span       span       span
+           0.5        0.6        0.7        0.8        0.9        1.0
 soil,k/ft     0.000      0.000      0.000      0.000      0.000      0.000
 shear,k      -1.00      -1.00      -1.00      -1.00      -1.00      -1.00
 bmom,kft      0.32       0.25       0.19       0.12       0.06       0.00
 tdisp,ft    0.74742    0.79073    0.83425    0.87793    0.92172    0.96557
 axial,k       0.00 AT 1st END and      0.00 AT 2nd END
1
```

(-Di + Dj)* GJ + Vj*Lij = Mj-Mi, where GJ=10, Lij=5

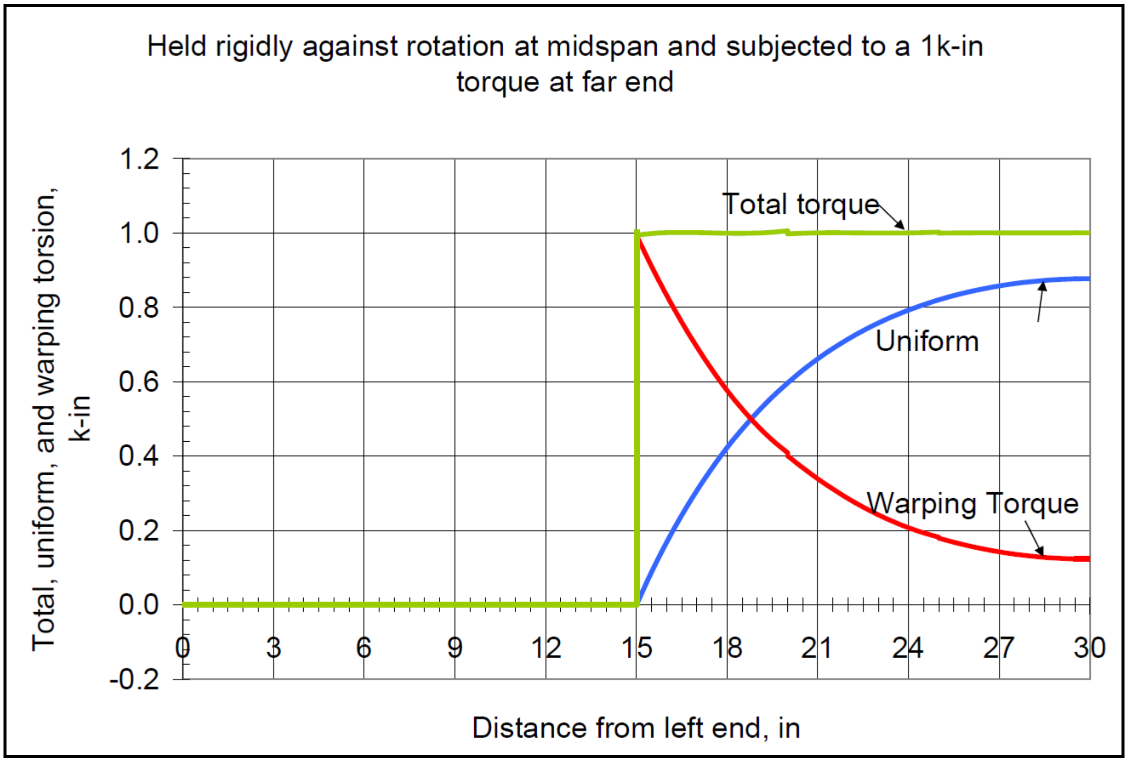
-0.53626 +0.96557)*10 -1*5 = 0.7069

0 + 0.71 = 0.71 OK

Output Checks, Last Page



Elastic Line of TWB Cantilevered at Midspan




Cross Section Torques Diagrams

Data Processed by Spreadsheet

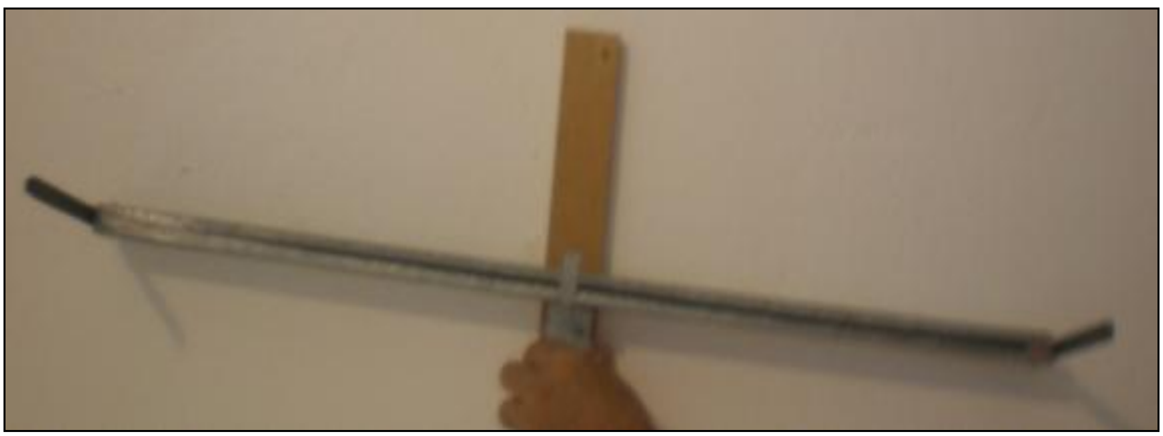
| elm | Z | Φ | T | B(z) | Φ' | Φ" | Φ"' | GJΦ' | -ECwΦ"' | T |
|---|---|---|---|---|---|---|---|---|---|---|
| 1 | 0 | 0 | 0 | 0 | 0.00E+00 | 0.00E+00 | 0.00E+00 | 0.00E+00 | 0.00E+00 | 0.00 |
| | 0.5 | 0 | 0 | 0 | 0.00E+00 | 0.00E+00 | 0.00E+00 | 0.00E+00 | 0.00E+00 | 0.00 |
| | 1 | 0 | 0 | 0 | 0.00E+00 | 0.00E+00 | 0.00E+00 | 0.00E+00 | 0.00E+00 | 0.00 |
| | 1.5 | 0 | 0 | 0 | 0.00E+00 | 0.00E+00 | 0.00E+00 | 0.00E+00 | 0.00E+00 | 0.00 |
| | 2 | 0 | 0 | 0 | 0.00E+00 | 0.00E+00 | 0.00E+00 | 0.00E+00 | 0.00E+00 | 0.00 |
| | 2.5 | 0 | 0 | 0 | 0.00E+00 | 0.00E+00 | 0.00E+00 | 0.00E+00 | 0.00E+00 | 0.00 |
| | 3 | 0 | 0 | 0 | 0.00E+00 | 0.00E+00 | 0.00E+00 | 0.00E+00 | 0.00E+00 | 0.00 |
| | 3.5 | 0 | 0 | 0 | 0.00E+00 | 0.00E+00 | 0.00E+00 | 0.00E+00 | 0.00E+00 | 0.00 |
| | 4 | 0 | 0 | 0 | 0.00E+00 | 0.00E+00 | 0.00E+00 | 0.00E+00 | 0.00E+00 | 0.00 |
| | 4.5 | 0 | 0 | 0 | 0.00E+00 | 0.00E+00 | 0.00E+00 | 0.00E+00 | 0.00E+00 | 0.00 |
| 2 | 5 | 0 | 0 | 0 | 0.00E+00 | 0.00E+00 | 0.00E+00 | 0.00E+00 | 0.00E+00 | 0.00 |
| | 5.5 | 0 | 0 | 0 | 0.00E+00 | 0.00E+00 | 0.00E+00 | 0.00E+00 | 0.00E+00 | 0.00 |
| | 6 | 0 | 0 | 0 | 0.00E+00 | 0.00E+00 | 0.00E+00 | 0.00E+00 | 0.00E+00 | 0.00 |
| | 6.5 | 0 | 0 | 0 | 0.00E+00 | 0.00E+00 | 0.00E+00 | 0.00E+00 | 0.00E+00 | 0.00 |
| | 7 | 0 | 0 | 0 | 0.00E+00 | 0.00E+00 | 0.00E+00 | 0.00E+00 | 0.00E+00 | 0.00 |
| | 7.5 | 0 | 0 | 0 | 0.00E+00 | 0.00E+00 | 0.00E+00 | 0.00E+00 | 0.00E+00 | 0.00 |
| | 8 | 0 | 0 | 0 | 0.00E+00 | 0.00E+00 | 0.00E+00 | 0.00E+00 | 0.00E+00 | 0.00 |
| | 8.5 | 0 | 0 | 0 | 0.00E+00 | 0.00E+00 | 0.00E+00 | 0.00E+00 | 0.00E+00 | 0.00 |
| | 9 | 0 | 0 | 0 | 0.00E+00 | 0.00E+00 | 0.00E+00 | 0.00E+00 | 0.00E+00 | 0.00 |
| | 9.5 | 0 | 0 | 0 | 0.00E+00 | 0.00E+00 | 0.00E+00 | 0.00E+00 | 0.00E+00 | 0.00 |

| | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|
| 3 | 10 | 0 | 0 | 0 | 0.00E+00 | 0.00E+00 | 0.00E+00 | 0.00E+00 | 0.00E+00 | 0.00 |
| | 10.5 | 0 | 0 | 0 | 0.00E+00 | 0.00E+00 | 0.00E+00 | 0.00E+00 | 0.00E+00 | 0.00 |
| | 11 | 0 | 0 | 0 | 0.00E+00 | 0.00E+00 | 0.00E+00 | 0.00E+00 | 0.00E+00 | 0.00 |
| | 11.5 | 0 | 0 | 0 | 0.00E+00 | 0.00E+00 | 0.00E+00 | 0.00E+00 | 0.00E+00 | 0.00 |
| | 12 | 0 | 0 | 0 | 0.00E+00 | 0.00E+00 | 0.00E+00 | 0.00E+00 | 0.00E+00 | 0.00 |
| | 12.5 | 0 | 0 | 0 | 0.00E+00 | 0.00E+00 | 0.00E+00 | 0.00E+00 | 0.00E+00 | 0.00 |
| | 13 | 0 | 0 | 0 | 0.00E+00 | 0.00E+00 | 0.00E+00 | 0.00E+00 | 0.00E+00 | 0.00 |
| | 13.5 | 0 | 0 | 0 | 0.00E+00 | 0.00E+00 | 0.00E+00 | 0.00E+00 | 0.00E+00 | 0.00 |
| | 14 | 0 | 0 | 0 | 0.00E+00 | 0.00E+00 | 0.00E+00 | 0.00E+00 | 0.00E+00 | 0.00 |
| | 14.5 | 0 | 0 | 0 | 0.00E+00 | 0.00E+00 | 0.00E+00 | 0.00E+00 | 0.00E+00 | 0.00 |
| 4 | 15 | 0 | -1 | 5.3443 | 0.00E+00 | 1.84E-02 | -3.43E-03 | 0.00E+00 | 9.94E-01 | 0.99 |
| | 15.5 | 0.0022 | -1 | 4.8666 | 8.80E-03 | 1.68E-02 | -3.14E-03 | 8.80E-02 | 9.10E-01 | 1.00 |
| | 16 | 0.0087 | -1 | 4.431 | 1.68E-02 | 1.53E-02 | -2.87E-03 | 1.68E-01 | 8.33E-01 | 1.00 |
| | 16.5 | 0.0189 | -1 | 4.0335 | 2.41E-02 | 1.39E-02 | -2.62E-03 | 2.41E-01 | 7.60E-01 | 1.00 |
| | 17 | 0.0327 | -1 | 3.6709 | 3.07E-02 | 1.27E-02 | -2.39E-03 | 3.07E-01 | 6.93E-01 | 1.00 |
| | 17.5 | 0.0496 | -1 | 3.3399 | 3.68E-02 | 1.15E-02 | -2.18E-03 | 3.68E-01 | 6.32E-01 | 1.00 |
| | 18 | 0.0693 | -1 | 3.0377 | 4.23E-02 | 1.05E-02 | -1.99E-03 | 4.23E-01 | 5.77E-01 | 1.00 |
| | 18.5 | 0.0917 | -1 | 2.7617 | 4.73E-02 | 9.52E-03 | -1.82E-03 | 4.73E-01 | 5.26E-01 | 1.00 |
| | 19 | 0.1165 | -1 | 2.5096 | 5.18E-02 | 8.66E-03 | -1.66E-03 | 5.18E-01 | 4.82E-01 | 1.00 |
| | 19.5 | 0.1435 | -1 | 2.2791 | 5.59E-02 | 7.86E-03 | -1.53E-03 | 5.59E-01 | 4.43E-01 | 1.00 |
| 5 | 20 | 0.1724 | -1 | 2.0682 | 5.97E-02 | 7.13E-03 | -1.38E-03 | 5.97E-01 | 4.01E-01 | 1.00 |
| | 20.5 | 0.2031 | -1 | 1.8752 | 6.31E-02 | 6.47E-03 | -1.27E-03 | 6.31E-01 | 3.69E-01 | 1.00 |
| | 21 | 0.2354 | -1 | 1.6984 | 6.62E-02 | 5.86E-03 | -1.17E-03 | 6.62E-01 | 3.39E-01 | 1.00 |
| | 21.5 | 0.2692 | -1 | 1.5362 | 6.89E-02 | 5.30E-03 | -1.07E-03 | 6.89E-01 | 3.11E-01 | 1.00 |
| | 22 | 0.3043 | -1 | 1.3873 | 7.15E-02 | 4.78E-03 | -9.85E-04 | 7.15E-01 | 2.86E-01 | 1.00 |
| | 22.5 | 0.3406 | -1 | 1.2504 | 7.37E-02 | 4.31E-03 | -9.06E-04 | 7.37E-01 | 2.63E-01 | 1.00 |
| | 23 | 0.378 | -1 | 1.1242 | 7.58E-02 | 3.88E-03 | -8.34E-04 | 7.58E-01 | 2.42E-01 | 1.00 |
| | 23.5 | 0.4163 | -1 | 1.0078 | 7.76E-02 | 3.48E-03 | -7.71E-04 | 7.76E-01 | 2.24E-01 | 1.00 |
| | 24 | 0.4556 | -1 | 0.9 | 7.93E-02 | 3.10E-03 | -7.15E-04 | 7.93E-01 | 2.07E-01 | 1.00 |
| | 24.5 | 0.4956 | -1 | 0.8 | 8.07E-02 | 2.76E-03 | -6.67E-04 | 8.07E-01 | 1.93E-01 | 1.00 |
| 6 | 25 | 0.5363 | -1 | 0.7069 | 8.20E-02 | 2.44E-03 | -6.18E-04 | 8.20E-01 | 1.79E-01 | 1.00 |
| | 25.5 | 0.5776 | -1 | 0.6199 | 8.32E-02 | 2.14E-03 | -5.80E-04 | 8.32E-01 | 1.68E-01 | 1.00 |
| | 26 | 0.6194 | -1 | 0.5382 | 8.42E-02 | 1.86E-03 | -5.47E-04 | 8.42E-01 | 1.58E-01 | 1.00 |
| | 26.5 | 0.6617 | -1 | 0.4612 | 8.50E-02 | 1.59E-03 | -5.17E-04 | 8.50E-01 | 1.50E-01 | 1.00 |
| | 27 | 0.7044 | -1 | 0.3882 | 8.58E-02 | 1.34E-03 | -4.92E-04 | 8.58E-01 | 1.43E-01 | 1.00 |
| | 27.5 | 0.7474 | -1 | 0.3185 | 8.64E-02 | 1.10E-03 | -4.70E-04 | 8.64E-01 | 1.36E-01 | 1.00 |
| | 28 | 0.7907 | -1 | 0.2516 | 8.69E-02 | 8.67E-04 | -4.53E-04 | 8.69E-01 | 1.31E-01 | 1.00 |
| | 28.5 | 0.8342 | -1 | 0.1868 | 8.72E-02 | 6.44E-04 | -4.40E-04 | 8.72E-01 | 1.28E-01 | 1.00 |
| | 29 | 0.8779 | -1 | 0.1236 | 8.75E-02 | 4.27E-04 | -4.31E-04 | 8.75E-01 | 1.25E-01 | 1.00 |
| | 29.5 | 0.9217 | -1 | 0.0616 | 8.77E-02 | 2.12E-04 | -4.26E-04 | 8.77E-01 | 1.24E-01 | 1.00 |
| | 30 | 0.9656 | -1 | 0 | 8.77E-02 | -4.35E-07 | -4.26E-04 | 8.77E-01 | 1.23E-01 | 1.00 |

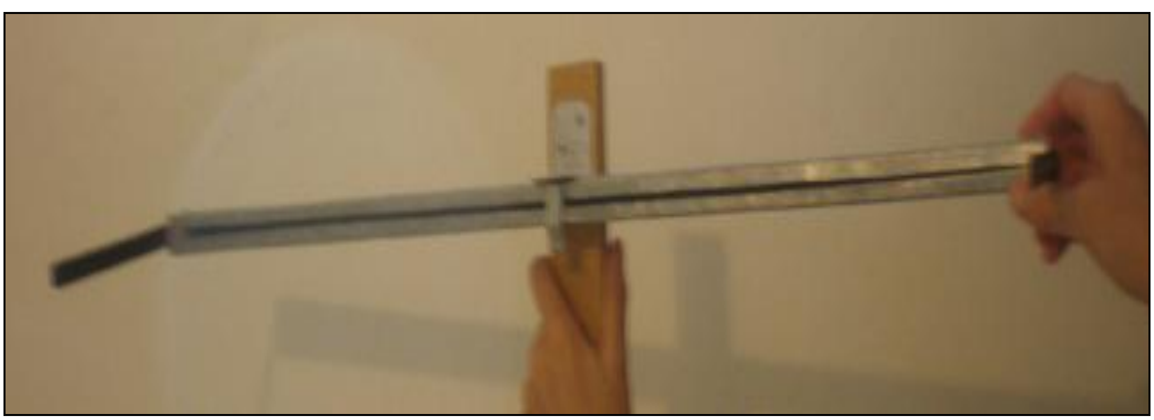
## APPENDIX B

## BAR RESTRAINED AGAINST TWIST AT MIDSPAN

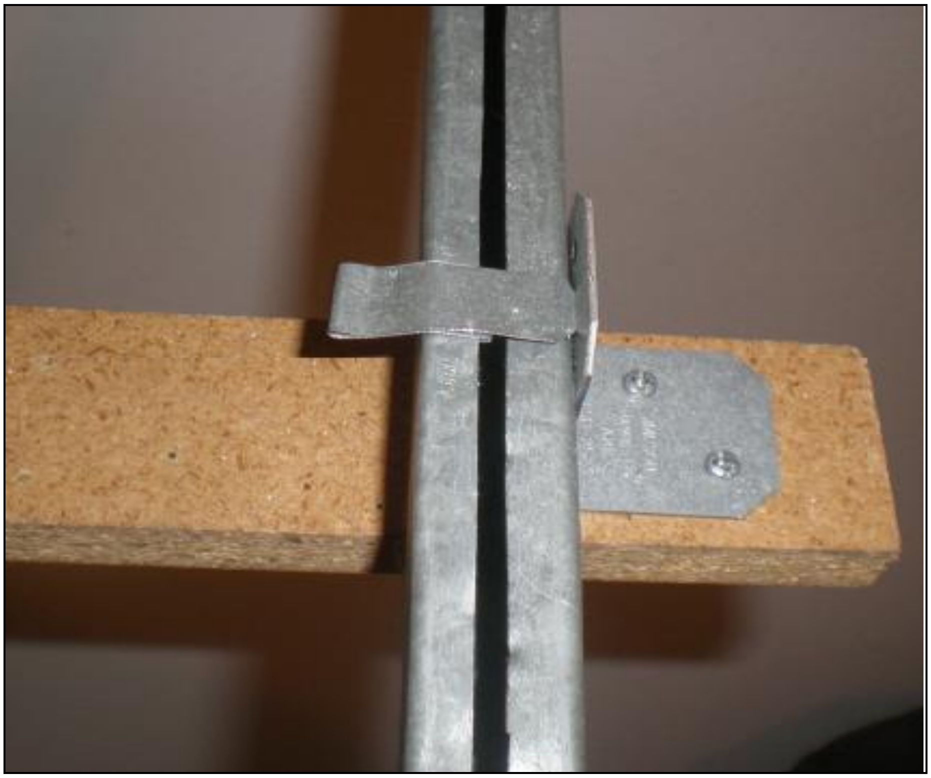
The present material contents the input model, input data, input forms, figures of output data in notepad version, and excel processed output data and charts for the corresponding case. The asymptotic behavior of the thin-walled beam elastic line is successfully shown in the charts, which evidences the efficiency of the high order finite element used by BMTORSWP. The cross section shown in the input form is not that of the real bar in the parametric experiment. It just provides an order of magnitude for the bogus cross section torsional properties in order to make verisimilar the input data.



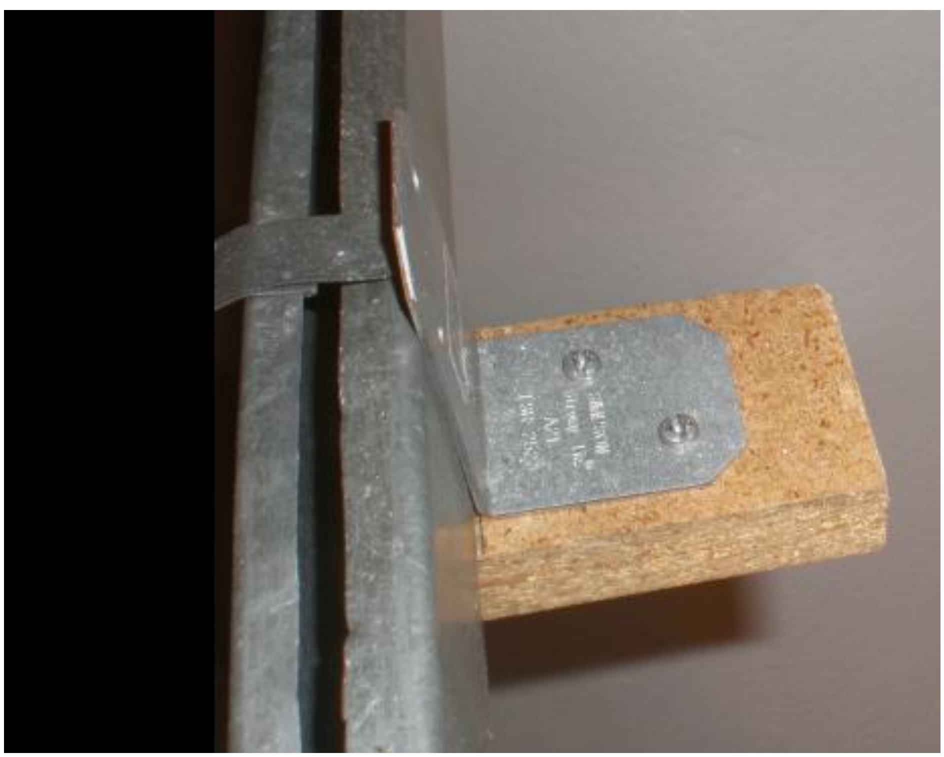
Straight TWB Restrained against Rotation at Midspan before Twisting Action



Straight TWB Restrained against Rotation at Midspan about to be Twisted

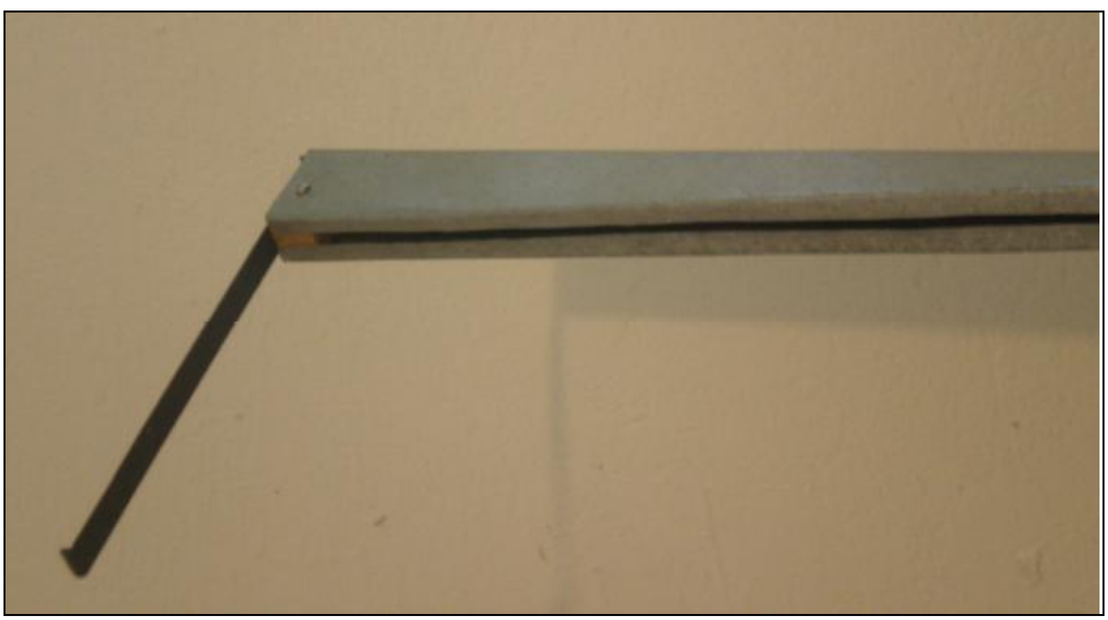


A Square Fastener at Right Side and a Separator at Top Prevent Rotation

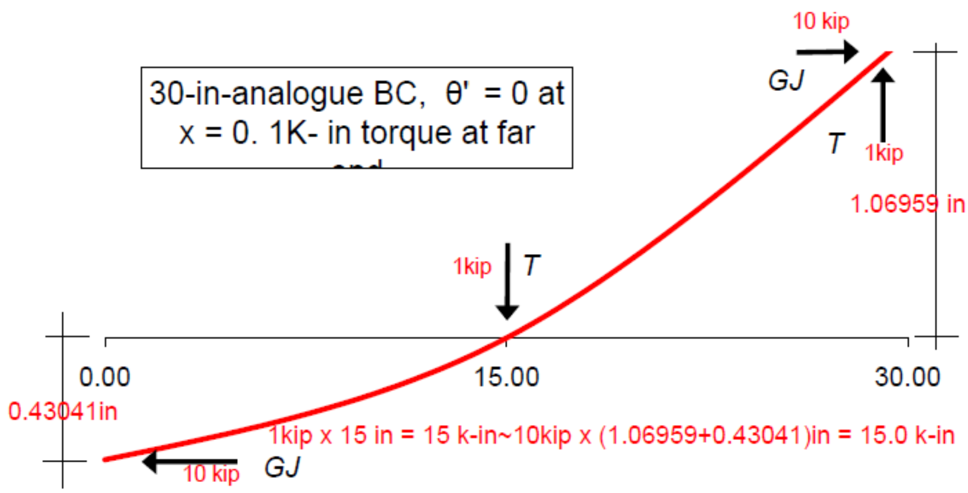


Detail of Right Fastener

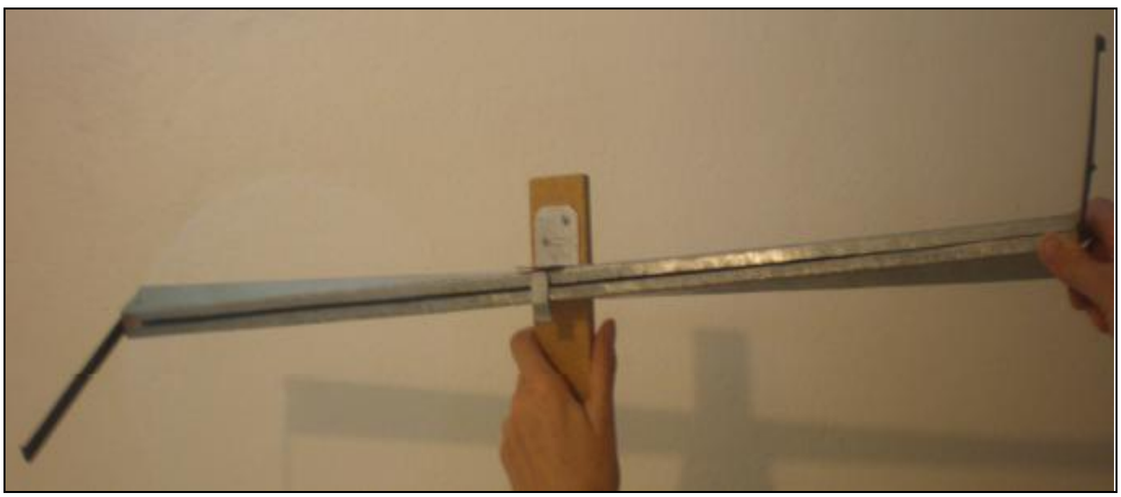
At Both Edges, Cross Section Distortion Prevented by a Piece of Wood Only Fastened at One Side with Centerline Guides for Measuring the Twist Angle



Elastic Line Detail at Free Near End without External Torque




Analogue EBC in 2nd Order Equilibrium with a 1 kip Reaction at Midspan Support



Zero External Torque with Restrained Warping at First Span Tw +Tt = T = 0.
Tw & Tt ≠ 0

```
                              6EL
6 Elements' beam. Torque angle restrained ad midspan. 1K-in torque at far end
  6 13  1     9  0  1  0  29000.
  1  1  2  3 0.03    1.   5.                         -10.
  2  3  4  5 0.03    1.   5.                         -10.
  3  5  6  7 0.03    1.   5.                  - GJ   -10.
  4  7  8  9 0.03    1.   5.                         -10.
  5  9 10 11 0.03    1.   5.                         -10.
  6 11 12 13 0.03    1.   5.                         -10.
 1 7 0 1 0 <-------------KX=KZ=0, KY=1 in Node 7
 113   1. <-- Total Torque
```

Input Data

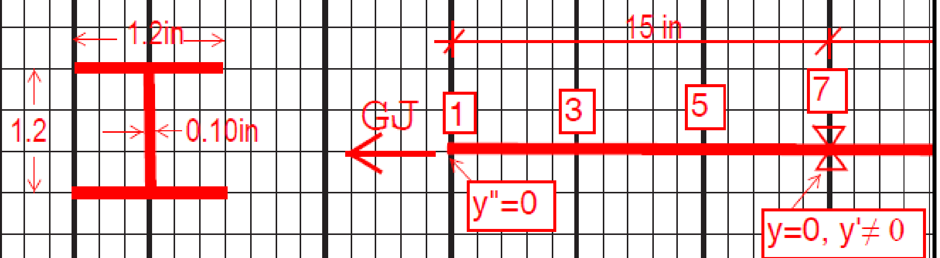
## PROGRAM BMTORSW INPUT FORM

ALPHAMERIC DESCRIPTION OF THE JOB    FILE NAME:

Beam with torsion angle restrained at midspan subjected

| NEL | NOD | NSUP | NSPD | JBW | NFX | NFY | NFZ | ELASTICITY (E11.4) | GENE |
|---|---|---|---|---|---|---|---|---|---|
| 6 | 13 | 1 | | 9 | 0 | 1 | 0 | 2900 0. | |

| NE | NI | NC | NJ | Cw | AREA | LENGTH | WTANG | SOIL NI | SOIL NJ |
|---|---|---|---|---|---|---|---|---|---|
| 1 | 1 | 2 | 3 | 0,03 | 1. | 5. | | | |
| 2 | 3 | 4 | 5 | 0,03 | 1. | 5. | | | |
| 3 | 5 | 6 | 7 | 0,03 | 1. | 5. | | | |
| 4 | 7 | 8 | 9 | 0,03 | 1. | 5. | | | |
| 5 | 9 | 10 | 11 | 0,03 | 1. | 5. | | | |
| 6 | 11 | 12 | 13 | 0,03 | 1. | 5. | | | |



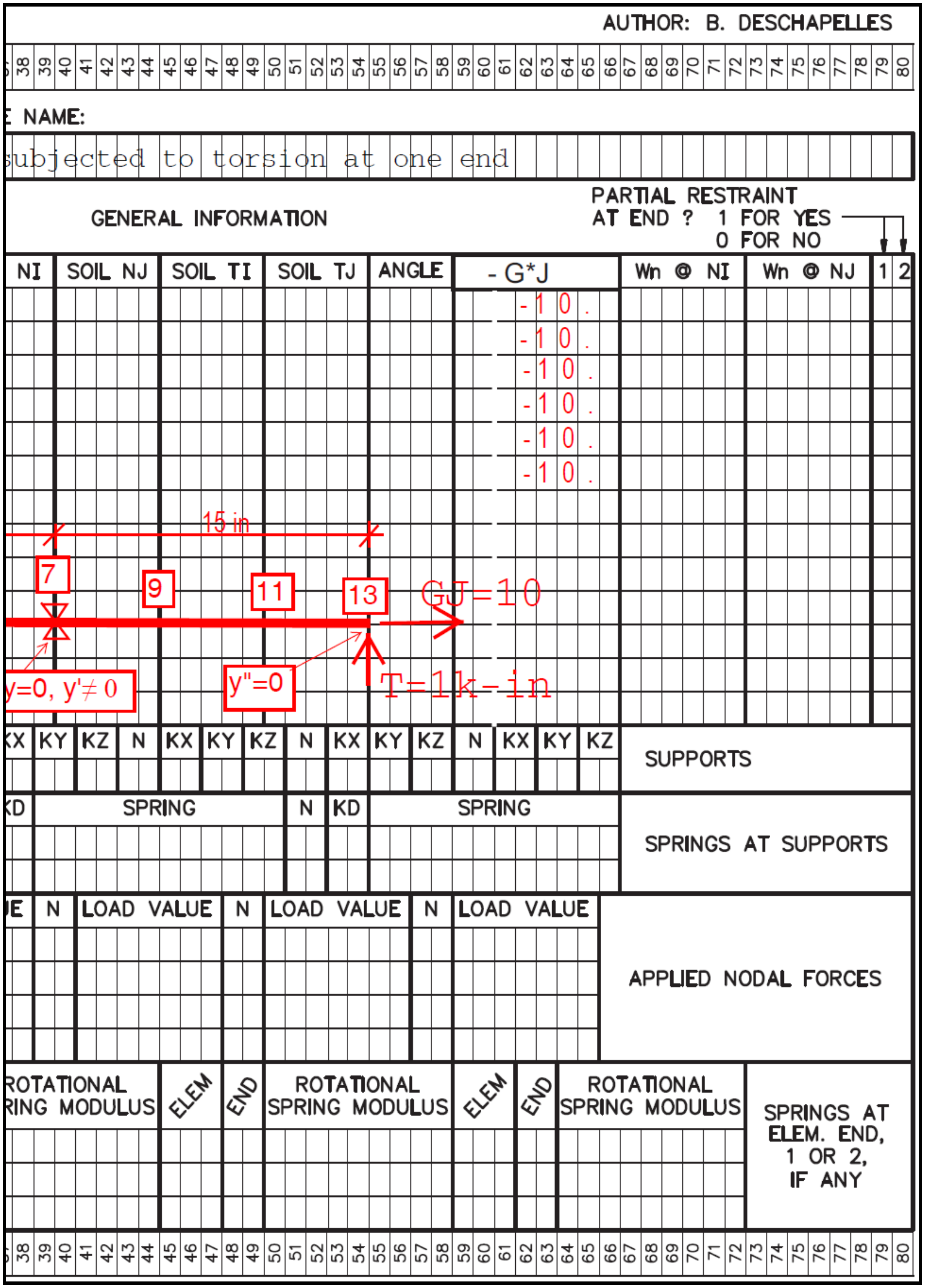

| NF | N | KX | KY | KZ | N | KX | KY | KZ | N | KX | KY | KZ | N | KX | KY | KZ | N | KX | KY | KZ | N |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| 1 | 7 | 0 | 1 | 0 | | | | | | | | | | | | | | | | | |

| NF | N | KD | SPRING | N | KD | SPRING | N | KD | SP |
|---|---|---|---|---|---|---|---|---|---|
| | | | | | | | | | |

| NF | N | LOAD VALUE | N | LOAD VALUE | N | LOAD VALUE | N | LOAD VALUE | N | LOAD |
|---|---|---|---|---|---|---|---|---|---|---|
| 1 | 13 | 1. | | | | | | | | |

| NF | ELEM | END | ROTATIONAL SPRING MODULUS | ELEM | END | ROTATIONAL SPRING MODULUS | ELEM | END | ROTATIONAL SPRING MODULU |
|---|---|---|---|---|---|---|---|---|---|
| | | | | | | | | | |

Input Form with Input Model, Left Side




Input Form with Input Model, Right Side

```
                                   6eout
 ------------------------------------------------------------------------
 YOU ARE USING COMPUTER PROGRAM BMTORSW,DEVELOPED BY DR. BERNARDO DESCHAPELLES
0        INPUT DATA FILE NAME IS = 6el.txt
0                OUTPUT FILE NAME IS = 6eout.txt
0          STORAGE FILE FOR POST-PROCESSING WITH EXCEL = 6egrf.grf

0------------------------------------------------------------------------
 6 Elements' beam. Torque angle restrained ad midspan. 1K-in torque at far end
0modulus of elasticity of the material=   29000. k/ft2
0ELEM nodes inertia length distrib. load  AXIAL  SOIL NORMAL MODULUS,Ksf  angle
      i   j  ft.4     ft     at i   at j  LOAD    1st END      2nd END    rad
  1   1   3 0.03000  5.00  0.000  0.000 -10.00      0.0          0.0   0.000 00
  2   3   5 0.03000  5.00  0.000  0.000 -10.00      0.0          0.0   0.000 00
  3   5   7 0.03000  5.00  0.000  0.000 -10.00      0.0          0.0   0.000 00
  4   7   9 0.03000  5.00  0.000  0.000 -10.00      0.0          0.0   0.000 00
  5   9  11 0.03000  5.00  0.000  0.000 -10.00      0.0          0.0   0.000 00
  6  11  13 0.03000  5.00  0.000  0.000 -10.00      0.0          0.0   0.000 00
0    INPUT DATA RELATED TO THE  1 SUPPORTS
  1 7 0 1 0
0    INPUT OF NODAL FORCES RELATED TO GLOBAL AXIS  2
         113   1.00
0      FINAL  SOLUTION  FOUND  AFTER    1   ITERATIONS
0     Output of nodal displacements in reference to  global axes
0node  displ.      displ.      displ.    node  displ.      displ.      displ.
     along  x    along  y    around  z        along  x    along  y    around  z
     or nonn1    or nonn2    or nonn 3        or nonn1    or nonn2    or nonn 3

+  1 0.0000E+00 -0.4304E+00  0.1925E-01
+                                         2 0.0000E+00 -0.3811E+00  0.8372E-02

+  3 0.0000E+00 -0.3295E+00  0.2209E-01
+                                         4 0.0000E+00 -0.2680E+00  0.1084E-01

+  5 0.0000E+00 -0.1988E+00  0.3142E-01
+                                         6 0.0000E+00 -0.1071E+00  0.1649E-01

+  7 0.0000E+00  0.0000E+00  0.5000E-01
+                                         8 0.0000E+00  0.1429E+00  0.2518E-01

+  9 0.0000E+00  0.3012E+00  0.6858E-01
+                                        10 0.0000E+00  0.4820E+00  0.3083E-01

+ 11 0.0000E+00  0.6705E+00  0.7791E-01
+                                        12 0.0000E+00  0.8689E+00  0.3329E-01
                                                 1kip x 15 in = 15 k-in~10kip x
+ 13 0.0000E+00  0.1070E+01  0.8075E-01
0---------------------------------------------------------(1.06959+0.43041)in = 15.0
      OUTPUT OF SOIL REACTIONS,STRESSES AND TRANSVERSE DISPLACEMENTS
0----------------------------------------------------------------------
0     ELEMENT   1  DISPLACEMENTS IN INCIDENCES   1   2   3
         NODE   1          0.00000E+00        -0.43041E+00         0.19253E-01
         NODE   2          0.00000E+00        -0.38111E+00         0.83720E-02
         NODE   3          0.00000E+00        -0.32946E+00         0.22086E-01
0     FORCES ACTING ALONG THE 9 DOF
         NODE   1          0.00000E+00        -0.46896E-12        -0.95923E-13
         NODE   2          0.00000E+00        -0.16342E-12        -0.38654E-11
         NODE   3          0.00000E+00        -0.85265E-12         0.10094E+01
0ELEMENT   1, FROM NODE  1, TO NODE  3 - LENGTH =  5.00 ft
0 left half of span,at tenth points of length
               span        span        span        span        span        span
+           0.0         0.1         0.2         0.3         0.4         0.5
 soil,k/ft    0.000       0.000       0.000       0.000       0.000       0.000
 shear,k       0.00        0.00        0.00        0.00        0.00        0.00
                                 Page 1
```

Page 1 Output Data

```
                                                6eout
 bmom,kft      0.00      0.10      0.19      0.29      0.39      0.49
 tdisp,ft  -0.43041  -0.42078  -0.41112  -0.40140  -0.39160  -0.38170
 axial,k       0.00 AT 1st END and      0.00 AT 2nd END
0right half of span,at tenth points of length
               span      span      span      span      span      span
+          0.5       0.6       0.7       0.8       0.9       1.0
 soil,k/ft    0.000     0.000     0.000     0.000     0.000     0.000
 shear,k       0.00      0.00      0.00      0.00      0.00      0.00
 bmom,kft      0.49      0.59      0.69      0.79      0.90      1.01
 tdisp,ft  -0.38170  -0.37165  -0.36143  -0.35101  -0.34037  -0.32946
 axial,k       0.00 AT 1st END and      0.00 AT 2nd END
0-----------------------------------------------------------------------
0      ELEMENT    2  DISPLACEMENTS IN INCIDENCES    3   4   5
          NODE    3         0.00000E+00        -0.32946E+00         0.22086E-01
          NODE    4         0.00000E+00        -0.26801E+00         0.10836E-01
          NODE    5         0.00000E+00        -0.19881E+00         0.31420E-01
0      FORCES ACTING ALONG THE 9 DOF
          NODE    3         0.00000E+00        -0.21316E-12        -0.10094E+01
          NODE    4         0.00000E+00        -0.56843E-13         0.34106E-11
          NODE    5         0.00000E+00        -0.28422E-13         0.23160E+01
0ELEMENT   2, FROM NODE   3, TO NODE   5 - LENGTH =   5.00 ft
0 left half of span,at tenth points of length
               span      span      span      span      span      span
+          0.0       0.1       0.2       0.3       0.4       0.5
 soil,k/ft    0.000     0.000     0.000     0.000     0.000     0.000
 shear,k       0.00      0.00      0.00      0.00      0.00      0.00
 bmom,kft      1.01      1.12      1.24      1.36      1.48      1.60
 tdisp,ft  -0.32946  -0.31827  -0.30675  -0.29488  -0.28262  -0.26994
 axial,k       0.00 AT 1st END and      0.00 AT 2nd END
0right half of span,at tenth points of length
               span      span      span      span      span      span
+          0.5       0.6       0.7       0.8       0.9       1.0
 soil,k/ft    0.000     0.000     0.000     0.000     0.000     0.000
 shear,k       0.00      0.00      0.00      0.00      0.00      0.00
 bmom,kft      1.60      1.74      1.87      2.01      2.16      2.32
 tdisp,ft  -0.26994  -0.25679  -0.24314  -0.22896  -0.21420  -0.19881
 axial,k       0.00 AT 1st END and      0.00 AT 2nd END
0-----------------------------------------------------------------------
0      ELEMENT    3  DISPLACEMENTS IN INCIDENCES    5   6   7
          NODE    5         0.00000E+00        -0.19881E+00         0.31420E-01
          NODE    6         0.00000E+00        -0.10711E+00         0.16489E-01
          NODE    7         0.00000E+00         0.00000E+00         0.50000E-01
0      FORCES ACTING ALONG THE 9 DOF
          NODE    5         0.00000E+00        -0.99476E-13        -0.23160E+01
          NODE    6         0.00000E+00         0.42633E-13        -0.22737E-12
          NODE    7         0.00000E+00        -0.28422E-13         0.43041E+01
0ELEMENT   3, FROM NODE   5, TO NODE   7 - LENGTH =   5.00 ft
0 left half of span,at tenth points of length
               span      span      span      span      span      span
+          0.0       0.1       0.2       0.3       0.4       0.5
 soil,k/ft    0.000     0.000     0.000     0.000     0.000     0.000
 shear,k       0.00      0.00      0.00      0.00      0.00      0.00
 bmom,kft      2.32      2.48      2.64      2.82      3.00      3.19
 tdisp,ft  -0.19881  -0.18276  -0.16600  -0.14848  -0.13014  -0.11095
 axial,k       0.00 AT 1st END and      0.00 AT 2nd END
0right half of span,at tenth points of length
               span      span      span      span      span      span
+          0.5       0.6       0.7       0.8       0.9       1.0
 soil,k/ft    0.000     0.000     0.000     0.000     0.000     0.000
 shear,k       0.00      0.00      0.00      0.00      0.00      0.00
 bmom,kft      3.19      3.40      3.61      3.83      4.06      4.30
 tdisp,ft  -0.11095  -0.09084  -0.06975  -0.04762  -0.02439   0.00000
 axial,k       0.00 AT 1st END and      0.00 AT 2nd END
                                               Page 2
```

0 -(-0.19881)*10 +0*5= 1.988

4.30- 2.32 = 1.98  OK

(-Di + Dj)* GJ +  Vj*L = Mj - Mi, where GJ=10

Page 2 Output Data

```
0-------------------------------------------------------------------------
0      ELEMENT   4  DISPLACEMENTS IN INCIDENCES    7    8    9
         NODE   7         0.00000E+00          0.00000E+00          0.50000E-01
         NODE   8         0.00000E+00          0.14289E+00          0.25178E-01
         NODE   9         0.00000E+00          0.30119E+00          0.68580E-01
0      FORCES ACTING ALONG THE 9 DOF
         NODE   7         0.00000E+00         -0.10000E+01         -0.43041E+01
         NODE   8         0.00000E+00          0.14211E-12          0.68212E-12
         NODE   9         0.00000E+00          0.10000E+01          0.23160E+01
0ELEMENT   4, FROM NODE   7, TO NODE   9 - LENGTH =   5.00 ft
0 left half of span,at tenth points of length
               span       span       span       span       span       span
+          0.0        0.1        0.2        0.3        0.4        0.5
 soil,k/ft     0.000      0.000      0.000      0.000      0.000      0.000
 shear,k      -1.00      -1.00      -1.00      -1.00      -1.00      -1.00
 bmom,kft      4.30       4.06       3.83       3.61       3.40       3.19
 tdisp,ft    0.00000    0.02561    0.05238    0.08025    0.10916    0.13905
 axial,k        0.00 AT 1st END and       0.00 AT 2nd END
0right half of span,at tenth points of length
               span       span       span       span       span       span
+          0.5        0.6        0.7        0.8        0.9        1.0
 soil,k/ft     0.000      0.000      0.000      0.000      0.000      0.000
 shear,k      -1.00      -1.00      -1.00      -1.00      -1.00      -1.00
 bmom,kft      3.19       3.00       2.82       2.64       2.48       2.32
 tdisp,ft    0.13905    0.16986    0.20152    0.23400    0.26724    0.30119
 axial,k        0.00 AT 1st END and       0.00 AT 2nd END
0-------------------------------------------------------------------------
0      ELEMENT   5  DISPLACEMENTS IN INCIDENCES    9   10   11
         NODE   9         0.00000E+00          0.30119E+00          0.68580E-01
         NODE  10         0.00000E+00          0.48199E+00          0.30831E-01
         NODE  11         0.00000E+00          0.67054E+00          0.77914E-01
0      FORCES ACTING ALONG THE 9 DOF
         NODE   9         0.00000E+00         -0.10000E+01         -0.23160E+01
         NODE  10         0.00000E+00         -0.34106E-12          0.22737E-11
         NODE  11         0.00000E+00          0.10000E+01          0.10094E+01
0ELEMENT   5, FROM NODE   9, TO NODE 11 - LENGTH =   5.00 ft
0 left half of span,at tenth points of length
               span       span       span       span       span       span
+          0.0        0.1        0.2        0.3        0.4        0.5
 soil,k/ft     0.000      0.000      0.000      0.000      0.000      0.000
 shear,k      -1.00      -1.00      -1.00      -1.00      -1.00      -1.00
 bmom,kft      2.32       2.16       2.01       1.87       1.74       1.60
 tdisp,ft    0.30119    0.33580    0.37104    0.40686    0.44321    0.48006
 axial,k        0.00 AT 1st END and       0.00 AT 2nd END
0right half of span,at tenth points of length
               span       span       span       span       span       span
+          0.5        0.6        0.7        0.8        0.9        1.0
 soil,k/ft     0.000      0.000      0.000      0.000      0.000      0.000
 shear,k      -1.00      -1.00      -1.00      -1.00      -1.00      -1.00
 bmom,kft      1.60       1.48       1.36       1.24       1.12       1.01
 tdisp,ft    0.48006    0.51738    0.55512    0.59325    0.63173    0.67054
 axial,k        0.00 AT 1st END and       0.00 AT 2nd END
0-------------------------------------------------------------------------
0      ELEMENT   6  DISPLACEMENTS IN INCIDENCES   11   12   13
         NODE  11         0.00000E+00          0.67054E+00          0.77914E-01
         NODE  12         0.00000E+00          0.86889E+00          0.33295E-01
         NODE  13         0.00000E+00          0.10696E+01          0.80747E-01
0      FORCES ACTING ALONG THE 9 DOF
         NODE  11         0.00000E+00         -0.10000E+01         -0.10094E+01
         NODE  12         0.00000E+00         -0.51159E-12         -0.10914E-10
         NODE  13         0.00000E+00          0.10000E+01         -0.68212E-12
0ELEMENT   6, FROM NODE 11, TO NODE 13 - LENGTH =   5.00 ft
0 left half of span,at tenth points of length
```



Page 3 Output Data

```
                                                   6eout
                 span        span        span        span        span        span
+           0.0         0.1         0.2         0.3         0.4         0.5
 soil,k/ft     0.000       0.000       0.000       0.000       0.000       0.000
 shear,k       -1.00       -1.00       -1.00       -1.00       -1.00       -1.00
 bmom,kft       1.01        0.90        0.79        0.69        0.59        0.49
 tdisp,ft    0.67054     0.70963     0.74899     0.78857     0.82835     0.86830
 axial,k        0.00 AT 1st END and       0.00 AT 2nd END
Oright half of span,at tenth points of length
                 span        span        span        span        span        span
+           0.5         0.6         0.7         0.8         0.9         1.0
 soil,k/ft     0.000       0.000       0.000       0.000       0.000       0.000
 shear,k       -1.00       -1.00       -1.00       -1.00       -1.00       -1.00
 bmom,kft       0.49        0.39        0.29        0.19        0.10        0.00
 tdisp,ft    0.86830     0.90840     0.94860     0.98888     1.02922     1.06959
 axial,k        0.00 AT 1st END and       0.00 AT 2nd END
1
```

1kip x 15 in = 15 k-in~10kip x (1.06959+0.43041)in = 15.0 k-in

T L GJ $\Phi_{13}$ $-\Phi_1$

Last Page Output Data

Table of Bimoment, Torque and Torque Angle and Derivatives

| | | | | | | | |
|---|---|---|---|---|---|---|---|
| | | | | GJ | 10 | ksi-in^2 | |
| 6 Elements-beam. Fork ad midspan. 1K-in torque at far end | | | | | | | |
| | | | | E | 29000 | ksi/in^2 | |
| | | | | Cw | 0.030 | in^6 | |
| elm | Z | Φ | Torque | Bimoment | Φ' | Φ" | Φ''' |
| 1 | 0.00 | -0.43 | 0.00 | 0.000 | 1.925E-02 | -2.452E-08 | 2.214E-04 |
| | 0.50 | -0.42 | 0 | 0.096 | 1.928E-02 | 1.107E-04 | 2.217E-04 |
| | 1.00 | -0.41 | 0 | 0.193 | 1.936E-02 | 2.217E-04 | 2.226E-04 |
| | 1.50 | -0.4 | 0 | 0.290 | 1.950E-02 | 3.334E-04 | 2.242E-04 |
| | 2.00 | -0.39 | 0 | 0.388 | 1.970E-02 | 4.460E-04 | 2.264E-04 |
| | 2.50 | -0.38 | 0 | 0.487 | 1.995E-02 | 5.599E-04 | 2.293E-04 |
| | 3.00 | -0.37 | 0 | 0.588 | 2.026E-02 | 6.754E-04 | 2.329E-04 |
| | 3.50 | -0.36 | 0 | 0.690 | 2.062E-02 | 7.929E-04 | 2.371E-04 |
| | 4.00 | -0.35 | 0 | 0.794 | 2.105E-02 | 9.126E-04 | 2.420E-04 |
| | 4.50 | -0.34 | 0 | 0.900 | 2.154E-02 | 1.035E-03 | 2.475E-04 |
| | 5.00 | -0.33 | 0 | 1.009 | 2.209E-02 | 1.160E-03 | 2.537E-04 |
| 2 | | | | | | | |
| | 5.00 | -0.33 | 0 | 1.009 | 2.209E-02 | 1.160E-03 | 2.542E-04 |
| | 5.50 | -0.32 | 0 | 1.121 | 2.270E-02 | 1.289E-03 | 2.610E-04 |
| | 6.00 | -0.31 | 0 | 1.237 | 2.338E-02 | 1.421E-03 | 2.687E-04 |
| | 6.50 | -0.29 | 0 | 1.355 | 2.412E-02 | 1.558E-03 | 2.772E-04 |

| elm | Z | Φ | T | Bimom | Φ' | Φ'' | Φ''' |
|---|---|---|---|---|---|---|---|
| | 7.00 | -0.28 | 0 | 1.478 | 2.493E-02 | 1.699E-03 | 2.866E-04 |
| | 7.50 | -0.27 | 0 | 1.605 | 2.582E-02 | 1.844E-03 | 2.968E-04 |
| | 8.00 | -0.26 | 0 | 1.736 | 2.678E-02 | 1.996E-03 | 3.079E-04 |
| | 8.50 | -0.24 | 0 | 1.873 | 2.782E-02 | 2.152E-03 | 3.198E-04 |
| | 9.00 | -0.23 | 0 | 2.015 | 2.893E-02 | 2.316E-03 | 3.326E-04 |
| | 9.50 | -0.21 | 0 | 2.162 | 3.013E-02 | 2.485E-03 | 3.462E-04 |
| | 10.00 | -0.2 | 0 | 2.316 | 3.142E-02 | 2.662E-03 | 3.608E-04 |
| 3 | | | | | | | |
| | 10.00 | -0.2 | 0 | 2.316 | 3.142E-02 | 2.662E-03 | 3.618E-04 |
| | 10.50 | -0.18 | 0 | 2.477 | 3.280E-02 | 2.847E-03 | 3.772E-04 |
| | 11.00 | -0.17 | 0 | 2.644 | 3.427E-02 | 3.039E-03 | 3.938E-04 |
| | 11.50 | -0.15 | 0 | 2.819 | 3.584E-02 | 3.241E-03 | 4.118E-04 |
| | 12.00 | -0.13 | 0 | 3.003 | 3.751E-02 | 3.451E-03 | 4.310E-04 |
| | 12.50 | -0.11 | 0 | 3.195 | 3.929E-02 | 3.672E-03 | 4.516E-04 |
| | 13.00 | -0.09 | 0 | 3.396 | 4.118E-02 | 3.903E-03 | 4.735E-04 |
| | 13.50 | -0.07 | 0 | 3.607 | 4.320E-02 | 4.146E-03 | 4.966E-04 |
| | 14.00 | -0.05 | 0 | 3.828 | 4.533E-02 | 4.400E-03 | 5.211E-04 |
| | 14.50 | -0.02 | 0 | 4.060 | 4.760E-02 | 4.667E-03 | 5.469E-04 |
| | 15.00 | 0 | 0 | 4.304 | 5.000E-02 | 4.947E-03 | 5.739E-04 |
| 4 | | | | | | | |
| | 15.00 | 0 | -1 | 4.304 | 5.000E-02 | 4.947E-03 | -5.739E-04 |
| | 15.50 | 0.026 | -1 | 4.060 | 5.240E-02 | 4.667E-03 | -5.469E-04 |
| | 16.00 | 0.052 | -1 | 3.828 | 5.467E-02 | 4.400E-03 | -5.211E-04 |
| | 16.50 | 0.08 | -1 | 3.607 | 5.680E-02 | 4.146E-03 | -4.966E-04 |
| | 17.00 | 0.109 | -1 | 3.396 | 5.882E-02 | 3.903E-03 | -4.735E-04 |
| | 17.50 | 0.139 | -1 | 3.195 | 6.071E-02 | 3.672E-03 | -4.516E-04 |
| | 18.00 | 0.17 | -1 | 3.003 | 6.249E-02 | 3.451E-03 | -4.310E-04 |
| | 18.50 | 0.202 | -1 | 2.819 | 6.416E-02 | 3.241E-03 | -4.118E-04 |
| | 19.00 | 0.234 | -1 | 2.644 | 6.573E-02 | 3.039E-03 | -3.938E-04 |
| | 19.50 | 0.267 | -1 | 2.477 | 6.720E-02 | 2.847E-03 | -3.772E-04 |
| | 20.00 | 0.301 | -1 | 2.316 | 6.858E-02 | 2.662E-03 | -3.618E-04 |
| 5 | | | | | | | |
| | 20.00 | 0.301 | -1 | 2.316 | 6.858E-02 | 2.662E-03 | -3.608E-04 |
| | 20.50 | 0.336 | -1 | 2.162 | 6.987E-02 | 2.485E-03 | -3.462E-04 |
| | 21.00 | 0.371 | -1 | 2.015 | 7.107E-02 | 2.316E-03 | -3.326E-04 |
| | 21.50 | 0.407 | -1 | 1.873 | 7.218E-02 | 2.152E-03 | -3.198E-04 |
| | 22.00 | 0.443 | -1 | 1.736 | 7.322E-02 | 1.996E-03 | -3.079E-04 |

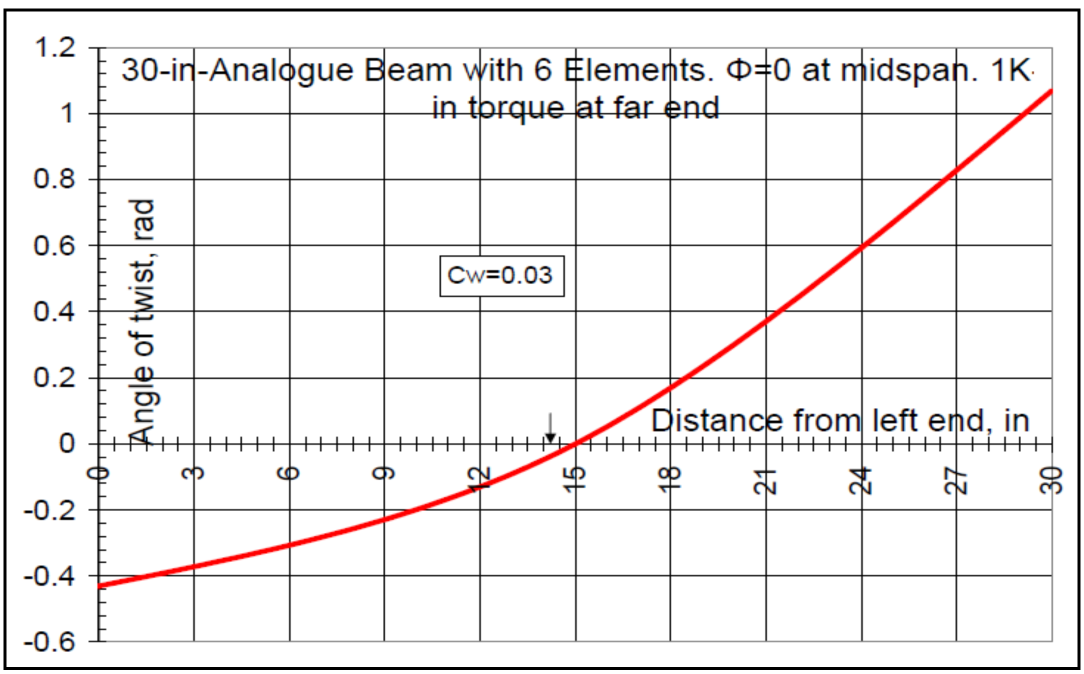
| elm | Z | Φ | T | Bimom | Φ' | Φ'' | Φ''' |
|---|---|---|---|---|---|---|---|
| | 22.50 | 0.48 | -1 | 1.605 | 7.418E-02 | 1.844E-03 | -2.968E-04 |
| | 23.00 | 0.517 | -1 | 1.478 | 7.507E-02 | 1.699E-03 | -2.866E-04 |
| | 23.50 | 0.555 | -1 | 1.355 | 7.588E-02 | 1.558E-03 | -2.772E-04 |
| | 24.00 | 0.593 | -1 | 1.237 | 7.662E-02 | 1.421E-03 | -2.687E-04 |
| | 24.50 | 0.632 | -1 | 1.121 | 7.730E-02 | 1.289E-03 | -2.610E-04 |
| | 25.00 | 0.671 | -1 | 1.009 | 7.791E-02 | 1.160E-03 | -2.542E-04 |
| 6 | | | | | | | |
| | 25.00 | 0.671 | -1 | 1.009 | 7.791E-02 | 1.160E-03 | -2.537E-04 |
| | 25.50 | 0.71 | -1 | 0.900 | 7.846E-02 | 1.035E-03 | -2.475E-04 |
| | 26.00 | 0.749 | -1 | 0.794 | 7.895E-02 | 9.126E-04 | -2.420E-04 |
| | 26.50 | 0.789 | -1 | 0.690 | 7.938E-02 | 7.929E-04 | -2.371E-04 |
| | 27.00 | 0.828 | -1 | 0.588 | 7.974E-02 | 6.754E-04 | -2.329E-04 |
| | 27.50 | 0.868 | -1 | 0.487 | 8.005E-02 | 5.599E-04 | -2.293E-04 |
| | 28.00 | 0.908 | -1 | 0.388 | 8.030E-02 | 4.460E-04 | -2.264E-04 |
| | 28.50 | 0.949 | -1 | 0.290 | 8.050E-02 | 3.334E-04 | -2.242E-04 |
| | 29.00 | 0.989 | -1 | 0.193 | 8.064E-02 | 2.217E-04 | -2.226E-04 |
| | 29.50 | 1.029 | -1 | 0.096 | 8.072E-02 | 1.107E-04 | -2.217E-04 |
| | 30.00 | 1.07 | -1 | 0.000 | 8.075E-02 | -2.452E-08 | -2.214E-04 |

Table of Excel Processed Data on Torques

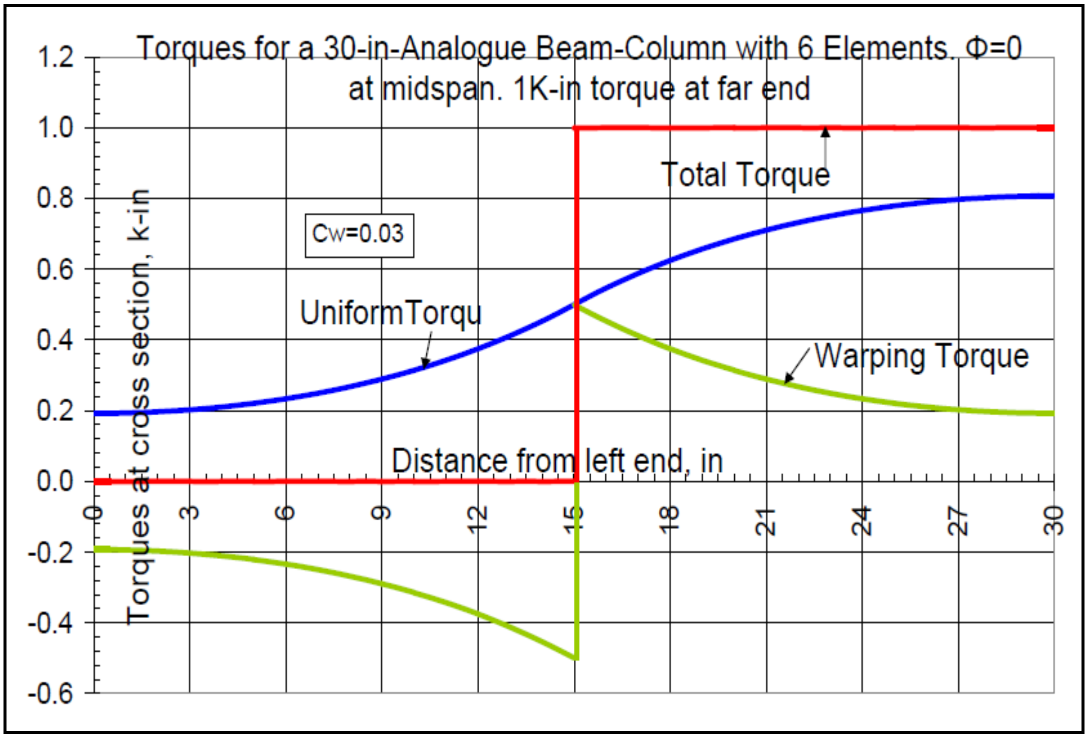
| 1st. order General Torsion Theory | | | | |
|---|---|---|---|---|
| elm | Z | GJΦ' | -ECwΦ''' | T |
| 1 | | St. Venant | Vlasov | Total |
| | 0.00 | 1.925E-01 | -1.926E-01 | -1.180E-04 |
| | 0.50 | 1.928E-01 | -1.929E-01 | -7.900E-05 |
| | 1.00 | 1.936E-01 | -1.937E-01 | -6.200E-05 |
| | 1.50 | 1.950E-01 | -1.951E-01 | -5.400E-05 |
| | 2.00 | 1.970E-01 | -1.970E-01 | 3.200E-05 |
| | 2.50 | 1.995E-01 | -1.995E-01 | 9.000E-06 |
| | 3.00 | 2.026E-01 | -2.026E-01 | -2.300E-05 |
| | 3.50 | 2.062E-01 | -2.063E-01 | -7.700E-05 |
| | 4.00 | 2.105E-01 | -2.105E-01 | -4.000E-05 |
| | 4.50 | 2.154E-01 | -2.153E-01 | 7.500E-05 |
| | 5.00 | 2.209E-01 | -2.207E-01 | 1.810E-04 |
| 2 | | | | |
| | 5.00 | 2.209E-01 | -2.212E-01 | -2.540E-04 |

| elm | Z | GJΦ' | -ECwΦ''' | T |
|---|---|---|---|---|
| | 5.50 | 2.270E-01 | -2.271E-01 | -7.000E-05 |
| | 6.00 | 2.338E-01 | -2.338E-01 | 3.100E-05 |
| | 6.50 | 2.412E-01 | -2.412E-01 | 3.600E-05 |
| | 7.00 | 2.493E-01 | -2.493E-01 | -4.200E-05 |
| | 7.50 | 2.582E-01 | -2.582E-01 | -1.600E-05 |
| | 8.00 | 2.678E-01 | -2.679E-01 | -7.300E-05 |
| | 8.50 | 2.782E-01 | -2.782E-01 | -2.600E-05 |
| | 9.00 | 2.893E-01 | -2.894E-01 | -6.200E-05 |
| | 9.50 | 3.013E-01 | -3.012E-01 | 1.060E-04 |
| | 10.00 | 3.142E-01 | -3.139E-01 | 3.040E-04 |
| 3 | | | | |
| | 10.00 | 3.142E-01 | -3.148E-01 | -5.660E-04 |
| | 10.50 | 3.280E-01 | -3.282E-01 | -1.640E-04 |
| | 11.00 | 3.427E-01 | -3.426E-01 | 9.400E-05 |
| | 11.50 | 3.584E-01 | -3.583E-01 | 1.340E-04 |
| | 12.00 | 3.751E-01 | -3.750E-01 | 1.300E-04 |
| | 12.50 | 3.929E-01 | -3.929E-01 | 8.000E-06 |
| | 13.00 | 4.118E-01 | -4.119E-01 | -1.450E-04 |
| | 13.50 | 4.320E-01 | -4.320E-01 | -4.200E-05 |
| | 14.00 | 4.533E-01 | -4.534E-01 | -5.700E-05 |
| | 14.50 | 4.760E-01 | -4.758E-01 | 1.970E-04 |
| | 15.00 | 5.000E-01 | -4.993E-01 | 7.070E-04 |
| 4 | | | | |
| | 15.00 | 5.000E-01 | 4.993E-01 | 9.993E-01 |
| | 15.50 | 5.240E-01 | 4.758E-01 | 9.998E-01 |
| | 16.00 | 5.467E-01 | 4.534E-01 | 1.000E+00 |
| | 16.50 | 5.680E-01 | 4.320E-01 | 1.000E+00 |
| | 17.00 | 5.882E-01 | 4.119E-01 | 1.000E+00 |
| | 17.50 | 6.071E-01 | 3.929E-01 | 1.000E+00 |
| | 18.00 | 6.249E-01 | 3.750E-01 | 9.999E-01 |
| | 18.50 | 6.416E-01 | 3.583E-01 | 9.999E-01 |
| | 19.00 | 6.573E-01 | 3.426E-01 | 9.999E-01 |
| | 19.50 | 6.720E-01 | 3.282E-01 | 1.000E+00 |
| | 20.00 | 6.858E-01 | 3.148E-01 | 1.001E+00 |
| 5 | 20.00 | 6.858E-01 | 3.139E-01 | 9.997E-01 |
| | 20.50 | 6.987E-01 | 3.012E-01 | 9.999E-01 |
| | 21.00 | 7.107E-01 | 2.894E-01 | 1.000E+00 |

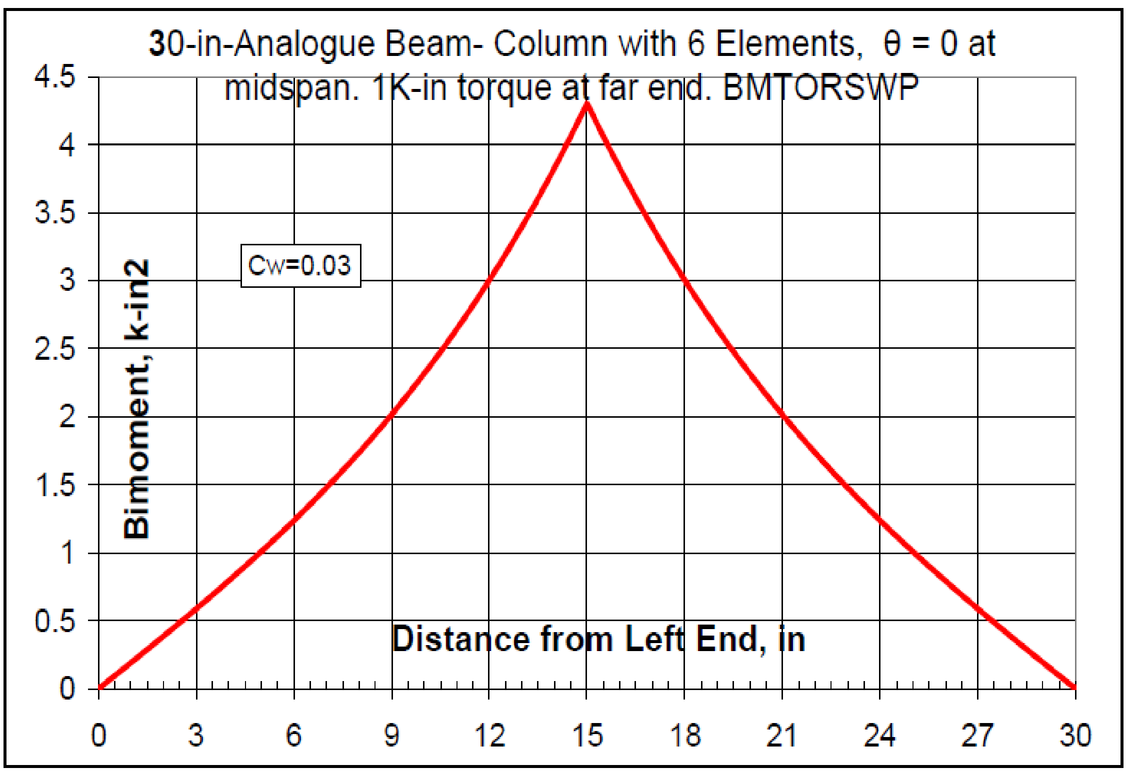
| elm | Z | GJΦ' | -ECwΦ''' | T |
|---|---|---|---|---|
| | 21.50 | 7.218E-01 | 2.782E-01 | 1.000E+00 |
| | 22.00 | 7.322E-01 | 2.679E-01 | 1.000E+00 |
| | 22.50 | 7.418E-01 | 2.582E-01 | 1.000E+00 |
| | 23.00 | 7.507E-01 | 2.493E-01 | 1.000E+00 |
| | 23.50 | 7.588E-01 | 2.412E-01 | 1.000E+00 |
| | 24.00 | 7.662E-01 | 2.338E-01 | 1.000E+00 |
| | 24.50 | 7.730E-01 | 2.271E-01 | 1.000E+00 |
| | 25.00 | 7.791E-01 | 2.212E-01 | 1.000E+00 |
| 6 | 25.00 | 7.791E-01 | 2.207E-01 | 9.998E-01 |
| | 25.50 | 7.846E-01 | 2.153E-01 | 9.999E-01 |
| | 26.00 | 7.895E-01 | 2.105E-01 | 1.000E+00 |
| | 26.50 | 7.938E-01 | 2.063E-01 | 1.000E+00 |
| | 27.00 | 7.974E-01 | 2.026E-01 | 1.000E+00 |
| | 27.50 | 8.005E-01 | 1.995E-01 | 1.000E+00 |
| | 28.00 | 8.030E-01 | 1.970E-01 | 1.000E+00 |
| | 28.50 | 8.050E-01 | 1.951E-01 | 1.000E+00 |
| | 29.00 | 8.064E-01 | 1.937E-01 | 1.000E+00 |
| | 29.50 | 8.072E-01 | 1.929E-01 | 1.000E+00 |
| | 30.00 | 8.075E-01 | 1.926E-01 | 1.000E+00 |




30-in-EBC with 6 Elements, θ=0 at Midspan, 1k-in Torque at Far End



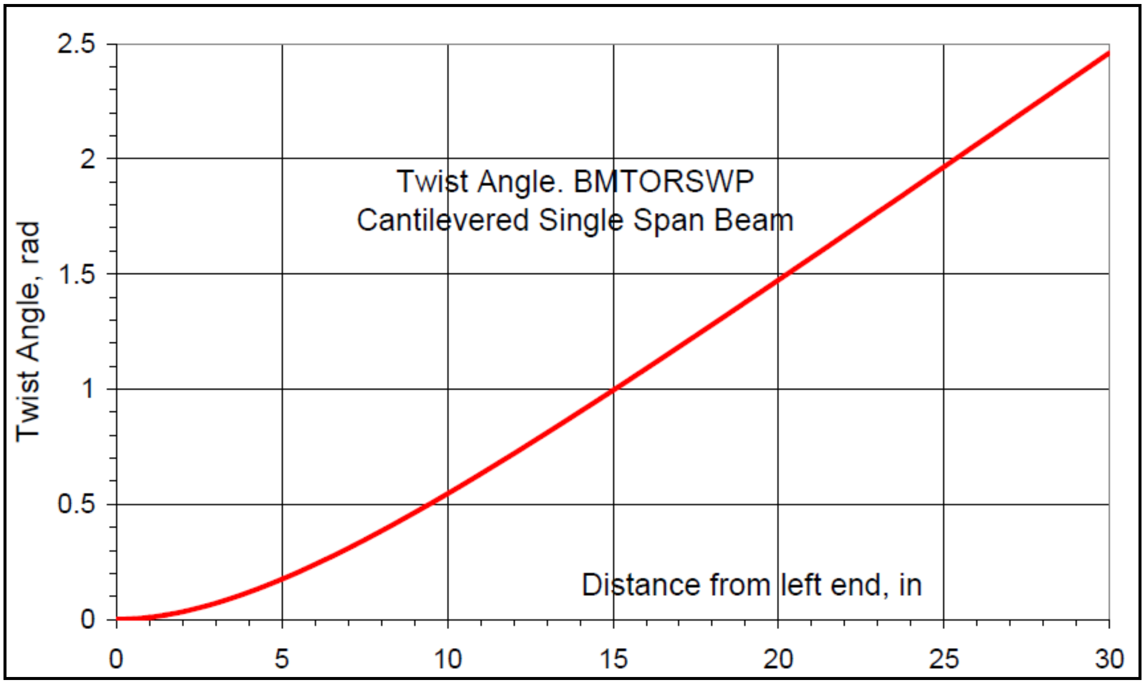

30-in-TWB with 6 Elements, θ=0 at Midspan, 1k-in Torque at Far End



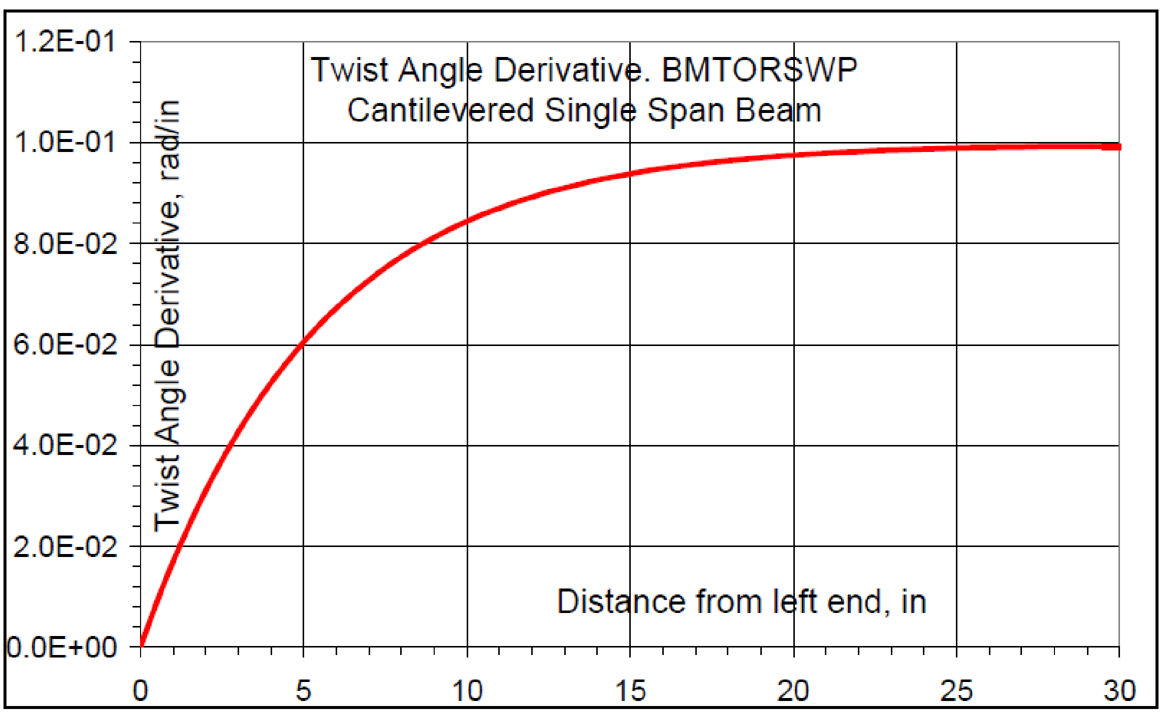

Bimoment 30-in-TWB with 6 Elements, θ=0 at Midspan, 1K-in at Far End

## APPENDIX C

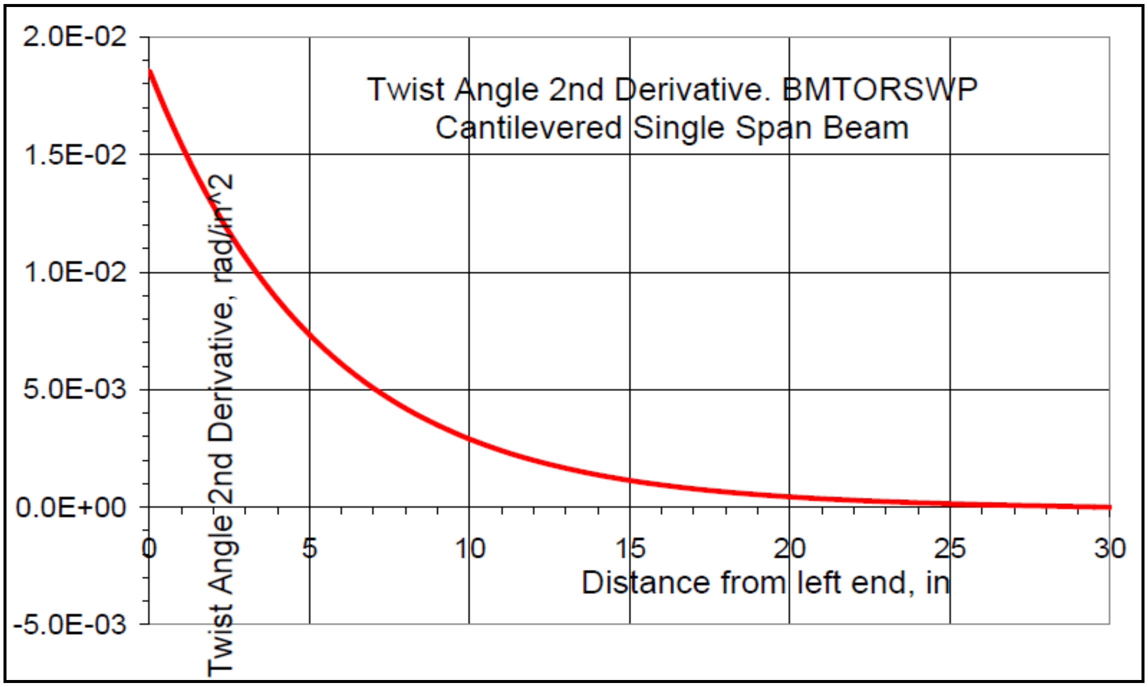
## CANTILEVERED SINGLE SPAN BAR



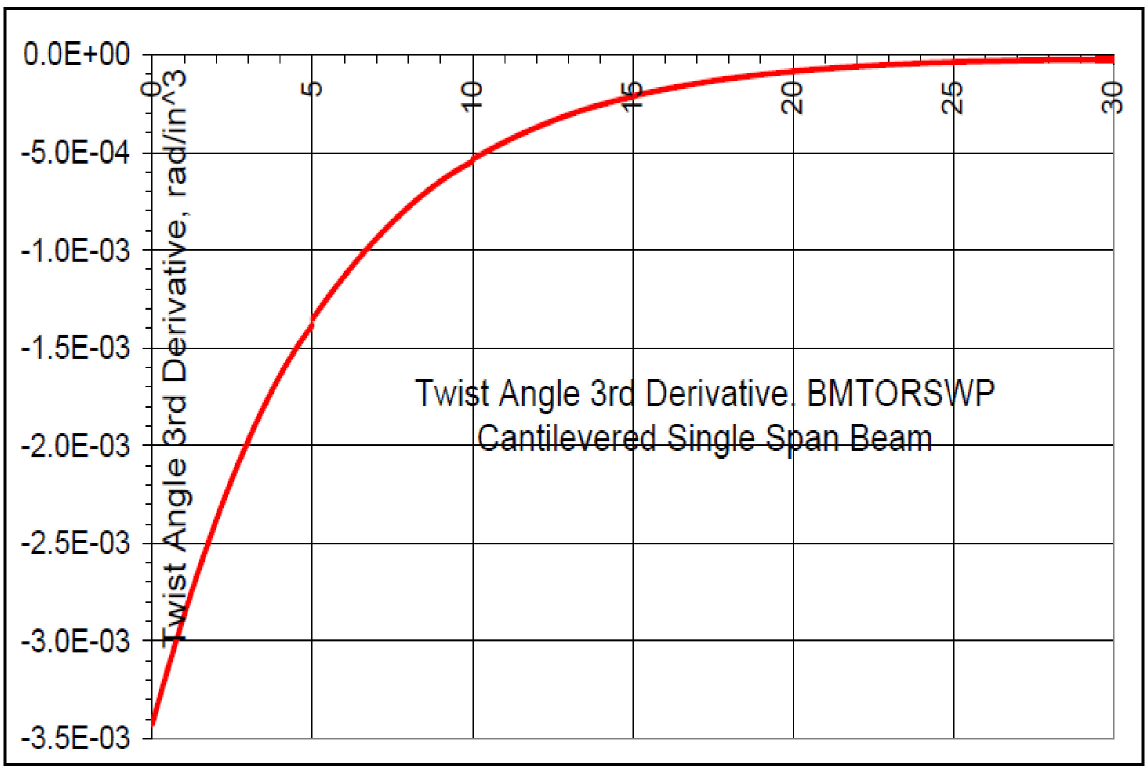

Twist Angle



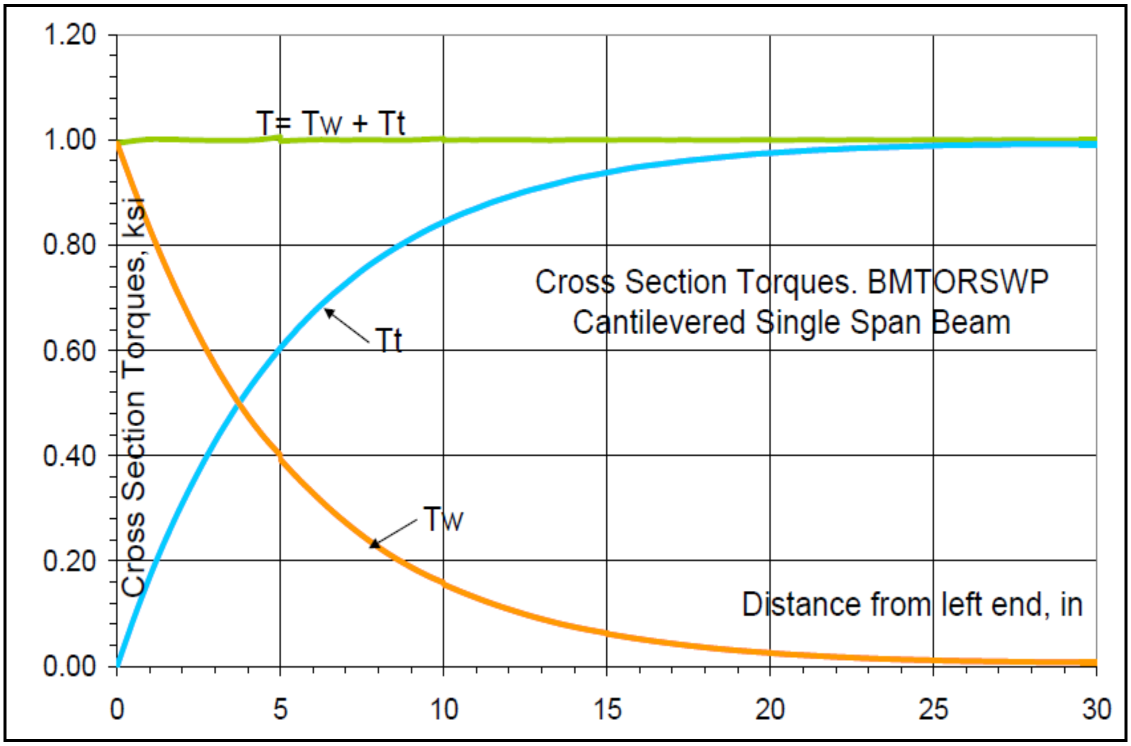

Twist Angle First Derivative



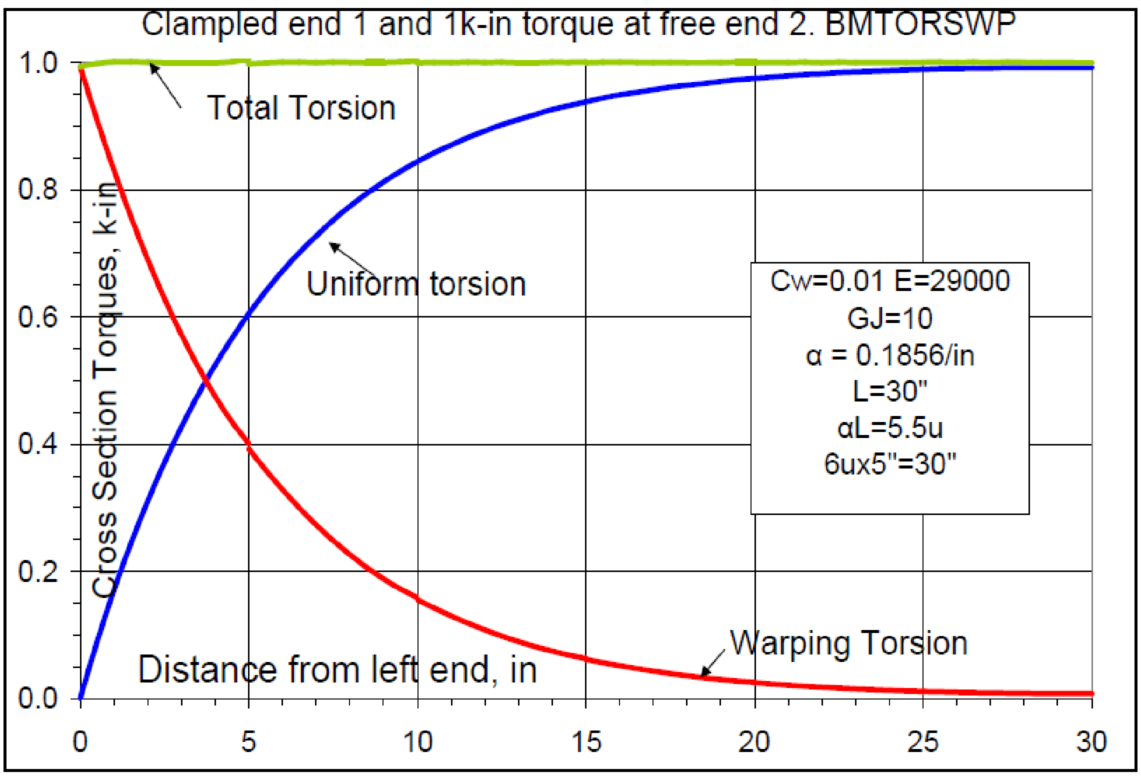

Twist Angle Second Derivative




Twist Angle Third Derivative




Torques at Cross Section




Torques at Cross Section with Total Torsion Computed from Derivatives

XS data

| | | | | | |
|---|---|---|---|---|---|
| GJ= | 10 | | | L= | 3.000E+01 |
| E= | 29000 | T= | 1k-in | λ | 1.857E-01 |
| Cw= | 0.01 | | | λ L= | 5.571E+00 |

Single Span Cantilevered Bar under 1k-in-Torque at Far End

| elm | Z | Φ | Φ' | Φ" | Φ''' | GJΦ' | ECwΦ''' | T |
|---|---|---|---|---|---|---|---|---|
| 1 | | | | | | | | |
| | 0 | 0 | 0.00E+00 | 1.86E-02 | -3.43E-03 | 0.000E+00 | 9.935E-01 | 0.994 |
| | 0.5 | 0.0023 | 8.87E-03 | 1.69E-02 | -3.14E-03 | 8.866E-02 | 9.094E-01 | 0.998 |
| | 1 | 0.0087 | 1.70E-02 | 1.54E-02 | -2.87E-03 | 1.695E-01 | 8.311E-01 | 1.001 |
| | 1.5 | 0.0191 | 2.43E-02 | 1.41E-02 | -2.61E-03 | 2.431E-01 | 7.581E-01 | 1.001 |
| | 2 | 0.0329 | 3.10E-02 | 1.28E-02 | -2.38E-03 | 3.102E-01 | 6.905E-01 | 1.001 |
| | 2.5 | 0.05 | 3.71E-02 | 1.17E-02 | -2.17E-03 | 3.714E-01 | 6.284E-01 | 1.000 |
| | 3 | 0.07 | 4.27E-02 | 1.06E-02 | -1.97E-03 | 4.271E-01 | 5.722E-01 | 0.999 |
| | 3.5 | 0.0926 | 4.78E-02 | 9.70E-03 | -1.80E-03 | 4.779E-01 | 5.211E-01 | 0.999 |
| | 4 | 0.1177 | 5.24E-02 | 8.84E-03 | -1.64E-03 | 5.242E-01 | 4.753E-01 | 1.000 |
| | 4.5 | 0.145 | 5.66E-02 | 8.05E-03 | -1.50E-03 | 5.664E-01 | 4.353E-01 | 1.002 |
| | 5 | 0.1743 | 6.05E-02 | 7.33E-03 | -1.38E-03 | 6.048E-01 | 4.008E-01 | 1.006 |
| 2 | | | | | | | | |
| | 5 | 0.1743 | 6.05E-02 | 7.34E-03 | -1.35E-03 | 6.048E-01 | 3.927E-01 | 0.997 |
| | 5.5 | 0.2054 | 6.40E-02 | 6.69E-03 | -1.24E-03 | 6.398E-01 | 3.593E-01 | 0.999 |
| | 6 | 0.2382 | 6.72E-02 | 6.09E-03 | -1.13E-03 | 6.718E-01 | 3.286E-01 | 1.000 |
| | 6.5 | 0.2725 | 7.01E-02 | 5.55E-03 | -1.03E-03 | 7.009E-01 | 2.996E-01 | 1.000 |
| | 7 | 0.3083 | 7.27E-02 | 5.06E-03 | -9.41E-04 | 7.274E-01 | 2.729E-01 | 1.000 |
| | 7.5 | 0.3452 | 7.52E-02 | 4.61E-03 | -8.57E-04 | 7.515E-01 | 2.484E-01 | 1.000 |
| | 8 | 0.3834 | 7.74E-02 | 4.20E-03 | -7.80E-04 | 7.736E-01 | 2.261E-01 | 1.000 |
| | 8.5 | 0.4226 | 7.94E-02 | 3.83E-03 | -7.10E-04 | 7.936E-01 | 2.059E-01 | 1.000 |
| | 9 | 0.4627 | 8.12E-02 | 3.49E-03 | -6.48E-04 | 8.119E-01 | 1.879E-01 | 1.000 |
| | 9.5 | 0.5037 | 8.29E-02 | 3.18E-03 | -5.94E-04 | 8.286E-01 | 1.721E-01 | 1.001 |
| | 10 | 0.5455 | 8.44E-02 | 2.90E-03 | -5.46E-04 | 8.438E-01 | 1.585E-01 | 1.002 |
| 3 | | | | | | | | |
| | 10 | 0.5455 | 8.44E-02 | 2.90E-03 | -5.35E-04 | 8.438E-01 | 1.552E-01 | 0.999 |
| | 10.5 | 0.5881 | 8.58E-02 | 2.64E-03 | -4.90E-04 | 8.576E-01 | 1.421E-01 | 1.000 |
| | 11 | 0.6313 | 8.70E-02 | 2.41E-03 | -4.48E-04 | 8.702E-01 | 1.299E-01 | 1.000 |
| | 11.5 | 0.6751 | 8.82E-02 | 2.19E-03 | -4.09E-04 | 8.817E-01 | 1.185E-01 | 1.000 |
| | 12 | 0.7194 | 8.92E-02 | 2.00E-03 | -3.72E-04 | 8.922E-01 | 1.080E-01 | 1.000 |
| | 12.5 | 0.7643 | 9.02E-02 | 1.82E-03 | -3.39E-04 | 9.017E-01 | 9.828E-02 | 1.000 |
| | 13 | 0.8096 | 9.10E-02 | 1.66E-03 | -3.09E-04 | 9.104E-01 | 8.949E-02 | 1.000 |
| | 13.5 | 0.8553 | 9.18E-02 | 1.51E-03 | -2.81E-04 | 9.183E-01 | 8.152E-02 | 1.000 |
| | 14 | 0.9014 | 9.26E-02 | 1.38E-03 | -2.57E-04 | 9.255E-01 | 7.444E-02 | 1.000 |
| | 14.5 | 0.9478 | 9.32E-02 | 1.25E-03 | -2.35E-04 | 9.321E-01 | 6.818E-02 | 1.000 |
| | 15 | 0.9946 | 9.38E-02 | 1.14E-03 | -2.17E-04 | 9.381E-01 | 6.281E-02 | 1.001 |
| 4 | | | | | | | | |

| | | | | | | | | |
|---|---|---|---|---|---|---|---|---|
| | 15 | 0.9946 | 9.38E-02 | 1.14E-03 | -2.12E-04 | 9.381E-01 | 6.154E-02 | 1.000 |
| | 15.5 | 1.0416 | 9.44E-02 | 1.04E-03 | -1.94E-04 | 9.435E-01 | 5.638E-02 | 1.000 |
| | 16 | 1.0889 | 9.49E-02 | 9.47E-04 | -1.78E-04 | 9.485E-01 | 5.156E-02 | 1.000 |
| elm | Z | Φ | Φ' | Φ'' | Φ''' | GJΦ' | ECwΦ''' | T |
| | | | | | | | | |
| | 16.5 | 1.1365 | 9.53E-02 | 8.62E-04 | -1.62E-04 | 9.530E-01 | 4.710E-02 | 1.000 |
| | 17 | 1.1842 | 9.57E-02 | 7.84E-04 | -1.48E-04 | 9.571E-01 | 4.295E-02 | 1.000 |
| | 17.5 | 1.2322 | 9.61E-02 | 7.13E-04 | -1.35E-04 | 9.608E-01 | 3.915E-02 | 1.000 |
| | 18 | 1.2803 | 9.64E-02 | 6.49E-04 | -1.23E-04 | 9.642E-01 | 3.570E-02 | 1.000 |
| | 18.5 | 1.3286 | 9.67E-02 | 5.90E-04 | -1.12E-04 | 9.673E-01 | 3.260E-02 | 1.000 |
| | 19 | 1.377 | 9.70E-02 | 5.36E-04 | -1.03E-04 | 9.701E-01 | 2.984E-02 | 1.000 |
| | 19.5 | 1.4256 | 9.73E-02 | 4.87E-04 | -9.45E-05 | 9.727E-01 | 2.740E-02 | 1.000 |
| | 20 | 1.4743 | 9.75E-02 | 4.41E-04 | -8.73E-05 | 9.750E-01 | 2.532E-02 | 1.000 |
| 5 | | | | | | | | |
| | 20 | 1.4743 | 9.75E-02 | 4.42E-04 | -8.56E-05 | 9.750E-01 | 2.482E-02 | 1.000 |
| | 20.5 | 1.5231 | 9.77E-02 | 4.01E-04 | -7.87E-05 | 9.771E-01 | 2.283E-02 | 1.000 |
| | 21 | 1.572 | 9.79E-02 | 3.63E-04 | -7.23E-05 | 9.790E-01 | 2.098E-02 | 1.000 |
| | 21.5 | 1.621 | 9.81E-02 | 3.28E-04 | -6.64E-05 | 9.808E-01 | 1.927E-02 | 1.000 |
| | 22 | 1.6701 | 9.82E-02 | 2.96E-04 | -6.10E-05 | 9.823E-01 | 1.770E-02 | 1.000 |
| | 22.5 | 1.7192 | 9.84E-02 | 2.67E-04 | -5.61E-05 | 9.837E-01 | 1.627E-02 | 1.000 |
| | 23 | 1.7685 | 9.85E-02 | 2.40E-04 | -5.17E-05 | 9.850E-01 | 1.499E-02 | 1.000 |
| | 23.5 | 1.8177 | 9.86E-02 | 2.15E-04 | -4.77E-05 | 9.861E-01 | 1.384E-02 | 1.000 |
| | 24 | 1.8671 | 9.87E-02 | 1.92E-04 | -4.43E-05 | 9.871E-01 | 1.284E-02 | 1.000 |
| | 24.5 | 1.9165 | 9.88E-02 | 1.71E-04 | -4.13E-05 | 9.881E-01 | 1.198E-02 | 1.000 |
| | 25 | 1.9659 | 9.89E-02 | 1.51E-04 | -3.89E-05 | 9.889E-01 | 1.127E-02 | 1.000 |
| 6 | | | | | | | | |
| | 25 | 1.9659 | 9.89E-02 | 1.51E-04 | -3.83E-05 | 9.889E-01 | 1.110E-02 | 1.000 |
| | 25.5 | 2.0153 | 9.90E-02 | 1.32E-04 | -3.59E-05 | 9.896E-01 | 1.042E-02 | 1.000 |
| | 26 | 2.0648 | 9.90E-02 | 1.15E-04 | -3.39E-05 | 9.902E-01 | 9.817E-03 | 1.000 |
| | 26.5 | 2.1144 | 9.91E-02 | 9.85E-05 | -3.20E-05 | 9.907E-01 | 9.286E-03 | 1.000 |
| | 27 | 2.1639 | 9.91E-02 | 8.29E-05 | -3.05E-05 | 9.912E-01 | 8.831E-03 | 1.000 |
| | 27.5 | 2.2135 | 9.92E-02 | 6.80E-05 | -2.91E-05 | 9.916E-01 | 8.448E-03 | 1.000 |
| | 28 | 2.2631 | 9.92E-02 | 5.37E-05 | -2.81E-05 | 9.919E-01 | 8.140E-03 | 1.000 |
| | 28.5 | 2.3127 | 9.92E-02 | 3.99E-05 | -2.73E-05 | 9.921E-01 | 7.905E-03 | 1.000 |
| | 29 | 2.3623 | 9.92E-02 | 2.64E-05 | -2.67E-05 | 9.923E-01 | 7.746E-03 | 1.000 |
| | 29.5 | 2.4119 | 9.92E-02 | 1.32E-05 | -2.64E-05 | 9.924E-01 | 7.656E-03 | 1.000 |
| | 30 | 2.4615 | 9.92E-02 | -2.69E-08 | -2.64E-05 | 9.924E-01 | 7.644E-03 | 1.000 |

The column of total torque was calculated from the twist angle derivatives with a negligible error. The computer produces a more exact output for the total torque but with opposite sign convention. The charts show how the asymptotic behavior of the twist angle derivatives increases as the derivative order does. This is a very suitable feature of the finite element used by the software.

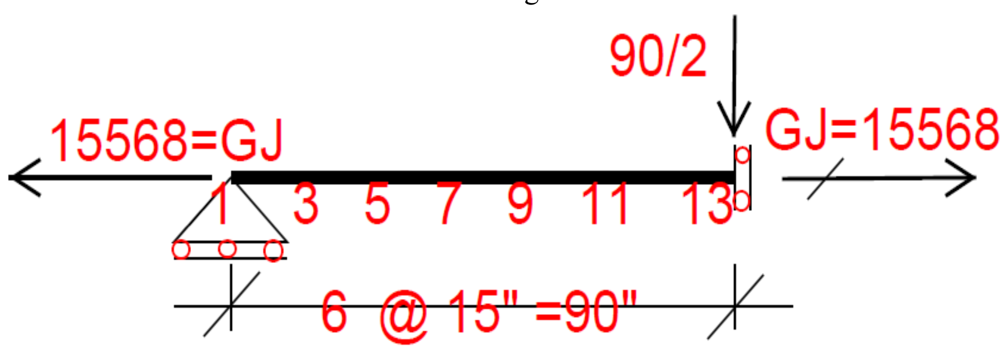
# APPENDIX D

## AISC-DG9-EXAMPLE 5.1

In this EBC of case study two, this material corresponds to the figure of the input model including its 2nd order equilibrium check, table of boundary conditions, stresses superposition strategy, input data, input forms, figures of output data in notepad version, and output charts and output data excel processed.

Normalized figures containing stresses computed by hand from the torque angle and derivatives, comparisons of combined flexure and torsional stresses, maximum twist angle for serviceability could be also found. Again, the asymptotic behavior of the thin-walled beam elastic line is successfully shown in the charts, which evidences the efficiency of the high order finite element used by BMTORSWP.

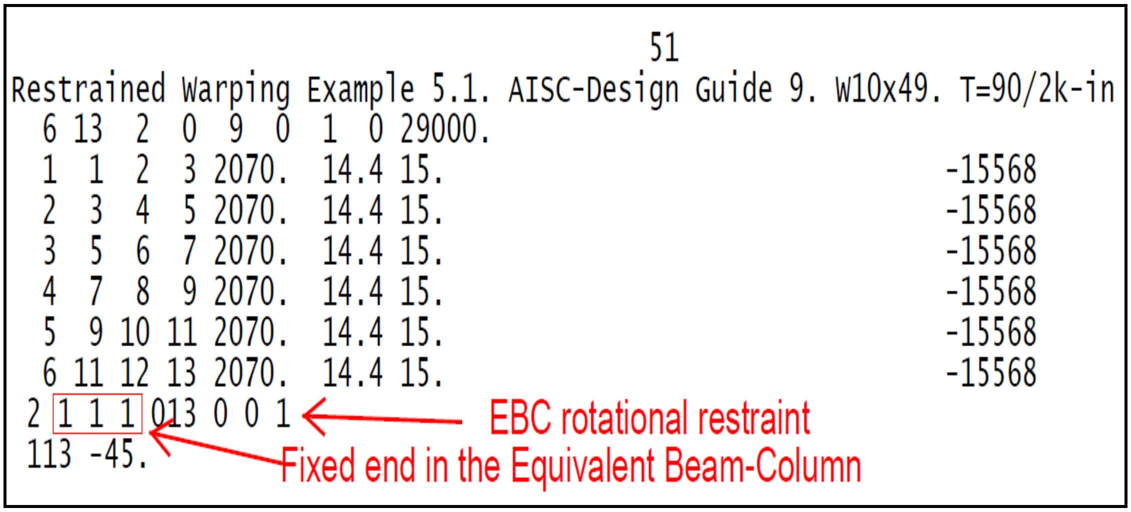
Summation of moments in the EBC: $GJ\cdot\Delta + B - T\cdot L = 0$.
Then, $15568\Delta + 2502.52 - 45 \times 90 = 0$;
$\Delta = 1547.48/15568 = 0.09940$ (o.k.);
where $\Delta$ is the deflection at EBC midspan, $T$ is the EBC vertical load, and $B$ is the bimoment or analog EBC moment




Equivalent Beam-Column Model for BMTORSWP

| Node | TWB | EBC |
|---|---|---|
| A | No torque angle, $\theta$ = 0 | No transverse displacement y = 0 |
| B | Maximum torsional rotation when ($d\theta/dx$) = 0 | Maximum vertical transverse displacement when ($dy/dx$) = 0 |

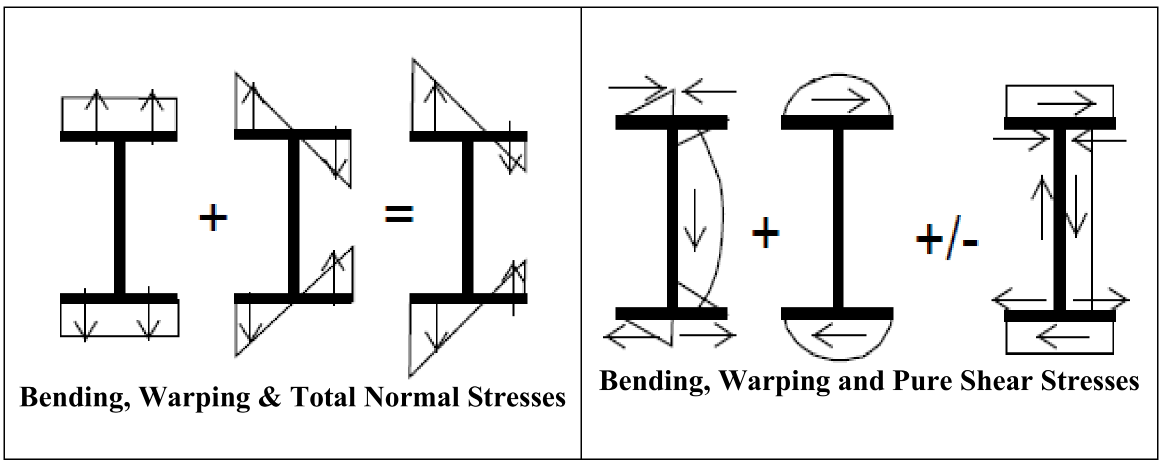
Boundary Conditions for TWB and Analog EBC



```
                                                  51
Restrained Warping Example 5.1. AISC-Design Guide 9. W10x49. T=90/2k-in
  6 13  2  0  9  0  1  0 29000.
  1  1  2  3 2070.  14.4 15.                                   -15568
  2  3  4  5 2070.  14.4 15.                                   -15568
  3  5  6  7 2070.  14.4 15.                                   -15568
  4  7  8  9 2070.  14.4 15.                                   -15568
  5  9 10 11 2070.  14.4 15.                                   -15568
  6 11 12 13 2070.  14.4 15.                                   -15568
 2 1 1 1 013 0 0 1
 113 -45.
```



Notepad Input




Stresses Superposition Strategy

The figures containing the Stresses Superposition Strategy correspond to a sign convention based upon the different expected force directions for each of the interest points in the cross section. The AISC uses a positive convention for the stresses that correspond to a thin walled beam that undergoes a flexural deformation.

The asymptotic behavior of the thin-walled beam elastic line produces radical variations in zones near applied loads and supports. This behavior is successfully shown in the charts, due to the suitable high order finite element used by BMTORSWP.

PROGRAM BMTORSW INPUT FORM

ALPHAMERIC DESCRIPTION OF THE JOB | FILE NAME:

Design Guide Example 5.1 - w10x49 with 90 k-in torque at m

| NEL | NOD | NSUP | NSPD | JBW | NFX | NFY | NFZ | ELASTICITY (E11.4) |
|---|---|---|---|---|---|---|---|---|
| 6 | 13 | 2 | 0 | 9 | 0 | 1 | 0 | 29000. |

| NE | NI | NC | NJ | $C_w$ | AREA | LENGTH | WTANG | SOIL NI | SOI |
|---|---|---|---|---|---|---|---|---|---|
| 1 | 1 | 2 | 3 | 2070. | 14.4 | 15. | | | |
| 2 | 3 | 4 | 5 | 2070. | 14.4 | 15. | | | |
| 3 | 5 | 6 | 7 | 2070. | 14.4 | 15. | | | |
| 4 | 7 | 8 | 9 | 2070. | 14.4 | 15. | | | |
| 5 | 9 | 10 | 11 | 2070. | 14.4 | 15. | | | |
| 6 | 11 | 12 | 13 | 2070. | 14.4 | 15. | | | |

| NF | N | KX | KY | KZ | N | KX | KY | KZ | N | KX | KY | KZ | N | KX | KY | KZ | N | KX | KY | K |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| 2 | 1 | 1 | 1 | 1 | 13 | 0 | 0 | 1 | | | | | | | | | | | | |

| NF | N | KD | SPRING | N | KD | SPRING | N | KD |
|---|---|---|---|---|---|---|---|---|
| | | | | | | | | |

| NF | N | LOAD VALUE | N | LOAD VALUE | N | LOAD VALUE | N | LOAD VALUE | N | LO |
|---|---|---|---|---|---|---|---|---|---|---|
| 1 | 13 | -45. | | | | | | | | |

| NF | ELEM | END | ROTATIONAL SPRING MODULUS | ELEM | END | ROTATIONAL SPRING MODULUS | ELEM | END | ROTATION SPRING MOD |
|---|---|---|---|---|---|---|---|---|---|
| | | | | | | | | | |

Ex. 5.1 DG-9, Left Side Input Form

AUTHOR: B. DESCHAPELLES

39 40 41 42 43 44 45 46 47 48 49 50 51 52 53 54 55 56 57 58 59 60 61 62 63 64 65 66 67 68 69 70 71 72 73 74 75 76 77 78 79 80

NAME:

at mid span

GENERAL INFORMATION

PARTIAL RESTRAINT AT END ? 1 FOR YES, 0 FOR NO

| I | SOIL NJ | SOIL TI | SOIL TJ | ANGLE | - G*J | Wn @ NI | Wn @ NJ | 1 | 2 |
|---|---|---|---|---|---|---|---|---|---|
| | | | | | -1 56 6 8 . | | | | |
| | | | | | S A M E | | | | |
| | | | | | -1 56 6 8 . | | | | |

| KY | KZ | N | KX | KY | KZ | N | KX | KY | KZ | N | KX | KY | KZ | SUPPORTS |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|

| SPRING | N | KD | SPRING | SPRINGS AT SUPPORTS |
|---|---|---|---|---|

| N | LOAD VALUE | N | LOAD VALUE | N | LOAD VALUE | APPLIED NODAL FORCES |
|---|---|---|---|---|---|---|

| TATIONAL G MODULUS | ELEM | END | ROTATIONAL SPRING MODULUS | ELEM | END | ROTATIONAL SPRING MODULUS | SPRINGS AT ELEM. END, 1 OR 2, IF ANY |
|---|---|---|---|---|---|---|---|

39 40 41 42 43 44 45 46 47 48 49 50 51 52 53 54 55 56 57 58 59 60 61 62 63 64 65 66 67 68 69 70 71 72 73 74 75 76 77 78 79 80

Ex. 5.1 DG-9, Right Side Input Form

```
                                 51OT

 ------------------------------------------------------------------------
YOU ARE USING COMPUTER PROGRAM BMTORSW,DEVELOPED BY DR. BERNARDO DESCHAPELLES

        INPUT DATA FILE NAME IS = 51.txt

                OUTPUT FILE NAME IS = 51ot.txt

          STORAGE FILE FOR POST-PROCESSING WITH EXCEL = 51grf.grf


 ------------------------------------------------------------------------
Restrained Warping Example 5.1. AISC-Design Guide 9. W10x49. T=90/2k-in

modulus of elasticity of the material=   29000. k/ft2

ELEM nodes inertia length distrib. load  AXIAL  SOIL NORMAL MODULUS,Ksf  angle
    i   j  ft.4     ft     at i    at j   LOAD    1st END       2nd END    rad
 1   1   3******** 15.00  0.000   0.000********     0.0          0.0    0.000 00
 2   3   5******** 15.00  0.000   0.000********     0.0          0.0    0.000 00
 3   5   7******** 15.00  0.000   0.000********     0.0          0.0    0.000 00
 4   7   9******** 15.00  0.000   0.000********     0.0          0.0    0.000 00
 5   9  11******** 15.00  0.000   0.000********     0.0          0.0    0.000 00
 6  11  13******** 15.00  0.000   0.000********     0.0          0.0    0.000 00

    INPUT DATA RELATED TO THE  2 SUPPORTS
 2 1 1 1 013 0 0 1

    INPUT OF NODAL FORCES RELATED TO GLOBAL AXIS  2
         113 -45.00

      FINAL  SOLUTION  FOUND  AFTER   1  ITERATIONS

     Output of nodal displacements in reference to  global axes

node  displ.      displ.      displ.     node  displ.      displ.      displ.
    along  x    along  y    around  z        along  x    along  y    around  z
    or nonn1    or nonn2    or nonn 3        or nonn1    or nonn2    or nonn 3
                                        Noise disappears in the Excel File
  1 0.0000E+00  0.0000E+00 -0.1604E-02
                  Max.                     2 0.0000E+00 -0.1199E-01 -0.1991E-02

  3 0.0000E+00 -0.2388E-01 -0.1567E-02
                                           4 0.0000E+00 -0.3539E-01 -0.1896E-02

  5 0.0000E+00 -0.4662E-01 -0.1451E-02
                                           6 0.0000E+00 -0.5706E-01 -0.1701E-02

  7 0.0000E+00 -0.6700E-01 -0.1252E-02
                                           8 0.0000E+00 -0.7571E-01 -0.1393E-02

  9 0.0000E+00 -0.8369E-01 -0.9559E-03
                                          10 0.0000E+00 -0.8991E-01 -0.9546E-03

 11 0.0000E+00 -0.9511E-01 -0.5467E-03
                                          12 0.0000E+00 -0.9794E-01 -0.3606E-03

 13 0.0000E+00 -0.9940E-01  0.0000E+00                Max.

 ------------------------------------------------------------------------
     OUTPUT OF SOIL REACTIONS,STRESSES AND TRANSVERSE DISPLACEMENTS

 ------------------------------------------------------------------------
                                Page 1
```

Page 1, Output File

```
                                510T
     ELEMENT    1  DISPLACEMENTS IN INCIDENCES    1   2   3
        NODE    1          0.00000E+00          0.00000E+00         -0.16045E-02
        NODE    2          0.00000E+00         -0.11986E-01         -0.19915E-02
        NODE    3          0.00000E+00         -0.23879E-01         -0.15667E-02
     FORCES ACTING ALONG THE 9 DOF
        NODE    1          0.00000E+00          0.45000E+02          0.23919E-10
        NODE    2          0.00000E+00          0.38613E-10         -0.89074E-10
        NODE    3          0.00000E+00         -0.45000E+02          0.30326E+03

ELEMENT   1, FROM NODE   1, TO NODE   3 - LENGTH = 15.00 ft

 left half of span,at tenth points of length
             span      span      span      span      span      span
         0.0       0.1       0.2       0.3       0.4       0.5
soil,k/ft    0.000     0.000     0.000     0.000     0.000     0.000
shear,k      45.00     45.00     45.00     45.00     45.00     45.00
bmom,kft      0.00     30.04     60.09     90.18    120.32    150.53
tdisp,ft   0.00000  -0.00241  -0.00481  -0.00721  -0.00961  -0.01201
axial,k       0.00 AT 1st END and      0.00 AT 2nd END

right half of span,at tenth points of length
             span      span      span      span      span      span
         0.5       0.6       0.7       0.8       0.9       1.0
soil,k/ft    0.000     0.000     0.000     0.000     0.000     0.000
shear,k      45.00     45.00     45.00     45.00     45.00     45.00
bmom,kft    150.53    180.83    211.23    241.76    272.43    303.26
tdisp,ft  -0.01201  -0.01440  -0.01678  -0.01916  -0.02152  -0.02388
axial,k       0.00 AT 1st END and      0.00 AT 2nd END

----------------------------------------------------------------------
     ELEMENT    2  DISPLACEMENTS IN INCIDENCES    3   4   5
        NODE    3          0.00000E+00         -0.23879E-01         -0.15667E-02
        NODE    4          0.00000E+00         -0.35391E-01         -0.18964E-02
        NODE    5          0.00000E+00         -0.46615E-01         -0.14514E-02
     FORCES ACTING ALONG THE 9 DOF
        NODE    3          0.00000E+00          0.45000E+02         -0.30326E+03
        NODE    4          0.00000E+00         -0.59101E-10          0.90580E-09
        NODE    5          0.00000E+00         -0.45000E+02          0.62430E+03

ELEMENT   2, FROM NODE   3, TO NODE   5 - LENGTH = 15.00 ft

 left half of span,at tenth points of length
             span      span      span      span      span      span
         0.0       0.1       0.2       0.3       0.4       0.5
soil,k/ft    0.000     0.000     0.000     0.000     0.000     0.000
shear,k      45.00     45.00     45.00     45.00     45.00     45.00
bmom,kft    303.26    334.26    365.46    396.87    428.52    460.41
tdisp,ft  -0.02388  -0.02622  -0.02855  -0.03087  -0.03318  -0.03546
axial,k       0.00 AT 1st END and      0.00 AT 2nd END

right half of span,at tenth points of length
             span      span      span      span      span      span
         0.5       0.6       0.7       0.8       0.9       1.0
soil,k/ft    0.000     0.000     0.000     0.000     0.000     0.000
shear,k      45.00     45.00     45.00     45.00     45.00     45.00
bmom,kft    460.41    492.58    525.03    557.79    590.87    624.30
tdisp,ft  -0.03546  -0.03773  -0.03998  -0.04222  -0.04443  -0.04662
axial,k       0.00 AT 1st END and      0.00 AT 2nd END

----------------------------------------------------------------------
     ELEMENT    3  DISPLACEMENTS IN INCIDENCES    5   6   7
        NODE    5          0.00000E+00         -0.46615E-01         -0.14514E-02
        NODE    6          0.00000E+00         -0.57057E-01         -0.17006E-02
                                Page 2
```

Page 2, Output File

```
                                          51OT
        NODE    7          0.00000E+00        -0.67000E-01         -0.12517E-02
     FORCES ACTING ALONG THE 9 DOF
        NODE    5          0.00000E+00         0.45000E+02         -0.62430E+03
        NODE    6          0.00000E+00        -0.10010E-09          0.24890E-08
        NODE    7          0.00000E+00        -0.45000E+02          0.98194E+03

ELEMENT  3, FROM NODE  5, TO NODE  7 - LENGTH = 15.00 ft

 left half of span,at tenth points of length
              span      span      span      span      span      span
          0.0       0.1       0.2       0.3       0.4       0.5
soil,k/ft    0.000     0.000     0.000     0.000     0.000     0.000
shear,k      45.00     45.00     45.00     45.00     45.00     45.00
bmom,kft    624.30    658.09    692.26    726.84    761.85    797.30
tdisp,ft  -0.04662  -0.04878  -0.05092  -0.05304  -0.05512  -0.05718
axial,k       0.00 AT 1st END and      0.00 AT 2nd END

right half of span,at tenth points of length
              span      span      span      span      span      span
          0.5       0.6       0.7       0.8       0.9       1.0
soil,k/ft    0.000     0.000     0.000     0.000     0.000     0.000
shear,k      45.00     45.00     45.00     45.00     45.00     45.00
bmom,kft    797.30    833.21    869.61    906.52    943.95    981.94
tdisp,ft  -0.05718  -0.05921  -0.06121  -0.06317  -0.06510  -0.06700
axial,k       0.00 AT 1st END and      0.00 AT 2nd END

----------------------------------------------------------------------
     ELEMENT    4  DISPLACEMENTS IN INCIDENCES    7   8   9
        NODE    7          0.00000E+00        -0.67000E-01         -0.12517E-02
        NODE    8          0.00000E+00        -0.75713E-01         -0.13927E-02
        NODE    9          0.00000E+00        -0.83687E-01         -0.95591E-03
     FORCES ACTING ALONG THE 9 DOF
        NODE    7          0.00000E+00         0.45000E+02         -0.98194E+03
        NODE    8          0.00000E+00         0.78843E-10          0.11085E-08
        NODE    9          0.00000E+00        -0.45000E+02          0.13972E+04

ELEMENT  4, FROM NODE  7, TO NODE  9 - LENGTH = 15.00 ft

 left half of span,at tenth points of length
              span      span      span      span      span      span
          0.0       0.1       0.2       0.3       0.4       0.5
soil,k/ft    0.000     0.000     0.000     0.000     0.000     0.000
shear,k      45.00     45.00     45.00     45.00     45.00     45.00
bmom,kft    981.94   1020.50   1059.66   1099.43   1139.85   1180.93
tdisp,ft  -0.06700  -0.06886  -0.07068  -0.07246  -0.07420  -0.07590
axial,k       0.00 AT 1st END and      0.00 AT 2nd END

right half of span,at tenth points of length
              span      span      span      span      span      span
          0.5       0.6       0.7       0.8       0.9       1.0
soil,k/ft    0.000     0.000     0.000     0.000     0.000     0.000
shear,k      45.00     45.00     45.00     45.00     45.00     45.00
bmom,kft   1180.93   1222.70   1265.18   1308.40   1352.39   1397.16
tdisp,ft  -0.07590  -0.07755  -0.07916  -0.08072  -0.08223  -0.08369
axial,k       0.00 AT 1st END and      0.00 AT 2nd END

----------------------------------------------------------------------
     ELEMENT    5  DISPLACEMENTS IN INCIDENCES    9  10  11
        NODE    9          0.00000E+00        -0.83687E-01         -0.95591E-03
        NODE   10          0.00000E+00        -0.89910E-01         -0.95463E-03
        NODE   11          0.00000E+00        -0.95112E-01         -0.54667E-03
     FORCES ACTING ALONG THE 9 DOF
        NODE    9          0.00000E+00         0.45000E+02         -0.13972E+04
                                         Page 3
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Page 3 Output File

```
                                          51OT
          NODE  10          0.00000E+00         0.12910E-09          -0.23032E-08
          NODE  11          0.00000E+00        -0.45000E+02           0.18943E+04

 ELEMENT   5, FROM NODE   9, TO NODE 11 - LENGTH = 15.00 ft

  left half of span,at tenth points of length
               span      span      span      span      span      span
           0.0       0.1       0.2       0.3       0.4       0.5
 soil,k/ft    0.000     0.000     0.000     0.000     0.000     0.000
 shear,k      45.00     45.00     45.00     45.00     45.00     45.00
 bmom,kft   1397.16   1442.75   1489.18   1536.48   1584.68   1633.80
 tdisp,ft  -0.08369  -0.08509  -0.08645  -0.08775  -0.08899  -0.09017
 axial,k       0.00 AT 1st END and      0.00 AT 2nd END

 right half of span,at tenth points of length
               span      span      span      span      span      span
           0.5       0.6       0.7       0.8       0.9       1.0
 soil,k/ft    0.000     0.000     0.000     0.000     0.000     0.000
 shear,k      45.00     45.00     45.00     45.00     45.00     45.00
 bmom,kft   1633.80   1683.88   1734.93   1787.00   1840.12   1894.30
 tdisp,ft  -0.09017  -0.09128  -0.09234  -0.09333  -0.09426  -0.09511
 axial,k       0.00 AT 1st END and      0.00 AT 2nd END

 -------------------------------------------------------------------- Max.Twist Angle
      ELEMENT    6  DISPLACEMENTS IN INCIDENCES  11  12  13
          NODE  11          0.00000E+00        -0.95112E-01          -0.54667E-03
          NODE  12          0.00000E+00        -0.97939E-01          -0.36065E-03
          NODE  13          0.00000E+00        -0.99401E-01           0.00000E+00
      FORCES ACTING ALONG THE 9 DOF
          NODE  11          0.00000E+00         0.45000E+02          -0.18943E+04
          NODE  12          0.00000E+00        -0.35084E-09          -0.22137E-08
          NODE  13          0.00000E+00        -0.45000E+02           0.25025E+04

 ELEMENT   6, FROM NODE 11, TO NODE 13 - LENGTH = 15.00 ft

  left half of span,at tenth points of length
               span      span      span      span      span      span
           0.0       0.1       0.2       0.3       0.4       0.5
 soil,k/ft    0.000     0.000     0.000     0.000     0.000     0.000
 shear,k      45.00     45.00     45.00     45.00     45.00     45.00
 bmom,kft   1894.30   1949.60   2006.03   2063.63   2122.43   2182.47
 tdisp,ft  -0.09511  -0.09590  -0.09661  -0.09724  -0.09780  -0.09828
 axial,k       0.00 AT 1st END and      0.00 AT 2nd END

 right half of span,at tenth points of length
               span      span      span      span      span      span
           0.5       0.6       0.7       0.8       0.9       1.0
 soil,k/ft    0.000     0.000     0.000     0.000     0.000     0.000
 shear,k      45.00     45.00     45.00     45.00     45.00     45.00
 bmom,kft   2182.47   2243.79   2306.42   2370.39   2435.74   2502.52
 tdisp,ft  -0.09828  -0.09868  -0.09899  -0.09922  -0.09935  -0.09940
 axial,k       0.00 AT 1st END and      0.00 AT 2nd END

                                        Page 4
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Page 4 Output File

3.- First entry row corresponds to the torque angle and the rest to each of its 3 dimensionless increasing derivatives

Notice that in the AISC calculations, the applied torque is negative with the sign convention used in AISC-DG 9: $T_u = -90 \cdot kip \cdot in$

From AISC-DG 9, Appendix B, Case 3, with α = 0.5, and L/a = 2.9:
***θmi* at midspan z/L=0.5** ***θsi* at left support z/L=0**

$$\theta_m := \frac{T_u}{G \cdot J} \cdot \begin{pmatrix} 0.09 \cdot L \\ 0 \cdot in \\ \frac{-0.44}{a} \cdot in^2 \\ \frac{-0.50}{a^2} \cdot in^3 \end{pmatrix} = \begin{pmatrix} -0.09 \\ 0 \\ 4.1 \times 10^{-5} \\ 7.5 \times 10^{-7} \end{pmatrix} \qquad \theta_s := \frac{T_u}{G \cdot J} \cdot \begin{pmatrix} 0 \cdot L \\ 0.28 in \\ \frac{0}{a} \cdot in^2 \\ \frac{-0.22}{a^2} \cdot in^3 \end{pmatrix} = \begin{pmatrix} 0 \\ -1.62 \times 10^{-3} \\ 0 \\ 3.3 \times 10^{-7} \end{pmatrix}$$

Normalized Torque Angle and Derivatives (Increasing as per the Row Number) from AISC (Left) and BMTORSW

Shear stress in web (w) at the TWB support:

$$\tau_{t\_wsBmtrs} := G \cdot t_w \cdot \left(\theta_{sBmtrs_1} \cdot \frac{1}{in}\right) = -6.11 \cdot ksi \quad \text{From BMTORSW}$$

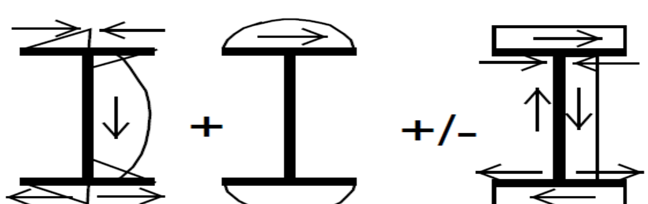
$$\tau_{t\_wsAisc} := G \cdot t_w \cdot \left(\theta_{sAisc_1} \cdot \frac{1}{in}\right) = -6.16 \cdot ksi \quad \text{From AISC-DG9}$$

Shear stress in flange (f) at the support (s):

$$\tau_{t\_fsBmtrs} := G \cdot t_f \cdot \left(\theta_{sBmtrs_1} \cdot \frac{1}{in}\right) = -10.06 \cdot ksi \quad \text{From BMTORSW}$$

$$\tau_{t\_fsAisc} := G \cdot t_f \cdot \left(\theta_{sAisc_1} \cdot \frac{1}{in}\right) = -10.15 \cdot ksi \quad \text{From AISC-DG9}$$

Shear Stress in Web and Flange at Midspan

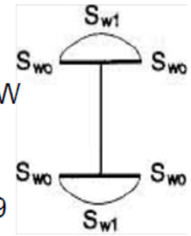



Two Solutions for Total Shear Stresses

Warping Shear Stress in flange at midspan (fm):

$$\tau_{w\_fmBmtrs} := -E \cdot S_{wl} \cdot \left(\theta_{mBmtrs_3} \cdot \frac{1}{in^3}\right) \cdot \frac{1}{t_f} = -1.28 \cdot ksi \quad \text{BMTORSW}$$

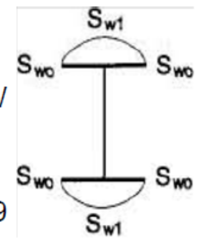
$$\tau_{w\_fmAisc} := -E \cdot S_{wl} \cdot \left(\theta_{mAisc_3} \cdot \frac{1}{in^3}\right) \cdot \frac{1}{t_f} = -1.28 \cdot ksi \quad \text{AISC-DG9}$$




Warping Shear Stress in flange at the support (fs):

$$\tau_{w\_fsBmtrs} := -E \cdot S_{wl} \cdot \left(\theta_{sBmtrs_3} \cdot \frac{1}{in^3}\right) \cdot \frac{1}{t_f} = -0.57 \cdot ksi \quad \text{BMTORSW}$$

$$\tau_{w\_fsAisc} := -E \cdot S_{wl} \cdot \left(\theta_{sAisc_3} \cdot \frac{1}{in^3}\right) \cdot \frac{1}{t_f} = -0.56 \cdot ksi \quad \text{AISC-DG9}$$

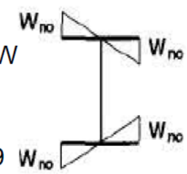



Shear Stress from Warping

Warping Normal Stresses at midspan due to warping

$$\sigma_{w\_mBmtrs} := E \cdot W_{no} \cdot \left(\theta_{mBmtrs_2} \cdot \frac{1}{in^2}\right) = 28.53 \cdot ksi \quad \text{BMTORSW}$$

$$\sigma_{w\_mAisc} := E \cdot W_{no} \cdot \left(\theta_{mAisc_2} \cdot \frac{1}{in^2}\right) = 28.04 \cdot ksi \quad \text{AISC-DG9}$$

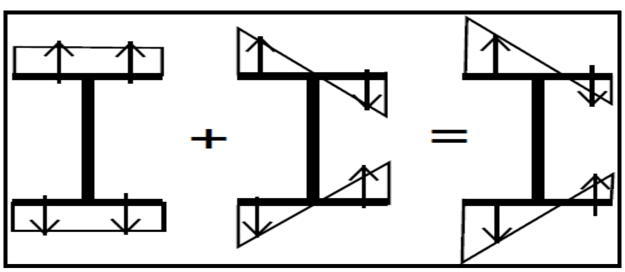



Warping Normal Stresses at the support $\sigma_w = 0$, since $\theta''=0$

$$\sigma_{w\_sBmtrs} := E \cdot W_{no} \cdot \left(\theta_{sBmtrs_2} \cdot \frac{1}{in^2}\right) = -0.003 \cdot psi \quad \text{BMTORSW}$$

$$\sigma_{w\_sAisc} := E \cdot W_{no} \cdot \left(\theta_{sAisc_2} \cdot \frac{1}{in^2}\right) = 0\,psi \quad \text{AISC-DG9}$$

Normal Stresses due to Warping

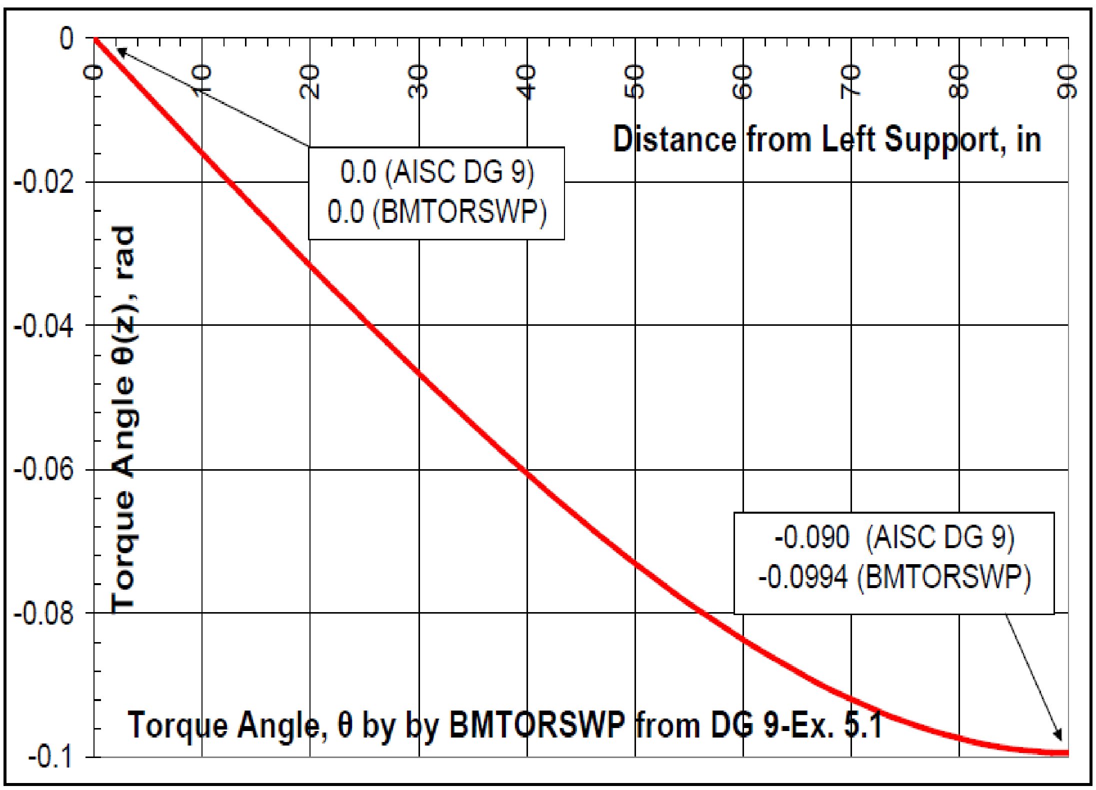


Superposition of Normal Stresses

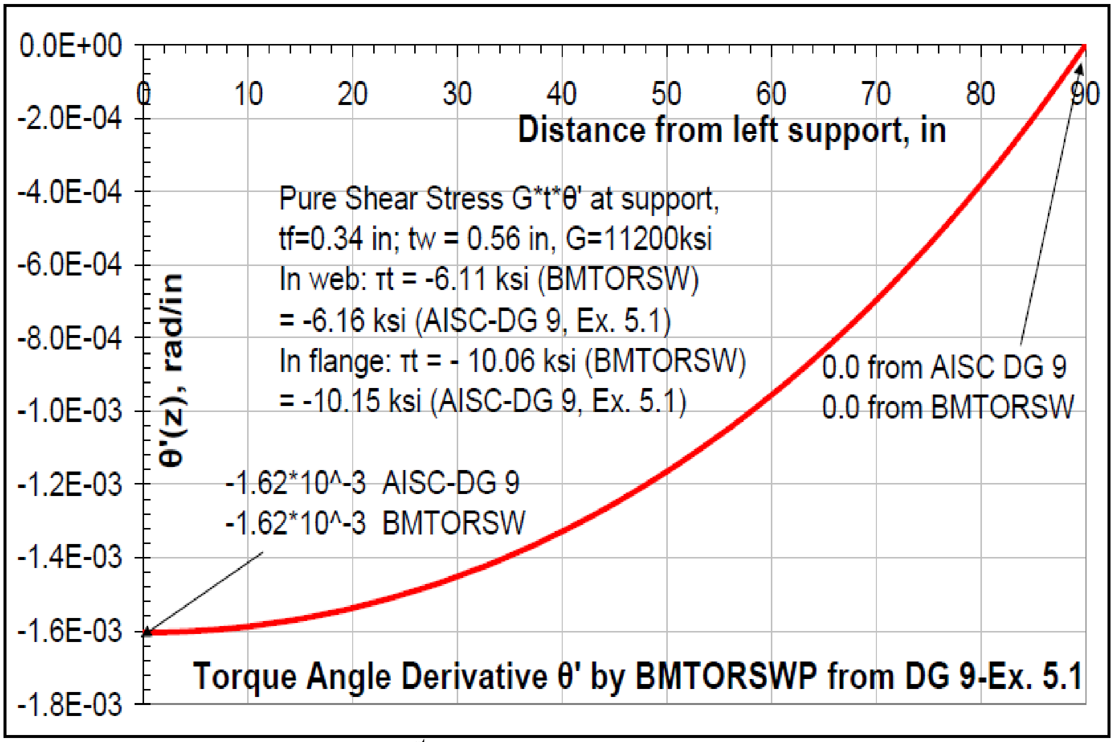
| Location | Normal Stresses | | | Shear Stresses | | | |
|---|---|---|---|---|---|---|---|
| | $\sigma_{uw}$ | $\sigma_{ub}$ | $f_{un}$ | $\tau_{ut}$ | $\tau_{uw}$ | $\tau_{ub}$ | $f_{uv}$ |
| Midspan | 28.53 | 12.36 | 40.89 | | | | |
| Flange | ±28.1 | ±12.4 | ±40.4 | 0 | -1.28 | ±0.640 | -1.92 |
| Web | ---- | ---- | ---- | 0 | ---- | ±2.45 | -2.45 |
| Support | | | | 10.06 | 0.57 | 0.64 | 11.27 |
| Flange | 0 | 0 | 0 | -10.2 | -0.564 | ±0.640 | -11.4 |
| Web | ---- | ---- | ---- | -6.16 | ---- | ±2.45 | -8.61 |
| Maximum | | 40. 89 | ±40.4 | | | 11.27 | -11.4 |

BMTORSWSP figures without sign, next to AISC DG9 figures

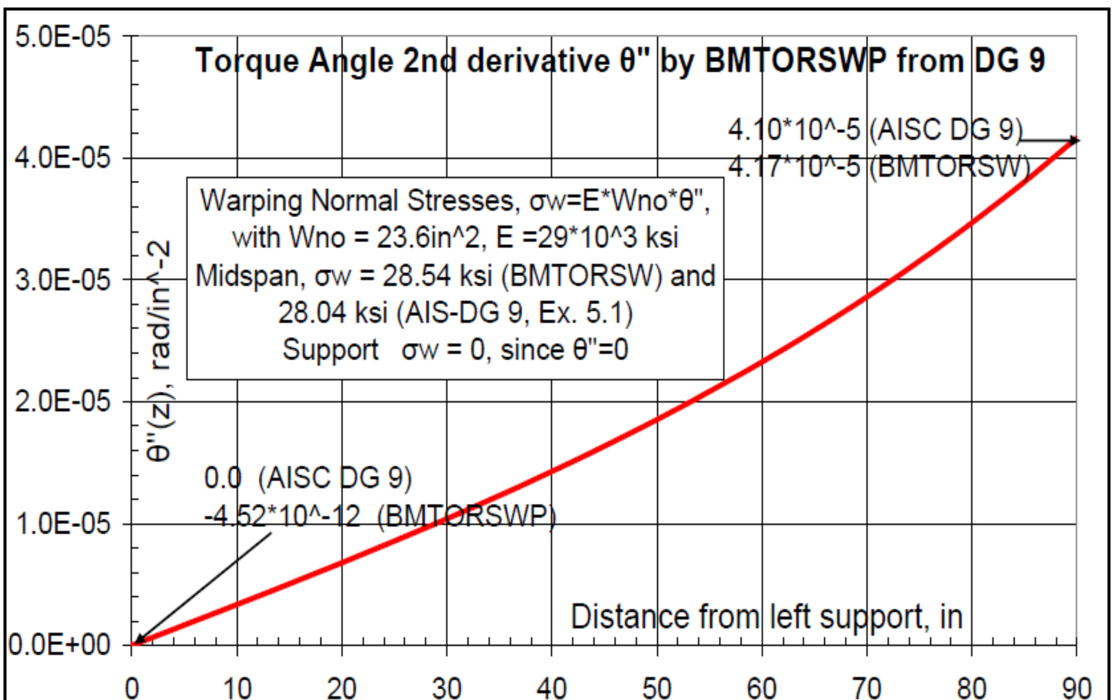
Comparisons Total of Flexure and Torsion Stresses



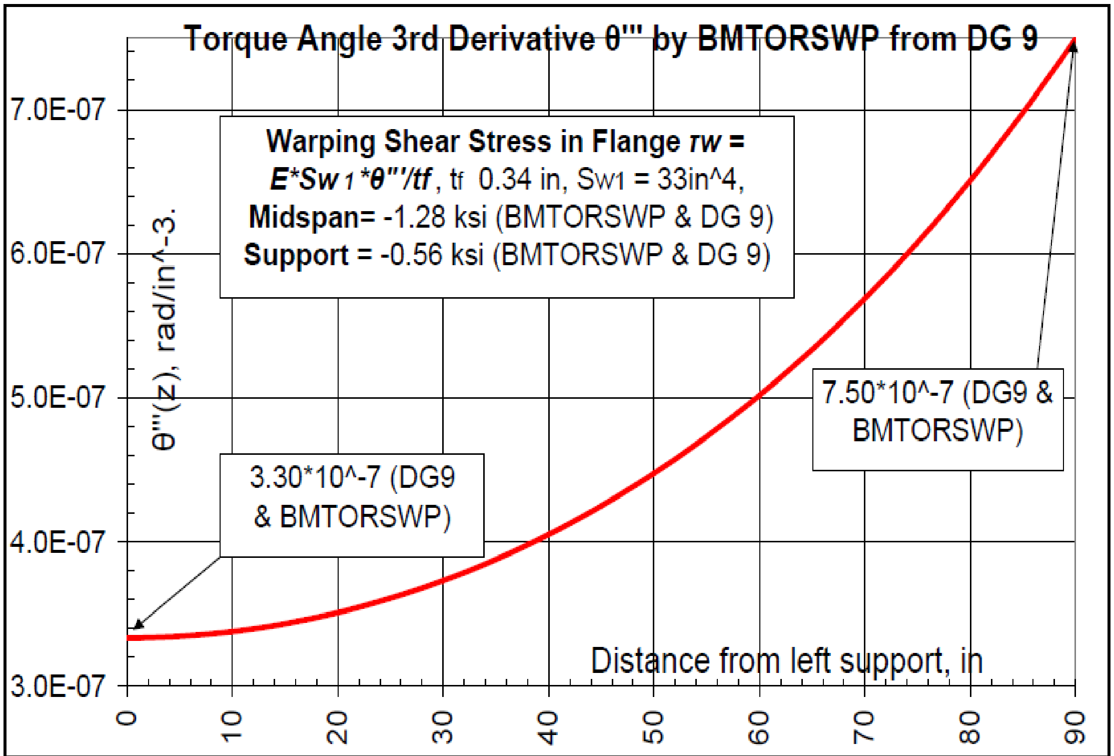

Torque Angle θ of AISC-Design Guide 9, Ex. 5.1



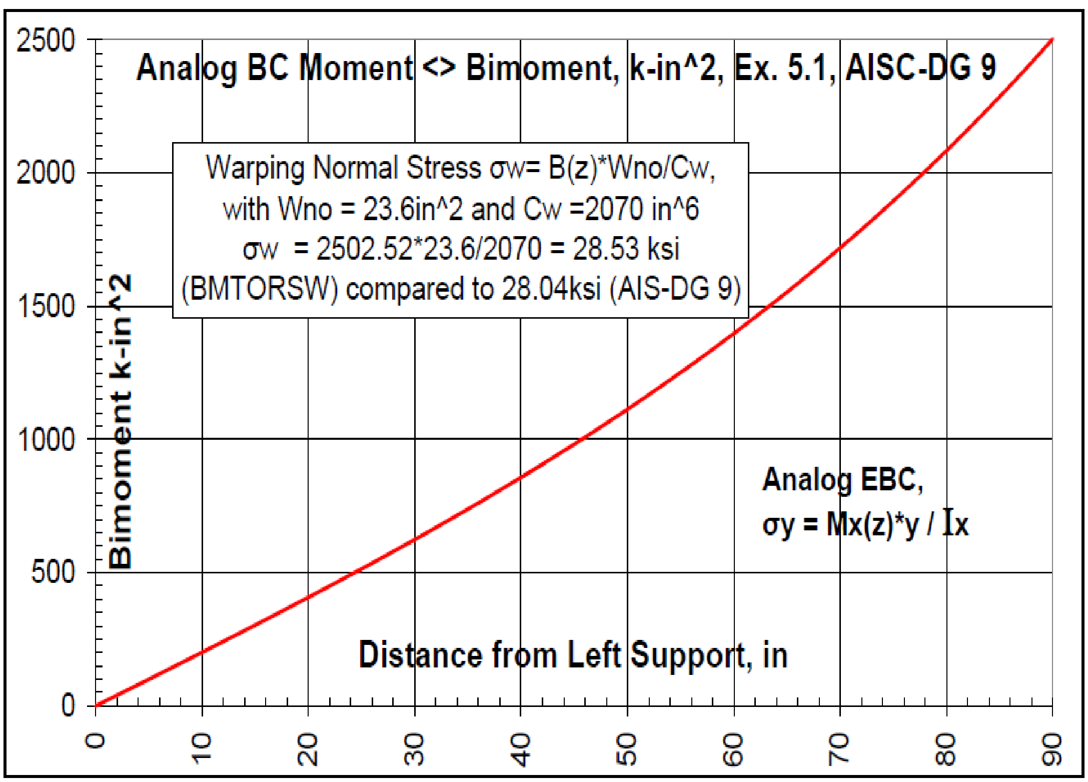

Torque Angle 1st Derivative θ' of AISC-DG 9, Ex. 5.1



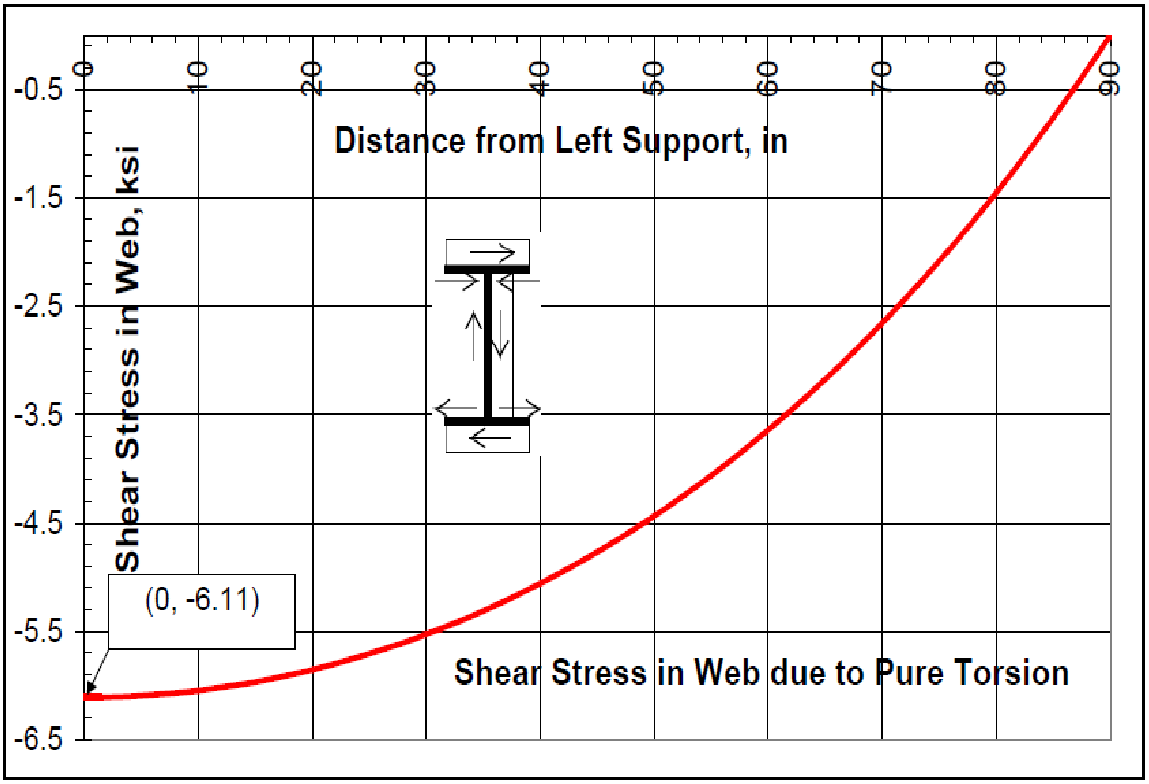

Torque Angle 2nd Derivative θ'' of AISC-DG 9, Ex. 5.1



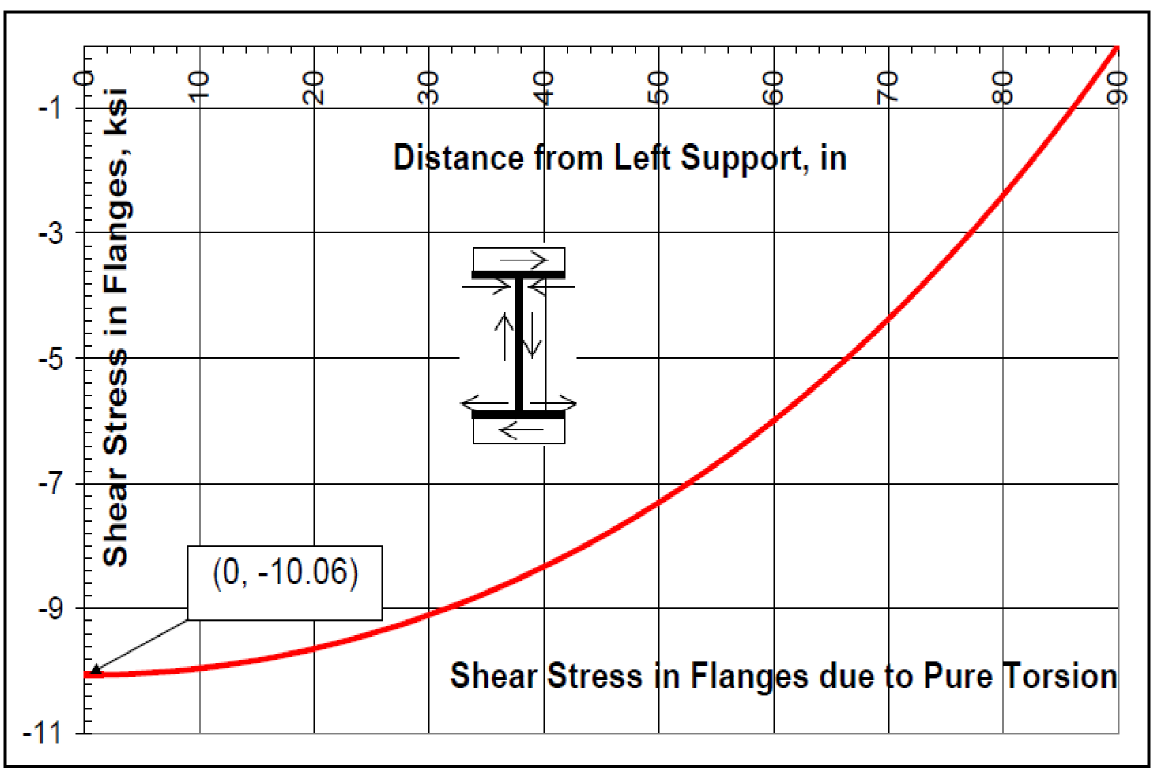

Torque Angle 3rd Derivative θ''' of AISC-DG 9, Ex. 5.1



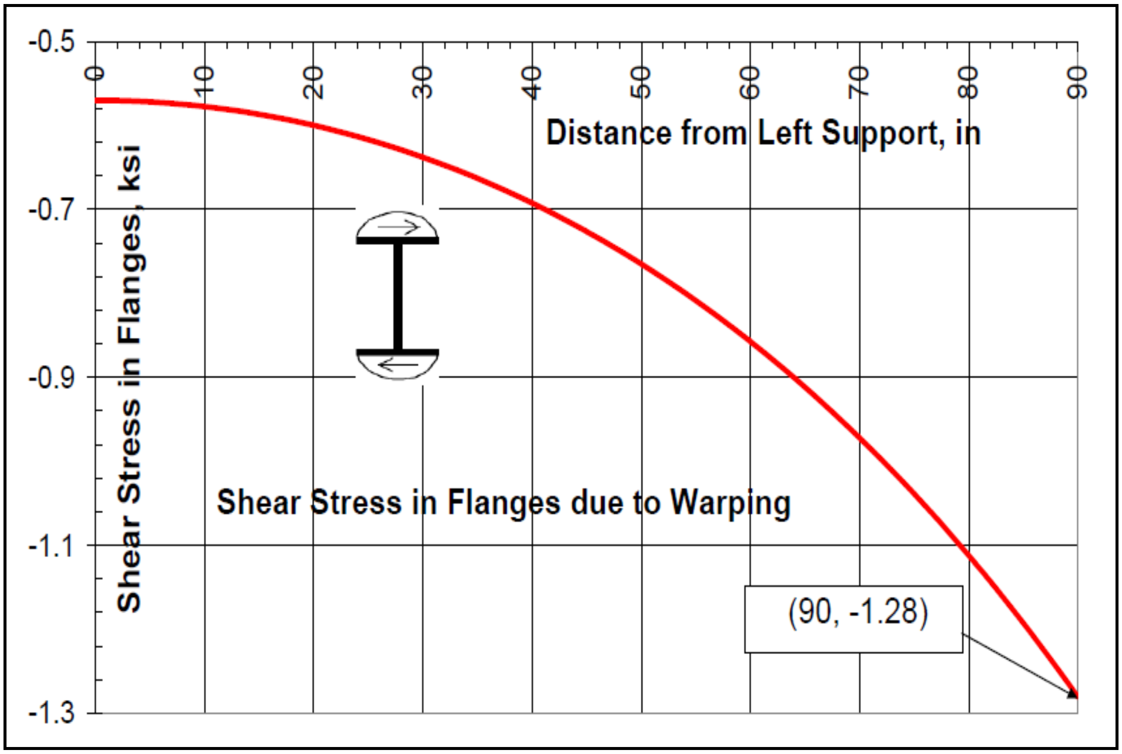

Bimoment from BMTORSWP of DG9-Ex. 5.1



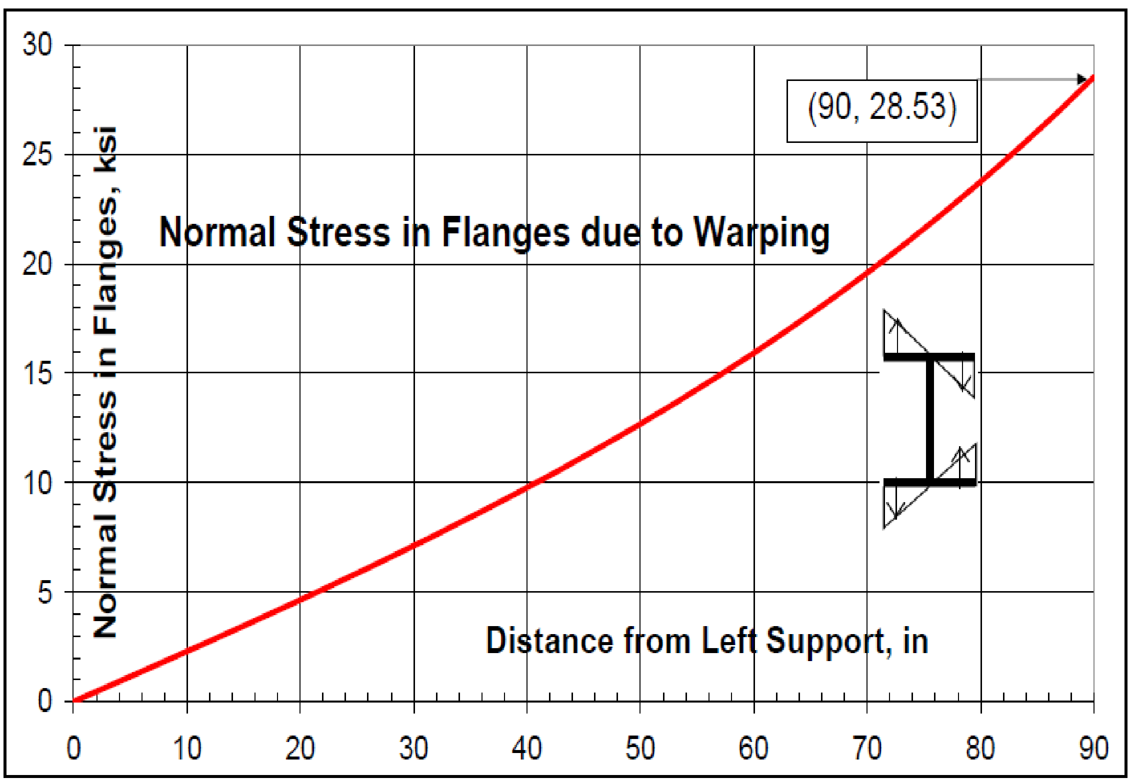

Shear Stress in Web Due to Pure Torsion




Shear Stress in Flanges Due to Pure Torsion




Shear Stress in Flanges Due to Warping




Normal Stress in Flanges Due to Warping

Table of Output for Graphics in Ex. 5.1

| | **BMTORSW OUTPUT FOR GRAPHICS** | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|
| **Elem** | **Z** | **θ** | **Z** | **Moment** | **Z** | **θ'** | **Z** | **θ''** | **Z** | **θ'''** |
| **1** | 0 | 0 | 0 | 0 | 0 | -1.60E-03 | 0 | -4.52E-12 | 0 | 3.34E-07 |
| | 1.5 | -0.0024 | 1.5 | 30.0357 | 1.5 | -1.60E-03 | 1.5 | 5.00E-07 | 1.5 | 3.34E-07 |
| | 3 | -0.0048 | 3 | 60.0889 | 3 | -1.60E-03 | 3 | 1.00E-06 | 3 | 3.34E-07 |
| | 4.5 | -0.0072 | 4.5 | 90.1772 | 4.5 | -1.60E-03 | 4.5 | 1.50E-06 | 4.5 | 3.34E-07 |
| | 6 | -0.0096 | 6 | 120.3182 | 6 | -1.60E-03 | 6 | 2.00E-06 | 6 | 3.35E-07 |
| | 7.5 | -0.012 | 7.5 | 150.5293 | 7.5 | -1.60E-03 | 7.5 | 2.51E-06 | 7.5 | 3.36E-07 |
| | 9 | -0.0144 | 9 | 180.8283 | 9 | -1.59E-03 | 9 | 3.01E-06 | 9 | 3.37E-07 |
| | 10.5 | -0.0168 | 10.5 | 211.2328 | 10.5 | -1.59E-03 | 10.5 | 3.52E-06 | 10.5 | 3.38E-07 |
| | 12 | -0.0192 | 12 | 241.7605 | 12 | -1.58E-03 | 12 | 4.03E-06 | 12 | 3.40E-07 |
| | 13.5 | -0.0215 | 13.5 | 272.4294 | 13.5 | -1.57E-03 | 13.5 | 4.54E-06 | 13.5 | 3.41E-07 |
| | 15 | -0.0239 | 15 | 303.2572 | 15 | -1.57E-03 | 15 | 5.05E-06 | 15 | 3.43E-07 |
| **2** | | | | | | | | | | |
| | 15 | -0.0239 | 15 | 303.2572 | 15 | -1.57E-03 | 15 | 5.05E-06 | 15 | 3.43E-07 |
| | 16.5 | -0.0262 | 16.5 | 334.2619 | 16.5 | -1.56E-03 | 16.5 | 5.57E-06 | 16.5 | 3.45E-07 |
| | 18 | -0.0286 | 18 | 365.4617 | 18 | -1.55E-03 | 18 | 6.09E-06 | 18 | 3.48E-07 |
| | 19.5 | -0.0309 | 19.5 | 396.8748 | 19.5 | -1.54E-03 | 19.5 | 6.61E-06 | 19.5 | 3.50E-07 |
| | 21 | -0.0332 | 21 | 428.5195 | 21 | -1.53E-03 | 21 | 7.14E-06 | 21 | 3.53E-07 |
| | 22.5 | -0.0355 | 22.5 | 460.4142 | 22.5 | -1.52E-03 | 22.5 | 7.67E-06 | 22.5 | 3.56E-07 |
| | 24 | -0.0377 | 24 | 492.5776 | 24 | -1.51E-03 | 24 | 8.21E-06 | 24 | 3.59E-07 |
| | 25.5 | -0.04 | 25.5 | 525.0284 | 25.5 | -1.49E-03 | 25.5 | 8.75E-06 | 25.5 | 3.62E-07 |
| | 27 | -0.0422 | 27 | 557.7856 | 27 | -1.48E-03 | 27 | 9.29E-06 | 27 | 3.66E-07 |
| | 28.5 | -0.0444 | 28.5 | 590.8683 | 28.5 | -1.47E-03 | 28.5 | 9.84E-06 | 28.5 | 3.69E-07 |
| | 30 | -0.0466 | 30 | 624.2958 | 30 | -1.45E-03 | 30 | 1.04E-05 | 30 | 3.73E-07 |
| **3** | | | | | | | | | | |
| | 30 | -0.0466 | 30 | 624.2958 | 30 | -1.45E-03 | 30 | 1.04E-05 | 30 | 3.73E-07 |
| | 31.5 | -0.0488 | 31.5 | 658.0876 | 31.5 | -1.44E-03 | 31.5 | 1.10E-05 | 31.5 | 3.77E-07 |
| | 33 | -0.0509 | 33 | 692.2634 | 33 | -1.42E-03 | 33 | 1.15E-05 | 33 | 3.82E-07 |
| | 34.5 | -0.053 | 34.5 | 726.8432 | 34.5 | -1.40E-03 | 34.5 | 1.21E-05 | 34.5 | 3.86E-07 |
| | 36 | -0.0551 | 36 | 761.8471 | 36 | -1.38E-03 | 36 | 1.27E-05 | 36 | 3.91E-07 |
| | 37.5 | -0.0572 | 37.5 | 797.2956 | 37.5 | -1.36E-03 | 37.5 | 1.33E-05 | 37.5 | 3.96E-07 |
| | 39 | -0.0592 | 39 | 833.2093 | 39 | -1.34E-03 | 39 | 1.39E-05 | 39 | 4.02E-07 |
| | 40.5 | -0.0612 | 40.5 | 869.6092 | 40.5 | -1.32E-03 | 40.5 | 1.45E-05 | 40.5 | 4.07E-07 |
| | 42 | -0.0632 | 42 | 906.5166 | 42 | -1.30E-03 | 42 | 1.51E-05 | 42 | 4.13E-07 |
| | 43.5 | -0.0651 | 43.5 | 943.953 | 43.5 | -1.28E-03 | 43.5 | 1.57E-05 | 43.5 | 4.19E-07 |
| | 45 | -0.067 | 45 | 981.9402 | 45 | -1.25E-03 | 45 | 1.64E-05 | 45 | 4.25E-07 |
| **4** | | | | | | | | | | |
| | 45 | -0.067 | 45 | 981.9402 | 45 | -1.25E-03 | 45 | 1.64E-05 | 45 | 4.25E-07 |
| | 46.5 | -0.0689 | 46.5 | 1020.5003 | 46.5 | -1.23E-03 | 46.5 | 1.70E-05 | 46.5 | 4.32E-07 |
| | 48 | -0.0707 | 48 | 1059.656 | 48 | -1.20E-03 | 48 | 1.77E-05 | 48 | 4.38E-07 |
| | 49.5 | -0.0725 | 49.5 | 1099.4301 | 49.5 | -1.17E-03 | 49.5 | 1.83E-05 | 49.5 | 4.45E-07 |

(continued) Table of Output for Graphics in Ex. 5.1

| | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|
| | 51 | -0.0742 | 51 | 1139.8457 | 51 | -1.15E-03 | 51 | 1.90E-05 | 51 | 4.53E-07 |
| | 52.5 | -0.0759 | 52.5 | 1180.9264 | 52.5 | -1.12E-03 | 52.5 | 1.97E-05 | 52.5 | 4.60E-07 |
| | 54 | -0.0776 | 54 | 1222.6963 | 54 | -1.09E-03 | 54 | 2.04E-05 | 54 | 4.68E-07 |
| | 55.5 | -0.0792 | 55.5 | 1265.1796 | 55.5 | -1.06E-03 | 55.5 | 2.11E-05 | 55.5 | 4.76E-07 |
| | 57 | -0.0807 | 57 | 1308.4012 | 57 | -1.02E-03 | 57 | 2.18E-05 | 57 | 4.84E-07 |
| | 58.5 | -0.0822 | 58.5 | 1352.3864 | 58.5 | -9.90E-04 | 58.5 | 2.25E-05 | 58.5 | 4.93E-07 |
| | 60 | -0.0837 | 60 | 1397.1606 | 60 | -9.56E-04 | 60 | 2.33E-05 | 60 | 5.02E-07 |
| **5** | | | | | | | | | | |
| | 60 | -0.0837 | 60 | 1397.1606 | 60 | -9.56E-04 | 60 | 2.33E-05 | 60 | 5.02E-07 |
| | 61.5 | -0.0851 | 61.5 | 1442.7502 | 61.5 | -9.20E-04 | 61.5 | 2.40E-05 | 61.5 | 5.11E-07 |
| | 63 | -0.0864 | 63 | 1489.1817 | 63 | -8.84E-04 | 63 | 2.48E-05 | 63 | 5.20E-07 |
| | 64.5 | -0.0877 | 64.5 | 1536.4822 | 64.5 | -8.46E-04 | 64.5 | 2.56E-05 | 64.5 | 5.30E-07 |
| | 66 | -0.089 | 66 | 1584.6793 | 66 | -8.07E-04 | 66 | 2.64E-05 | 66 | 5.40E-07 |
| | 67.5 | -0.0902 | 67.5 | 1633.801 | 67.5 | -7.67E-04 | 67.5 | 2.72E-05 | 67.5 | 5.51E-07 |
| | 69 | -0.0913 | 69 | 1683.8762 | 69 | -7.25E-04 | 69 | 2.81E-05 | 69 | 5.62E-07 |
| | 70.5 | -0.0923 | 70.5 | 1734.934 | 70.5 | -6.83E-04 | 70.5 | 2.89E-05 | 70.5 | 5.73E-07 |
| | 72 | -0.0933 | 72 | 1787.0041 | 72 | -6.39E-04 | 72 | 2.98E-05 | 72 | 5.84E-07 |
| | 73.5 | -0.0943 | 73.5 | 1840.1171 | 73.5 | -5.93E-04 | 73.5 | 3.07E-05 | 73.5 | 5.96E-07 |
| | 75 | -0.0951 | 75 | 1894.3038 | 75 | -5.47E-04 | 75 | 3.16E-05 | 75 | 6.08E-07 |
| **6** | | | | | | | | | | |
| | 75 | -0.0951 | 75 | 1894.3038 | 75 | -5.47E-04 | 75 | 3.16E-05 | 75 | 6.08E-07 |
| | 76.5 | -0.0959 | 76.5 | 1949.5959 | 76.5 | -4.99E-04 | 76.5 | 3.25E-05 | 76.5 | 6.20E-07 |
| | 78 | -0.0966 | 78 | 2006.0257 | 78 | -4.49E-04 | 78 | 3.34E-05 | 78 | 6.33E-07 |
| | 79.5 | -0.0972 | 79.5 | 2063.6261 | 79.5 | -3.98E-04 | 79.5 | 3.44E-05 | 79.5 | 6.46E-07 |
| | 81 | -0.0978 | 81 | 2122.4307 | 81 | -3.46E-04 | 81 | 3.54E-05 | 81 | 6.60E-07 |
| | 82.5 | -0.0983 | 82.5 | 2182.4738 | 82.5 | -2.92E-04 | 82.5 | 3.64E-05 | 82.5 | 6.74E-07 |
| | 84 | -0.0987 | 84 | 2243.7904 | 84 | -2.37E-04 | 84 | 3.74E-05 | 84 | 6.88E-07 |
| | 85.5 | -0.099 | 85.5 | 2306.4164 | 85.5 | -1.80E-04 | 85.5 | 3.84E-05 | 85.5 | 7.03E-07 |
| | 87 | -0.0992 | 87 | 2370.3882 | 87 | -1.22E-04 | 87 | 3.95E-05 | 87 | 7.18E-07 |
| | 88.5 | -0.0994 | 88.5 | 2435.7433 | 88.5 | -6.17E-05 | 88.5 | 4.06E-05 | 88.5 | 7.34E-07 |
| | 90 | -0.0994 | 90 | 2502.5197 | 90 | 0.00E+00 | 90 | 4.17E-05 | 90 | 7.50E-07 |
| **Elem** | **Z** | **θ** | **Z** | **Moment** | **Z** | **θ'** | **Z** | **θ''** | **Z** | **θ'''** |

$$\frac{\theta_{mAisc_0}}{LF} = -0.0624 \cdot rad$$ Maximum rotation as per AISC

$$\frac{\theta_{mBmtrs_0}}{LF} = -0.0663 \cdot rad$$ Maximum rotation as per BMTORSW

Maximum Twist Angle

Table of Stresses. Ex. 5.5

PROPERTIES FOR SPREADSHEET

| | | |
|---|---|---|
| G = | 11200 | ksi |
| tw = | 0.34 | in |
| tf = | 0.56 | in |
| E = | 29000 | ksi |
| S.wl | 33 | in^4 |
| W.no | 23.6 | in^2 |

STRESSES COMPUTED FROM BMTORSW OUTPUT

| | | | | Pure torsion shear | | | | Warping stresses | | |
|---|---|---|---|---|---|---|---|---|---|---|
| | | | | stress | | stress | | axial | | shear |
| Elem | Z | θ | Z | web | Z | flange | Z | flange | Z | flange |
| 1 | | | | | | | | | | |
| | 0 | 0 | 0 | -6.11 | 0 | -10.06 | 0 | 0.000 | 0 | -0.5699 |
| | 1.5 | -0.0024 | 1.5 | -6.11 | 1.5 | -10.06 | 1.5 | 0.342 | 1.5 | -0.5701 |
| | 3 | -0.0048 | 3 | -6.10 | 3 | -10.05 | 3 | 0.685 | 3 | -0.5706 |
| | 4.5 | -0.0072 | 4.5 | -6.10 | 4.5 | -10.04 | 4.5 | 1.028 | 4.5 | -0.5715 |
| | 6 | -0.0096 | 6 | -6.09 | 6 | -10.02 | 6 | 1.372 | 6 | -0.5727 |
| | 7.5 | -0.012 | 7.5 | -6.07 | 7.5 | -10.00 | 7.5 | 1.716 | 7.5 | -0.5742 |
| | 9 | -0.0144 | 9 | -6.06 | 9 | -9.98 | 9 | 2.061 | 9 | -0.5759 |
| | 10.5 | -0.0168 | 10.5 | -6.04 | 10.5 | -9.95 | 10.5 | 2.408 | 10.5 | -0.5781 |
| | 12 | -0.0192 | 12 | -6.02 | 12 | -9.91 | 12 | 2.756 | 12 | -0.5807 |
| | 13.5 | -0.0215 | 13.5 | -5.99 | 13.5 | -9.87 | 13.5 | 3.106 | 13.5 | -0.5834 |
| | 15 | -0.0239 | 15 | -5.97 | 15 | -9.83 | 15 | 3.458 | 15 | -0.5867 |
| 2 | | | | 0.00 | | 0.00 | | 0.000 | | 0.0000 |
| | 15 | -0.0239 | 15 | -5.97 | 15 | -9.83 | 15 | 3.458 | 15 | -0.5867 |
| | 16.5 | -0.0262 | 16.5 | -5.94 | 16.5 | -9.78 | 16.5 | 3.811 | 16.5 | -0.5903 |
| | 18 | -0.0286 | 18 | -5.90 | 18 | -9.72 | 18 | 4.167 | 18 | -0.5940 |
| | 19.5 | -0.0309 | 19.5 | -5.87 | 19.5 | -9.67 | 19.5 | 4.525 | 19.5 | -0.5983 |
| | 21 | -0.0332 | 21 | -5.83 | 21 | -9.60 | 21 | 4.885 | 21 | -0.6029 |
| | 22.5 | -0.0355 | 22.5 | -5.78 | 22.5 | -9.53 | 22.5 | 5.249 | 22.5 | -0.6079 |
| | 24 | -0.0377 | 24 | -5.74 | 24 | -9.45 | 24 | 5.616 | 24 | -0.6132 |
| | 25.5 | -0.04 | 25.5 | -5.69 | 25.5 | -9.37 | 25.5 | 5.986 | 25.5 | -0.6188 |
| | 27 | -0.0422 | 27 | -5.64 | 27 | -9.29 | 27 | 6.359 | 27 | -0.6248 |
| | 28.5 | -0.0444 | 28.5 | -5.59 | 28.5 | -9.20 | 28.5 | 6.737 | 28.5 | -0.6311 |
| | 30 | -0.0466 | 30 | -5.53 | 30 | -9.10 | 30 | 7.118 | 30 | -0.6378 |
| 3 | | | | 0.00 | | 0.00 | | 0.000 | | 0.0000 |
| | 30 | -0.0466 | 30 | -5.53 | 30 | -9.10 | 30 | 7.118 | 30 | -0.6379 |
| | 31.5 | -0.0488 | 31.5 | -5.46 | 31.5 | -9.00 | 31.5 | 7.501 | 31.5 | -0.6449 |
| | 33 | -0.0509 | 33 | -5.40 | 33 | -8.90 | 33 | 7.891 | 33 | -0.6523 |
| | 34.5 | -0.053 | 34.5 | -5.34 | 34.5 | -8.79 | 34.5 | 8.288 | 34.5 | -0.6602 |

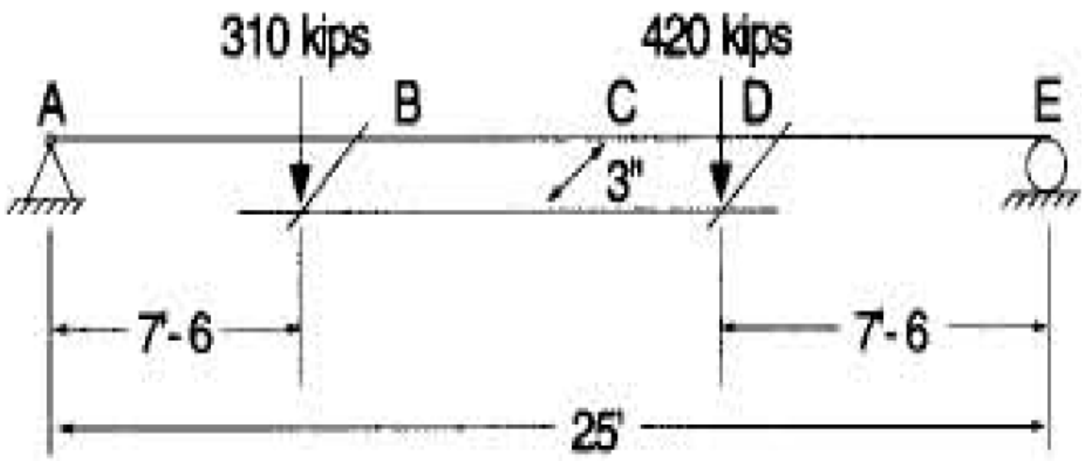
(continued) Table of Stresses. Ex. 5.5

| | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|
| 4 | 36 | -0.0551 | 36 | -5.26 | 36 | -8.67 | 36 | 8.685 | 36 | -0.6685 |
| | 37.5 | -0.0572 | 37.5 | -5.19 | 37.5 | -8.55 | 37.5 | 9.089 | 37.5 | -0.6771 |
| | 39 | -0.0592 | 39 | -5.11 | 39 | -8.42 | 39 | 9.499 | 39 | -0.6861 |
| | 40.5 | -0.0612 | 40.5 | -5.03 | 40.5 | -8.29 | 40.5 | 9.917 | 40.5 | -0.6955 |
| | 42 | -0.0632 | 42 | -4.95 | 42 | -8.15 | 42 | 10.334 | 42 | -0.7054 |
| | 43.5 | -0.0651 | 43.5 | -4.86 | 43.5 | -8.00 | 43.5 | 10.759 | 43.5 | -0.7157 |
| | 45 | -0.067 | 45 | -4.77 | 45 | -7.85 | 45 | 11.197 | 45 | -0.7265 |
| | 46.5 | -0.0689 | 46.5 | -4.67 | 46.5 | -7.70 | 46.5 | 11.635 | 46.5 | -0.7374 |
| | 48 | -0.0707 | 48 | -4.57 | 48 | -7.53 | 48 | 12.080 | 48 | -0.7489 |
| | 49.5 | -0.0725 | 49.5 | -4.47 | 49.5 | -7.36 | 49.5 | 12.531 | 49.5 | -0.7608 |
| | 51 | -0.0742 | 51 | -4.36 | 51 | -7.19 | 51 | 12.997 | 51 | -0.7733 |
| | 52.5 | -0.0759 | 52.5 | -4.25 | 52.5 | -7.01 | 52.5 | 13.462 | 52.5 | -0.7861 |
| | 54 | -0.0776 | 54 | -4.14 | 54 | -6.82 | 54 | 13.941 | 54 | -0.7994 |
| | 55.5 | -0.0792 | 55.5 | -4.02 | 55.5 | -6.62 | 55.5 | 14.427 | 55.5 | -0.8133 |
| | 57 | -0.0807 | 57 | -3.90 | 57 | -6.42 | 57 | 14.920 | 57 | -0.8275 |
| | 58.5 | -0.0822 | 58.5 | -3.77 | 58.5 | -6.21 | 58.5 | 15.420 | 58.5 | -0.8422 |
| | 60 | -0.0837 | 60 | -3.64 | 60 | -6.00 | 60 | 15.926 | 60 | -0.8574 |
| 5 | | | | 0.00 | | 0.00 | | 0.000 | | 0.0000 |
| | 60 | -0.0837 | 60 | -3.64 | 60 | -6.00 | 60 | 15.926 | 60 | -0.8575 |
| | 61.5 | -0.0851 | 61.5 | -3.50 | 61.5 | -5.77 | 61.5 | 16.446 | 61.5 | -0.8731 |
| | 63 | -0.0864 | 63 | -3.37 | 63 | -5.54 | 63 | 16.980 | 63 | -0.8893 |
| | 64.5 | -0.0877 | 64.5 | -3.22 | 64.5 | -5.31 | 64.5 | 17.521 | 64.5 | -0.9061 |
| | 66 | -0.089 | 66 | -3.07 | 66 | -5.06 | 66 | 18.068 | 66 | -0.9233 |
| | 67.5 | -0.0902 | 67.5 | -2.92 | 67.5 | -4.81 | 67.5 | 18.629 | 67.5 | -0.9413 |
| | 69 | -0.0913 | 69 | -2.76 | 69 | -4.55 | 69 | 19.197 | 69 | -0.9596 |
| | 70.5 | -0.0923 | 70.5 | -2.60 | 70.5 | -4.28 | 70.5 | 19.779 | 70.5 | -0.9785 |
| | 72 | -0.0933 | 72 | -2.43 | 72 | -4.01 | 72 | 20.375 | 72 | -0.9980 |
| | 73.5 | -0.0943 | 73.5 | -2.26 | 73.5 | -3.72 | 73.5 | 20.977 | 73.5 | -1.0180 |
| | 75 | -0.0951 | 75 | -2.08 | 75 | -3.43 | 75 | 21.600 | 75 | -1.0387 |
| 6 | | | | 0.00 | | 0.00 | | 0.000 | | 0.0000 |
| | 75 | -0.0951 | 75 | -2.08 | 75 | -3.43 | 75 | 21.600 | 75 | -1.0389 |
| | 76.5 | -0.0959 | 76.5 | -1.90 | 76.5 | -3.13 | 76.5 | 22.229 | 76.5 | -1.0600 |
| | 78 | -0.0966 | 78 | -1.71 | 78 | -2.82 | 78 | 22.873 | 78 | -1.0819 |
| | 79.5 | -0.0972 | 79.5 | -1.52 | 79.5 | -2.50 | 79.5 | 23.530 | 79.5 | -1.1045 |
| | 81 | -0.0978 | 81 | -1.32 | 81 | -2.17 | 81 | 24.200 | 81 | -1.1277 |
| | 82.5 | -0.0983 | 82.5 | -1.11 | 82.5 | -1.83 | 82.5 | 24.885 | 82.5 | -1.1515 |
| | 84 | -0.0987 | 84 | -0.90 | 84 | -1.49 | 84 | 25.583 | 84 | -1.1761 |
| | 85.5 | -0.099 | 85.5 | -0.69 | 85.5 | -1.13 | 85.5 | 26.295 | 85.5 | -1.2012 |
| | 87 | -0.0992 | 87 | -0.46 | 87 | -0.76 | 87 | 27.027 | 87 | -1.2272 |
| | 88.5 | -0.0994 | 88.5 | -0.23 | 88.5 | -0.39 | 88.5 | 27.773 | 88.5 | -1.2537 |
| | 90 | -0.0994 | 90 | 0.00 | 90 | 0.00 | 90 | 28.533 | 90 | -1.2808 |

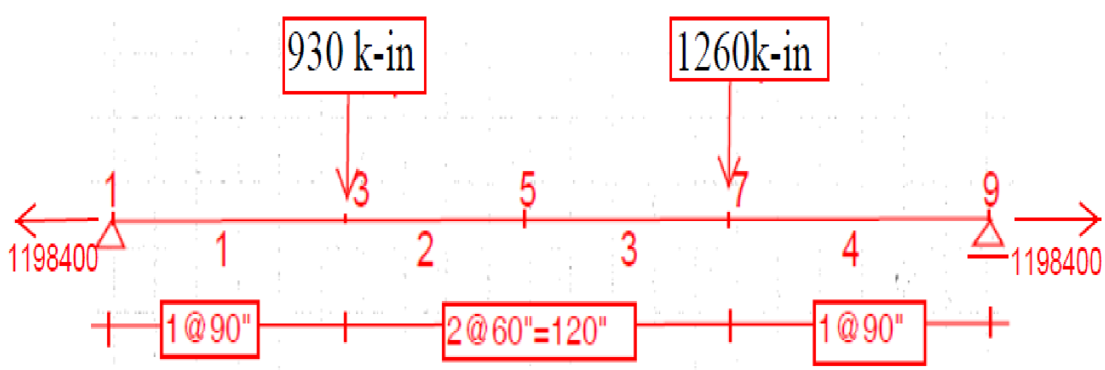
## APPENDIX E

## AISC-DG9-EXAMPLE 5.4

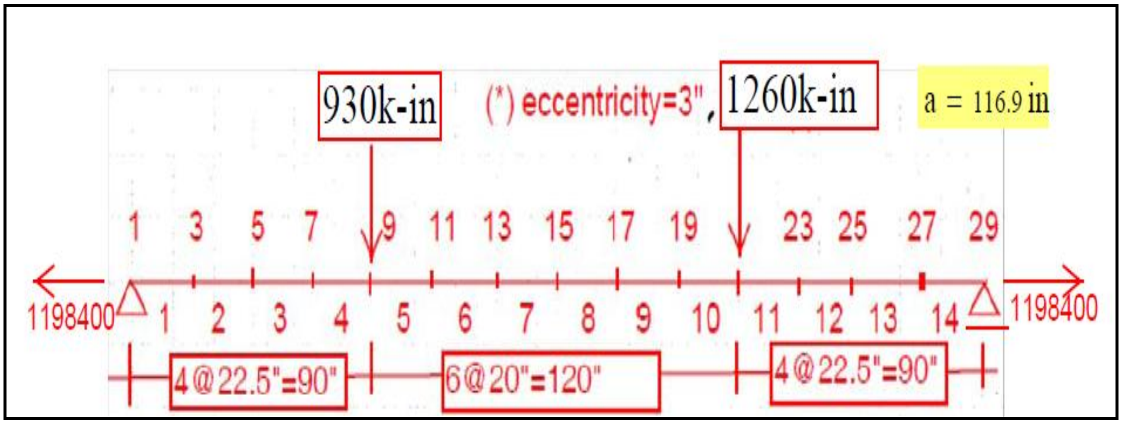
This material corresponds to the input model, input data, input forms, figures of output data with checks in notepad version, and excel processed output data and charts. An unsuccessful EBC model used before convergence study is presented. Charts are presented containing both partial and combined tresses along interest points of the beam and cross section profile. Again, the asymptotic behavior of the thin-walled beam elastic line is successfully shown in the charts, which evidences the efficiency of the high order finite element used by BMTORSWP.



Original TWB from AISC DG9



Unsuccessful EBC Model Used Before Convergence Study



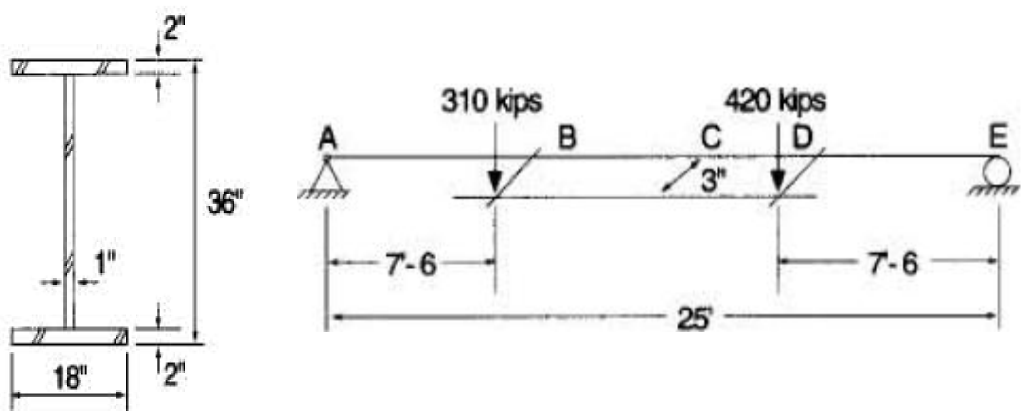
One Span between Consecutive Loads or Restraints and 3 FE in Spans

```
                                        54b
AISC Design Guide 9, Ex. 5.4
 14 29  2  0  9  0  2  0 29000.
  1  1  2  3564000. 106. 22.5                                 -1198400
  2  3  4  5564000. 106. 22.5                                 -1198400
  3  5  6  7564000. 106. 22.5                                 -1198400
  4  7  8  9564000. 106. 22.5                                 -1198400
  5  9 10 11564000. 106. 20.0                                 -1198400
  6 11 12 13564000. 106. 20.0                                 -1198400
  7 13 14 15564000. 106. 20.0                                 -1198400
  8 15 16 17564000. 106. 20.0                                 -1198400
  9 17 18 19564000. 106. 20.0                                 -1198400
 10 19 20 21564000. 106. 20.0                                 -1198400
 11 21 22 23564000. 106. 22.5                                 -1198400
 12 23 24 25564000. 106. 22.5                                 -1198400
 13 25 26 27564000. 106. 22.5                                 -1198400
 14 27 28 29564000. 106. 22.5                                 -1198400
 2 1 1 1 029 0 1 0
 2 9  930. 21  1260.
```

$-GJ$

$C_w$

Input Notepad, Ex. 5.4 DG9



TWB Cross Section and Interest Points along Beam

## PROGRAM BMTORSW INPUT FORM

ALPHAMERIC DESCRIPTION OF THE JOB — FILE NAME:

AISC Design Guide 9, Ex. 5.4

| NEL | NOD | NSUP | NSPD | JBW | NFX | NFY | NFZ | ELASTICITY (E11.4) |
|---|---|---|---|---|---|---|---|---|
| 14 | 29 | 2 | 0 | 9 | 0 | 2 | 0 | 29 000. |

| NE | NI | NC | NJ | Cw | AREA | LENGTH | WTANG | SOIL NI | SOI |
|---|---|---|---|---|---|---|---|---|---|
| 1 | 1 | 2 | 3 | 56400. | 106. | 22.5 | | | |
| 2 | 3 | 4 | 5 | SAME | SAME | 22.5 | | | |
| 3 | 5 | 6 | 7 | SAME | SAME | 22.5 | | | |
| 4 | 7 | 8 | 9 | SAME | SAME | 22.5 | | | |
| 5 | 9 | 10 | 11 | 56400. | 106. | 20.0 | | | |
| 6 | 11 | 12 | 13 | 56400. | 106. | 20.0 | | | |
| 7 | 13 | 14 | 15 | 56400. | 106. | 20.0 | | | |
| 8 | 15 | 16 | 17 | 56400. | 106. | 20.0 | | | |
| 9 | 17 | 18 | 19 | 56400. | 106. | 20.0 | | | |
| 10 | 19 | 20 | 21 | 56400. | 106. | 20.0 | | | |
| 11 | 21 | 22 | 23 | 56400. | 106. | 22.5 | | | |
| 12 | 23 | 24 | 25 | 56400. | 106. | 22.5 | | | |
| 13 | 25 | 26 | 27 | 56400. | 106. | 22.5 | | | |
| 14 | 27 | 28 | 29 | 56400. | 106. | 22.5 | | | |

| NF | N | KX | KY | KZ | N | KX | KY | KZ | N | KX | KY | KZ | N | KX | KY | KZ | N | KX | KY | K |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| 2 | 1 | 1 | 1 | 0 | 29 | 0 | 1 | 0 | | | | | | | | | | | | |

| NF | N | KD | SPRING | N | KD | SPRING | N | KD |
|---|---|---|---|---|---|---|---|---|
| | | | | | | | | |
| | | | | | | | | |

| NF | N | LOAD VALUE | N | LOAD VALUE | N | LOAD VALUE | N | LOAD VALUE | N | L |
|---|---|---|---|---|---|---|---|---|---|---|
| 2 | 9 | 930. | 21 | 1260. | | | | | | |
| | | | | | | | | | | |
| | | | | | | | | | | |
| | | | | | | | | | | |

| NF | ELEM | END | ROTATIONAL SPRING MODULUS | ELEM | END | ROTATIONAL SPRING MODULUS | ELEM | END | ROTATION SPRING MOD |
|---|---|---|---|---|---|---|---|---|---|
| | | | | | | | | | |
| | | | | | | | | | |
| | | | | | | | | | |

Input Form, Left Side, Ex. 5.4 DG9

AUTHOR: B. DESCHAPELLES

ME:

GENERAL INFORMATION

PARTIAL RESTRAINT AT END ? 1 FOR YES, 0 FOR NO

| SOIL NJ | SOIL TI | SOIL TJ | ANGLE | - G*J | Wn @ NI | Wn @ NJ | 1 | 2 |
|---|---|---|---|---|---|---|---|---|
| | | | | -1198400 | | | | |
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| | | | | -1198400 | | | | |

SUPPORTS

| Y | KZ | N | KX | KY | KZ | N | KX | KY | KZ | N | KX | KY | KZ |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| | | | | | | | | | | | | | |

SPRINGS AT SUPPORTS

| SPRING | N | KD | SPRING |
|---|---|---|---|
| | | | |

APPLIED NODAL FORCES

| N | LOAD VALUE | N | LOAD VALUE | N | LOAD VALUE |
|---|---|---|---|---|---|
| | | | | | |

SPRINGS AT ELEM. END, 1 OR 2, IF ANY

| TIONAL MODULUS | ELEM | END | ROTATIONAL SPRING MODULUS | ELEM | END | ROTATIONAL SPRING MODULUS |
|---|---|---|---|---|---|---|
| | | | | | | |

Input Form, Right Side, Ex. 5.4 DG9

```
                                 54OUT

--------------------------------------------------------------------------
YOU ARE USING COMPUTER PROGRAM BMTORSW,DEVELOPED BY DR. BERNARDO DESCHAPELLES

        INPUT DATA FILE NAME IS = 54b.txt

                OUTPUT FILE NAME IS = 54bout.txt

          STORAGE FILE FOR POST-PROCESSING WITH EXCEL = 54bgrf.grf


--------------------------------------------------------------------------
AISC Design Guide 9, Ex. 5.4

modulus of elasticity of the material=   29000. k/ft2

ELEM nodes inertia length distrib. load  AXIAL  SOIL NORMAL MODULUS,Ksf  angle
     i    j  ft.4     ft    at i    at j  LOAD   1st END       2nd END   rad
 1   1    3******** 22.50  0.000  0.000********      0.0          0.0   0.000 00
 2   3    5******** 22.50  0.000  0.000********      0.0          0.0   0.000 00
 3   5    7******** 22.50  0.000  0.000********      0.0          0.0   0.000 00
 4   7    9******** 22.50  0.000  0.000********      0.0          0.0   0.000 00
 5   9   11******** 20.00  0.000  0.000********      0.0          0.0   0.000 00
 6  11   13******** 20.00  0.000  0.000********      0.0          0.0   0.000 00
 7  13   15******** 20.00  0.000  0.000********      0.0          0.0   0.000 00
 8  15   17******** 20.00  0.000  0.000********      0.0          0.0   0.000 00
 9  17   19******** 20.00  0.000  0.000********      0.0          0.0   0.000 00
10  19   21******** 20.00  0.000  0.000********      0.0          0.0   0.000 00
11  21   23******** 22.50  0.000  0.000********      0.0          0.0   0.000 00
12  23   25******** 22.50  0.000  0.000********      0.0          0.0   0.000 00
13  25   27******** 22.50  0.000  0.000********      0.0          0.0   0.000 00
14  27   29******** 22.50  0.000  0.000********      0.0          0.0   0.000 00

    INPUT DATA RELATED TO THE  2 SUPPORTS      KX=KY=KZ restrained at Node 1
 2 1 1 1 029 0 1 0   <-----------------------  KY restrained at Node 29

    INPUT OF NODAL FORCES RELATED TO GLOBAL AXIS  2
          2 9 930.00211260.00

      FINAL  SOLUTION  FOUND  AFTER   1  ITERATIONS

     Output of nodal displacements in reference to  global axes

node  displ.      displ.      displ.     node  displ.      displ.      displ.
    along  x    along  y    around  z        along  x    along  y    around  z
    or nonn1    or nonn2    or nonn 3        or nonn1    or nonn2    or nonn 3

  1 0.0000E+00  0.0000E+00  0.3696E-03
                                          2 0.0000E+00  0.4140E-02  0.6878E-03

  3 0.0000E+00  0.8247E-02  0.3605E-03
                                          4 0.0000E+00  0.1222E-01  0.6535E-03

  5 0.0000E+00  0.1608E-01  0.3328E-03
                                          6 0.0000E+00  0.1967E-01  0.5836E-03

  7 0.0000E+00  0.2308E-01  0.2856E-03
                                          8 0.0000E+00  0.2605E-01  0.4755E-03

  9 0.0000E+00  0.2877E-01  0.2171E-03
                                         10 0.0000E+00  0.3071E-01  0.3035E-03

 11 0.0000E+00  0.3242E-01  0.1476E-03
                                 Page 1
```

Page 1, Output Notepad, AISC Ex 5.4

```
                                                   54OUT
                                                    12 0.0000E+00  0.3367E-01  0.1896E-03
  13 0.0000E+00  0.3469E-01  0.8011E-04
                                                    14 0.0000E+00  0.3527E-01  0.7735E-04
  15 0.0000E+00  0.3562E-01  0.1250E-04
                                                    16 0.0000E+00  0.3552E-01 -0.3672E-04
  17 0.0000E+00  0.3518E-01 -0.5717E-04
                                                    18 0.0000E+00  0.3437E-01 -0.1559E-03
  19 0.0000E+00  0.3331E-01 -0.1310E-03
                                                    20 0.0000E+00  0.3174E-01 -0.2837E-03
  21 0.0000E+00  0.2990E-01 -0.2110E-03
                                                    22 0.0000E+00  0.2720E-01 -0.4765E-03
  23 0.0000E+00  0.2419E-01 -0.2919E-03
                                                    24 0.0000E+00  0.2068E-01 -0.6042E-03
  25 0.0000E+00  0.1695E-01 -0.3477E-03
                                                    26 0.0000E+00  0.1290E-01 -0.6867E-03
  27 0.0000E+00  0.8719E-02 -0.3803E-03
                                                    28 0.0000E+00  0.4380E-02 -0.7272E-03
  29 0.0000E+00  0.0000E+00 -0.3911E-03

----------------------------------------------------------------------
     OUTPUT OF SOIL REACTIONS,STRESSES AND TRANSVERSE DISPLACEMENTS

----------------------------------------------------------------------
     ELEMENT    1  DISPLACEMENTS IN INCIDENCES    1    2    3
        NODE    1          0.00000E+00         0.00000E+00         0.36955E-03
        NODE    2          0.00000E+00         0.41404E-02         0.68780E-03
        NODE    3          0.00000E+00         0.82468E-02         0.36045E-03
     FORCES ACTING ALONG THE 9 DOF
        NODE    1          0.00000E+00        -0.10290E+04        -0.14755E-08
        NODE    2          0.00000E+00         0.36738E-09         0.11494E-07
        NODE    3          0.00000E+00         0.10290E+04        -0.13270E+05

ELEMENT   1, FROM NODE   1, TO NODE  3 - LENGTH = 22.50 ft

 left half of span,at tenth points of length
              span       span       span       span       span       span
          0.0        0.1        0.2        0.3        0.4        0.5
soil,k/ft     0.000      0.000      0.000      0.000      0.000      0.000
shear,k    -1029.00   -1029.00   -1029.00   -1029.00   -1029.00   -1029.00
bmom,kft       0.00   -1318.87   -2638.23   -3958.57   -5280.37   -6604.14
tdisp,ft    0.00000    0.00083    0.00166    0.00249    0.00332    0.00415
axial,k        0.00 AT 1st END and      0.00 AT 2nd END

right half of span,at tenth points of length
              span       span       span       span       span       span
          0.5        0.6        0.7        0.8        0.9        1.0
soil,k/ft     0.000      0.000      0.000      0.000      0.000      0.000
shear,k    -1029.00   -1029.00   -1029.00   -1029.00   -1029.00   -1029.00
bmom,kft   -6604.14   -7930.35   -9259.51 -10592.10 -11928.62 -13269.57
tdisp,ft    0.00415    0.00497    0.00580    0.00662    0.00743    0.00825
axial,k        0.00 AT 1st END and      0.00 AT 2nd END

----------------------------------------------------------------------
     ELEMENT    2  DISPLACEMENTS IN INCIDENCES    3    4    5
                                                Page 2
```

(-Di + Dj)* GJ + Vj*L = Mj-Mi, where GJ=1198400

0.00825*1198400+(-1029)*22.5 = 9886.8 - 23152.25 = - 13265.7

Mj-Mi = -13269.57-0 = -13269.6

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```
                                                54OUT
           NODE    3          0.00000E+00          0.82468E-02          0.36045E-03
           NODE    4          0.00000E+00          0.12216E-01          0.65349E-03
           NODE    5          0.00000E+00          0.16082E-01          0.33282E-03
        FORCES ACTING ALONG THE 9 DOF
           NODE    3          0.00000E+00         -0.10290E+04          0.13270E+05
           NODE    4          0.00000E+00         -0.17447E-08          0.66930E-08
           NODE    5          0.00000E+00          0.10290E+04         -0.27033E+05

   ELEMENT   2, FROM NODE   3, TO NODE   5 - LENGTH = 22.50 ft

    left half of span,at tenth points of length
                span      span      span      span      span      span
             0.0       0.1       0.2       0.3       0.4       0.5
   soil,k/ft    0.000     0.000     0.000     0.000     0.000     0.000
   shear,k   -1029.00  -1029.00  -1029.00  -1029.00  -1029.00  -1029.00
   bmom,kft -13269.57 -14615.43 -15966.72 -17323.93 -18687.57 -20058.14
   tdisp,ft   0.00825   0.00906   0.00986   0.01066   0.01145   0.01224
   axial,k        0.00 AT 1st END and      0.00 AT 2nd END

   right half of span,at tenth points of length
                span      span      span      span      span      span
             0.5       0.6       0.7       0.8       0.9       1.0
   soil,k/ft    0.000     0.000     0.000     0.000     0.000     0.000
   shear,k   -1029.00  -1029.00  -1029.00  -1029.00  -1029.00  -1029.00
   bmom,kft -20058.14 -21436.15 -22822.11 -24216.54 -25619.95 -27032.86
   tdisp,ft   0.01224   0.01302   0.01380   0.01457   0.01533   0.01608
   axial,k        0.00 AT 1st END and      0.00 AT 2nd END

   ----------------------------------------------------------------------
        ELEMENT    3  DISPLACEMENTS IN INCIDENCES    5    6    7
           NODE    5          0.00000E+00          0.16082E-01          0.33282E-03
           NODE    6          0.00000E+00          0.19668E-01          0.58359E-03
           NODE    7          0.00000E+00          0.23077E-01          0.28562E-03
        FORCES ACTING ALONG THE 9 DOF
           NODE    5          0.00000E+00         -0.10290E+04          0.27033E+05
           NODE    6          0.00000E+00         -0.72288E-09         -0.50648E-07
           NODE    7          0.00000E+00          0.10290E+04         -0.41802E+05

   ELEMENT   3, FROM NODE   5, TO NODE   7 - LENGTH = 22.50 ft

    left half of span,at tenth points of length
                span      span      span      span      span      span
             0.0       0.1       0.2       0.3       0.4       0.5
   soil,k/ft    0.000     0.000     0.000     0.000     0.000     0.000
   shear,k   -1029.00  -1029.00  -1029.00  -1029.00  -1029.00  -1029.00
   bmom,kft -27032.86 -28455.80 -29889.30 -31333.88 -32790.09 -34258.46
   tdisp,ft   0.01608   0.01683   0.01756   0.01829   0.01901   0.01971
   axial,k        0.00 AT 1st END and      0.00 AT 2nd END

   right half of span,at tenth points of length
                span      span      span      span      span      span
             0.5       0.6       0.7       0.8       0.9       1.0
   soil,k/ft    0.000     0.000     0.000     0.000     0.000     0.000
   shear,k   -1029.00  -1029.00  -1029.00  -1029.00  -1029.00  -1029.00
   bmom,kft -34258.46 -35739.54 -37233.87 -38742.02 -40264.53 -41801.99
   tdisp,ft   0.01971   0.02041   0.02109   0.02177   0.02243   0.02308
   axial,k        0.00 AT 1st END and      0.00 AT 2nd END

   ----------------------------------------------------------------------
        ELEMENT    4  DISPLACEMENTS IN INCIDENCES    7    8    9
           NODE    7          0.00000E+00          0.23077E-01          0.28562E-03
           NODE    8          0.00000E+00          0.26054E-01          0.47550E-03
           NODE    9          0.00000E+00          0.28775E-01          0.21710E-03
                                                Page 3
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(-Di + Dj)* GJ + Vj*L = Mj-Mi, where GJ=1198400

-0.01608+0.02308)1198400 -1029*22.5= 8388.8-23152.5=14763.7

-41801.99+27032.86 =-14769.13 OK

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                                                  54OUT
     FORCES ACTING ALONG THE 9 DOF
        NODE    7          0.00000E+00          -0.10290E+04          0.41802E+05
        NODE    8          0.00000E+00          -0.63679E-09          0.64662E-07
        NODE    9          0.00000E+00           0.10290E+04         -0.58126E+05

ELEMENT   4, FROM NODE   7, TO NODE   9 - LENGTH = 22.50 ft  Warp. Torque   Bimoment

 left half of span,at tenth points of length
              span      span      span      span      span      span
          0.0       0.1       0.2       0.3       0.4       0.5
soil,k/ft    0.000     0.000     0.000     0.000     0.000     0.000
shear,k   -1029.00  -1029.00  -1029.00  -1029.00  -1029.00  -1029.00
bmom,kft -41801.99 -43354.95 -44923.99 -46509.69 -48112.65 -49733.45
tdisp,ft   0.02308   0.02371   0.02434   0.02494   0.02554   0.02612
axial,k       0.00 AT 1st END and      0.00 AT 2nd END

right half of span,at tenth points of length
              span      span      span      span      span      span
          0.5       0.6       0.7       0.8       0.9       1.0
soil,k/ft    0.000     0.000     0.000     0.000     0.000     0.000
shear,k   -1029.00  -1029.00  -1029.00  -1029.00  -1029.00  -1029.00
bmom,kft -49733.45 -51372.71 -53031.01 -54708.99 -56407.27 -58126.46
tdisp,ft   0.02612   0.02668   0.02723   0.02776   0.02828   0.02877
axial,k       0.00 AT 1st END and      0.00 AT 2nd END

----------------------------------------------------------------------
     ELEMENT    5  DISPLACEMENTS IN INCIDENCES    9  10  11
        NODE    9          0.00000E+00           0.28775E-01          0.21710E-03
        NODE   10          0.00000E+00           0.30712E-01          0.30347E-03
        NODE   11          0.00000E+00           0.32417E-01          0.14765E-03
     FORCES ACTING ALONG THE 9 DOF
        NODE    9          0.00000E+00          -0.99000E+02          0.58126E+05
        NODE   10          0.00000E+00           0.99470E-09          0.23257E-07
        NODE   11          0.00000E+00           0.99000E+02         -0.55741E+05

ELEMENT   5, FROM NODE   9, TO NODE 11 - LENGTH = 20.00 ft

 left half of span,at tenth points of length
              span      span      span      span      span      span
          0.0       0.1       0.2       0.3       0.4       0.5
soil,k/ft    0.000     0.000     0.000     0.000     0.000     0.000
shear,k     -99.00    -99.00    -99.00    -99.00    -99.00    -99.00
bmom,kft -58126.46 -57812.63 -57515.74 -57235.70 -56972.45 -56725.89
tdisp,ft   0.02877   0.02920   0.02961   0.03001   0.03040   0.03077
axial,k       0.00 AT 1st END and      0.00 AT 2nd END

right half of span,at tenth points of length
              span      span      span      span      span      span
          0.5       0.6       0.7       0.8       0.9       1.0
soil,k/ft    0.000     0.000     0.000     0.000     0.000     0.000
shear,k     -99.00    -99.00    -99.00    -99.00    -99.00    -99.00
bmom,kft -56725.89 -56495.95 -56282.57 -56085.69 -55905.25 -55741.19
tdisp,ft   0.03077   0.03113   0.03147   0.03180   0.03212   0.03242
axial,k       0.00 AT 1st END and      0.00 AT 2nd END

----------------------------------------------------------------------
     ELEMENT    6  DISPLACEMENTS IN INCIDENCES   11  12  13
        NODE   11          0.00000E+00           0.32417E-01          0.14765E-03
        NODE   12          0.00000E+00           0.33668E-01          0.18965E-03
        NODE   13          0.00000E+00           0.34693E-01          0.80110E-04
     FORCES ACTING ALONG THE 9 DOF
        NODE   11          0.00000E+00          -0.99000E+02          0.55741E+05
        NODE   12          0.00000E+00           0.72169E-08         -0.27496E-07
                                                            Page 4
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(-Di + Dj)* GJ + Vj*L = Mj-Mi, where GJ=1198400

-0.02877+0.03242)*1198400-99*20=4374.16-1980=2394.16

-55741.19+58126.46=2385.27 OK

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                                    54OUT
      NODE  13          0.00000E+00         0.99000E+02        -0.54994E+05

ELEMENT   6, FROM NODE 11, TO NODE 13 - LENGTH = 20.00 ft

 left half of span,at tenth points of length
              span      span      span      span      span      span
          0.0       0.1       0.2       0.3       0.4       0.5
soil,k/ft    0.000     0.000     0.000     0.000     0.000     0.000
shear,k     -99.00    -99.00    -99.00    -99.00    -99.00    -99.00
bmom,kft -55741.19 -55593.47 -55462.04 -55346.87 -55247.92 -55165.16
tdisp,ft   0.03242   0.03271   0.03298   0.03324   0.03349   0.03372
axial,k       0.00 AT 1st END and      0.00 AT 2nd END

right half of span,at tenth points of length
              span      span      span      span      span      span
          0.5       0.6       0.7       0.8       0.9       1.0
soil,k/ft    0.000     0.000     0.000     0.000     0.000     0.000
shear,k     -99.00    -99.00    -99.00    -99.00    -99.00    -99.00
bmom,kft -55165.16 -55098.57 -55048.13 -55013.82 -54995.63 -54993.57
tdisp,ft   0.03372   0.03394   0.03415   0.03435   0.03453   0.03469
axial,k       0.00 AT 1st END and      0.00 AT 2nd END

----------------------------------------------------------------------
     ELEMENT    7  DISPLACEMENTS IN INCIDENCES  13  14  15   Max. Twist Angle
      NODE  13          0.00000E+00         0.34693E-01         0.80110E-04
      NODE  14          0.00000E+00         0.35270E-01         0.77350E-04
      NODE  15          0.00000E+00         0.35621E-01         0.12498E-04
     FORCES ACTING ALONG THE 9 DOF
      NODE  13          0.00000E+00        -0.99000E+02         0.54994E+05
      NODE  14          0.00000E+00        -0.81297E-08         0.37656E-07
      NODE  15          0.00000E+00         0.99000E+02        -0.55862E+05

ELEMENT   7, FROM NODE 13, TO NODE 15 - LENGTH = 20.00 ft

 left half of span,at tenth points of length
              span      span      span      span      span      span
          0.0       0.1       0.2       0.3       0.4       0.5
soil,k/ft    0.000     0.000     0.000     0.000     0.000     0.000
shear,k     -99.00    -99.00    -99.00    -99.00    -99.00    -99.00
bmom,kft -54993.57 -55007.62 -55037.79 -55084.10 -55146.55 -55225.16
tdisp,ft   0.03469   0.03485   0.03499   0.03511   0.03523   0.03533
axial,k       0.00 AT 1st END and      0.00 AT 2nd END

right half of span,at tenth points of length
              span      span      span      span      span      span
          0.5       0.6       0.7       0.8       0.9       1.0
soil,k/ft    0.000     0.000     0.000     0.000     0.000     0.000
shear,k     -99.00    -99.00    -99.00    -99.00    -99.00    -99.00
bmom,kft -55225.16 -55319.96 -55430.97 -55558.22 -55701.76 -55861.63
tdisp,ft   0.03533   0.03541   0.03548   0.03554   0.03559   0.03562
axial,k       0.00 AT 1st END and      0.00 AT 2nd END

----------------------------------------------------------------------
     ELEMENT    8  DISPLACEMENTS IN INCIDENCES  15  16  17
      NODE  15          0.00000E+00         0.35621E-01         0.12498E-04
      NODE  16          0.00000E+00         0.35516E-01        -0.36718E-04
      NODE  17          0.00000E+00         0.35180E-01        -0.57173E-04
     FORCES ACTING ALONG THE 9 DOF
      NODE  15          0.00000E+00        -0.99000E+02         0.55862E+05
      NODE  16          0.00000E+00        -0.11674E-08        -0.73443E-07
      NODE  17          0.00000E+00         0.99000E+02        -0.58371E+05

ELEMENT   8, FROM NODE 15, TO NODE 17 - LENGTH = 20.00 ft
                                   Page 5
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                                   54OUT
 left half of span,at tenth points of length
              span      span      span      span      span      span
          0.0       0.1       0.2       0.3       0.4       0.5
soil,k/ft     0.000     0.000     0.000     0.000     0.000     0.000
shear,k      -99.00    -99.00    -99.00    -99.00    -99.00    -99.00
bmom,kft -55861.63 -56037.87 -56230.53 -56439.67 -56665.36 -56907.65
tdisp,ft    0.03562   0.03564   0.03564   0.03563   0.03561   0.03557
axial,k        0.00 AT 1st END and      0.00 AT 2nd END

right half of span,at tenth points of length
              span      span      span      span      span      span
          0.5       0.6       0.7       0.8       0.9       1.0
soil,k/ft     0.000     0.000     0.000     0.000     0.000     0.000
shear,k      -99.00    -99.00    -99.00    -99.00    -99.00    -99.00
bmom,kft -56907.65 -57166.62 -57442.34 -57734.91 -58044.39 -58370.88
tdisp,ft    0.03557   0.03552   0.03546   0.03538   0.03529   0.03518
axial,k        0.00 AT 1st END and      0.00 AT 2nd END

----------------------------------------------------------------------
     ELEMENT   9  DISPLACEMENTS IN INCIDENCES  17  18  19
        NODE  17         0.00000E+00         0.35180E-01        -0.57173E-04
        NODE  18         0.00000E+00         0.34366E-01        -0.15591E-03
        NODE  19         0.00000E+00         0.33307E-01        -0.13095E-03
     FORCES ACTING ALONG THE 9 DOF
        NODE  17         0.00000E+00        -0.99000E+02         0.58371E+05
        NODE  18         0.00000E+00         0.75810E-08         0.12084E-06
        NODE  19         0.00000E+00         0.99000E+02        -0.62595E+05

ELEMENT   9, FROM NODE 17, TO NODE 19 - LENGTH = 20.00 ft

 left half of span,at tenth points of length
              span      span      span      span      span      span
          0.0       0.1       0.2       0.3       0.4       0.5
soil,k/ft     0.000     0.000     0.000     0.000     0.000     0.000
shear,k      -99.00    -99.00    -99.00    -99.00    -99.00    -99.00
bmom,kft -58370.88 -58714.48 -59075.30 -59453.42 -59848.97 -60262.06
tdisp,ft    0.03518   0.03506   0.03492   0.03477   0.03461   0.03443
axial,k        0.00 AT 1st END and      0.00 AT 2nd END

right half of span,at tenth points of length
              span      span      span      span      span      span
          0.5       0.6       0.7       0.8       0.9       1.0
soil,k/ft     0.000     0.000     0.000     0.000     0.000     0.000
shear,k      -99.00    -99.00    -99.00    -99.00    -99.00    -99.00
bmom,kft -60262.06 -60692.81 -61141.35 -61607.81 -62092.33 -62595.05
tdisp,ft    0.03443   0.03423   0.03402   0.03380   0.03356   0.03331
axial,k        0.00 AT 1st END and      0.00 AT 2nd END

----------------------------------------------------------------------
     ELEMENT  10  DISPLACEMENTS IN INCIDENCES  19  20  21
        NODE  19         0.00000E+00         0.33307E-01        -0.13095E-03
        NODE  20         0.00000E+00         0.31737E-01        -0.28373E-03
        NODE  21         0.00000E+00         0.29900E-01        -0.21100E-03
     FORCES ACTING ALONG THE 9 DOF
        NODE  19         0.00000E+00        -0.99000E+02         0.62595E+05
        NODE  20         0.00000E+00        -0.78221E-08         0.13106E-06
        NODE  21         0.00000E+00         0.99000E+02        -0.68658E+05

ELEMENT 10, FROM NODE 19, TO NODE 21 - LENGTH = 20.00 ft
                                                       Warp. Torque  Bimoment
 left half of span,at tenth points of length
              span      span      span      span      span      span
                                  Page 6
```

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```
                                          54OUT
          0.0          0.1          0.2          0.3          0.4          0.5
soil,k/ft     0.000        0.000        0.000        0.000        0.000        0.000
shear,k      -99.00       -99.00       -99.00       -99.00       -99.00       -99.00
bmom,kft -62595.05 -63116.11 -63655.67 -64213.88 -64790.92 -65386.94
tdisp,ft    0.03331     0.03304     0.03275     0.03245     0.03214     0.03180
axial,k           0.00 AT 1st END and       0.00 AT 2nd END

right half of span,at tenth points of length
              span         span         span         span         span         span
          0.5          0.6          0.7          0.8          0.9          1.0
soil,k/ft     0.000        0.000        0.000        0.000        0.000        0.000
shear,k      -99.00       -99.00       -99.00       -99.00       -99.00       -99.00
bmom,kft -65386.94 -66002.13 -66636.67 -67290.73 -67964.52 -68658.22
tdisp,ft    0.03180     0.03146     0.03109     0.03071     0.03031     0.02990
axial,k           0.00 AT 1st END and       0.00 AT 2nd END
                                                                  Warp. Torque  Bimoment
-----------------------------------------------------------------------
     ELEMENT  11  DISPLACEMENTS IN INCIDENCES  21  22  23
        NODE  21           0.00000E+00           0.29900E-01           -0.21100E-03
        NODE  22           0.00000E+00           0.27197E-01           -0.47648E-03
        NODE  23           0.00000E+00           0.24192E-01           -0.29194E-03
     FORCES ACTING ALONG THE 9 DOF
        NODE  21           0.00000E+00           0.11610E+04            0.68658E+05
        NODE  22           0.00000E+00           0.55354E-09            0.35221E-07
        NODE  23           0.00000E+00          -0.11610E+04           -0.49376E+05

ELEMENT 11, FROM NODE 21, TO NODE 23 - LENGTH = 22.50 ft

 left half of span,at tenth points of length
              span         span         span         span         span         span
          0.0          0.1          0.2          0.3          0.4          0.5
soil,k/ft     0.000        0.000        0.000        0.000        0.000        0.000
shear,k     1161.00      1161.00      1161.00      1161.00      1161.00      1161.00
bmom,kft -68658.22 -66627.53 -64621.55 -62639.54 -60680.77 -58744.51
tdisp,ft    0.02990     0.02941     0.02891     0.02838     0.02784     0.02727
axial,k           0.00 AT 1st END and       0.00 AT 2nd END

right half of span,at tenth points of length
              span         span         span         span         span         span
          0.5          0.6          0.7          0.8          0.9          1.0
soil,k/ft     0.000        0.000        0.000        0.000        0.000        0.000
shear,k     1161.00      1161.00      1161.00      1161.00      1161.00      1161.00
bmom,kft -58744.51 -56830.03 -54936.64 -53063.63 -51210.30 -49375.96
tdisp,ft    0.02727     0.02669     0.02609     0.02547     0.02484     0.02419
axial,k           0.00 AT 1st END and       0.00 AT 2nd END

-----------------------------------------------------------------------
     ELEMENT  12  DISPLACEMENTS IN INCIDENCES  23  24  25
        NODE  23           0.00000E+00           0.24192E-01           -0.29194E-03
        NODE  24           0.00000E+00           0.20676E-01           -0.60415E-03
        NODE  25           0.00000E+00           0.16951E-01           -0.34769E-03
     FORCES ACTING ALONG THE 9 DOF
        NODE  23           0.00000E+00           0.11610E+04            0.49376E+05
        NODE  24           0.00000E+00           0.16209E-08           -0.21178E-07
        NODE  25           0.00000E+00          -0.11610E+04           -0.31931E+05

ELEMENT 12, FROM NODE 23, TO NODE 25 - LENGTH = 22.50 ft

 left half of span,at tenth points of length
              span         span         span         span         span         span
          0.0          0.1          0.2          0.3          0.4          0.5
soil,k/ft     0.000        0.000        0.000        0.000        0.000        0.000
shear,k     1161.00      1161.00      1161.00      1161.00      1161.00      1161.00
                                          Page 7
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```
                                          54OUT
bmom,kft -49375.96 -47559.94 -45761.57 -43980.16 -42215.08 -40465.65
tdisp,ft   0.02419   0.02353   0.02285   0.02216   0.02145   0.02073
axial,k        0.00 AT 1st END and      0.00 AT 2nd END

right half of span,at tenth points of length
              span      span      span      span      span      span
          0.5       0.6       0.7       0.8       0.9       1.0
soil,k/ft    0.000     0.000     0.000     0.000     0.000     0.000
shear,k    1161.00   1161.00   1161.00   1161.00   1161.00   1161.00
bmom,kft -40465.65 -38731.23 -37011.18 -35304.85 -33611.63 -31930.87
tdisp,ft   0.02073   0.02000   0.01925   0.01850   0.01773   0.01695
axial,k        0.00 AT 1st END and      0.00 AT 2nd END

----------------------------------------------------------------------
     ELEMENT  13  DISPLACEMENTS IN INCIDENCES  25  26  27
        NODE  25          0.00000E+00          0.16951E-01         -0.34769E-03
        NODE  26          0.00000E+00          0.12896E-01         -0.68672E-03
        NODE  27          0.00000E+00          0.87188E-02         -0.38033E-03
     FORCES ACTING ALONG THE 9 DOF
        NODE  25          0.00000E+00          0.11610E+04          0.31931E+05
        NODE  26          0.00000E+00          0.97435E-09         -0.15432E-07
        NODE  27          0.00000E+00         -0.11610E+04         -0.15674E+05

ELEMENT 13, FROM NODE 25, TO NODE 27 - LENGTH = 22.50 ft

 left half of span,at tenth points of length
              span      span      span      span      span      span
          0.0       0.1       0.2       0.3       0.4       0.5
soil,k/ft    0.000     0.000     0.000     0.000     0.000     0.000
shear,k    1161.00   1161.00   1161.00   1161.00   1161.00   1161.00
bmom,kft -31930.87 -30261.95 -28604.26 -26957.18 -25320.10 -23692.42
tdisp,ft   0.01695   0.01616   0.01537   0.01456   0.01375   0.01293
axial,k        0.00 AT 1st END and      0.00 AT 2nd END

right half of span,at tenth points of length
              span      span      span      span      span      span
          0.5       0.6       0.7       0.8       0.9       1.0
soil,k/ft    0.000     0.000     0.000     0.000     0.000     0.000
shear,k    1161.00   1161.00   1161.00   1161.00   1161.00   1161.00
bmom,kft -23692.42 -22073.52 -20462.81 -18859.69 -17263.56 -15673.84
tdisp,ft   0.01293   0.01210   0.01126   0.01042   0.00957   0.00872
axial,k        0.00 AT 1st END and      0.00 AT 2nd END

----------------------------------------------------------------------
     ELEMENT  14  DISPLACEMENTS IN INCIDENCES  27  28  29
        NODE  27          0.00000E+00          0.87188E-02         -0.38033E-03
        NODE  28          0.00000E+00          0.43796E-02         -0.72724E-03
        NODE  29          0.00000E+00          0.00000E+00         -0.39108E-03
     FORCES ACTING ALONG THE 9 DOF
        NODE  27          0.00000E+00          0.11610E+04          0.15674E+05
        NODE  28          0.00000E+00         -0.33970E-09         -0.77334E-08
        NODE  29          0.00000E+00         -0.11610E+04         -0.17972E-08

ELEMENT 14, FROM NODE 27, TO NODE 29 - LENGTH = 22.50 ft

 left half of span,at tenth points of length
              span      span      span      span      span      span
          0.0       0.1       0.2       0.3       0.4       0.5
soil,k/ft    0.000     0.000     0.000     0.000     0.000     0.000
shear,k    1161.00   1161.00   1161.00   1161.00   1161.00   1161.00
bmom,kft -15673.84 -14089.93 -12511.25 -10937.21  -9367.23  -7800.72
tdisp,ft   0.00872   0.00786   0.00700   0.00613   0.00526   0.00439
axial,k        0.00 AT 1st END and      0.00 AT 2nd END
                                         Page 8
```

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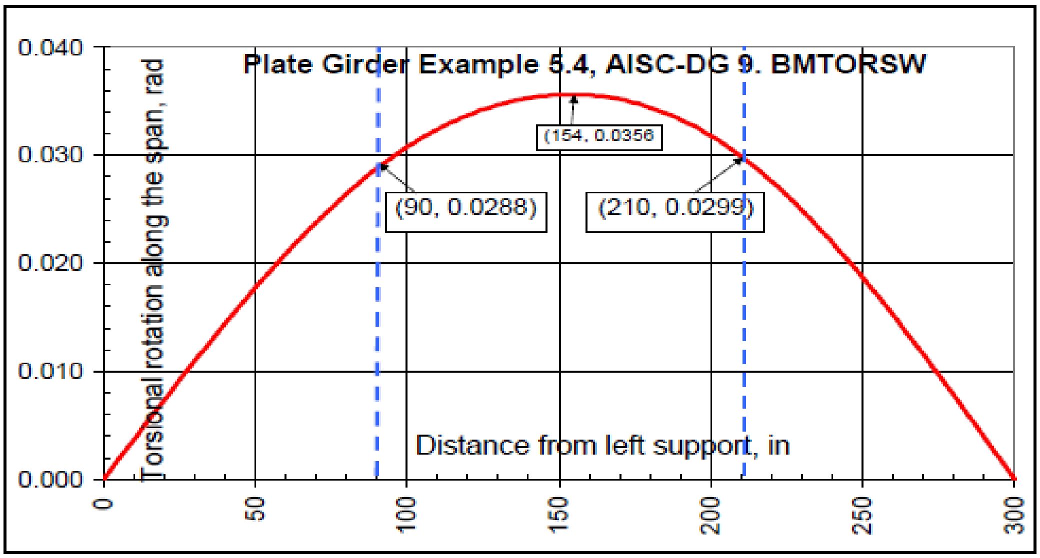
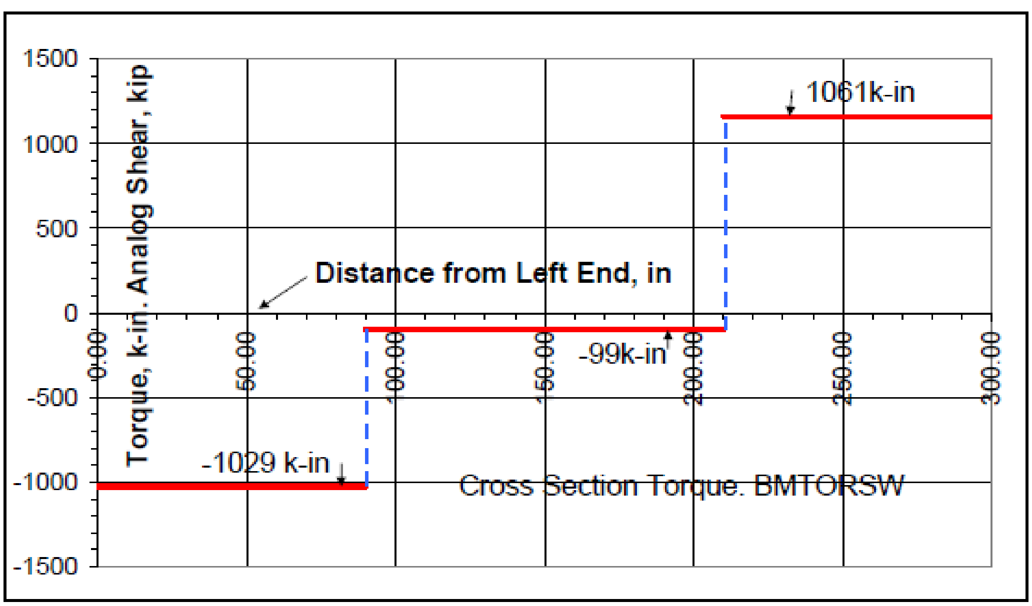
```
(-Di + Dj)* GJ +  Vj*L = Mj-Mi, where GJ=1198400
(-0.00872 + 0)*1198400 +  1161*22.5 = 0- -15673.84  54OUT
-10450.05 + 26122.5 = 15675.45 OK
right half of span,at tenth points of length
                span        span        span        span        span        span
            0.5         0.6         0.7         0.8         0.9         1.0
soil,k/ft     0.000       0.000       0.000       0.000       0.000       0.000
shear,k     1161.00     1161.00     1161.00     1161.00     1161.00     1161.00
bmom,kft   -7800.72    -6237.11    -4675.81    -3116.24    -1557.83        0.00
tdisp,ft    0.00439     0.00351     0.00264     0.00176     0.00088     0.00000
axial,k         0.00 AT 1st END and        0.00 AT 2nd END
```

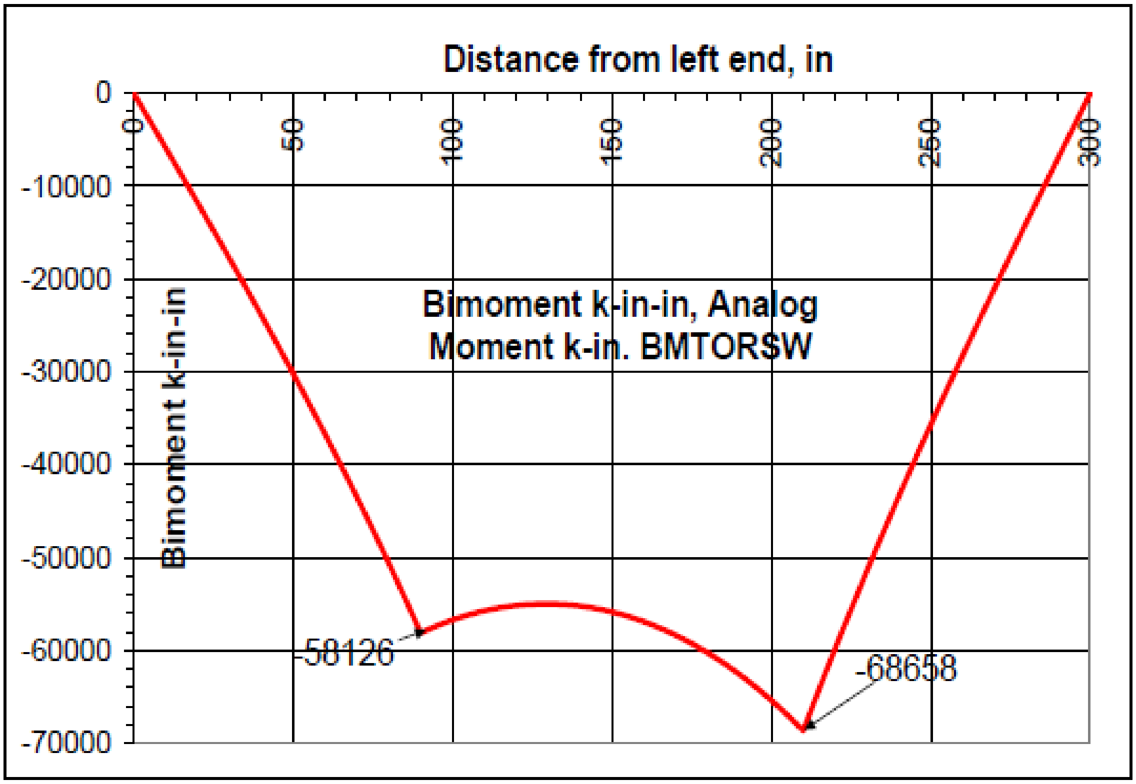
Last Page. Output Notepad. AISC Ex 5.4



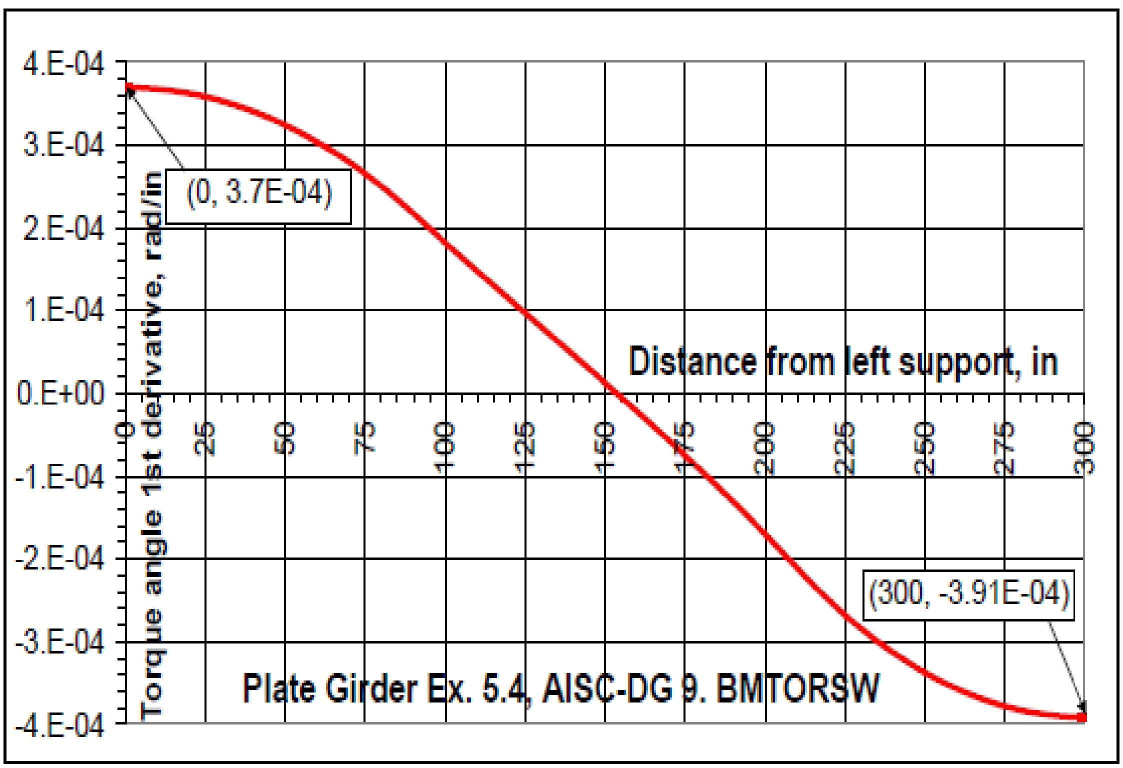

Twist Angle for Ultimate Loads in the TWB



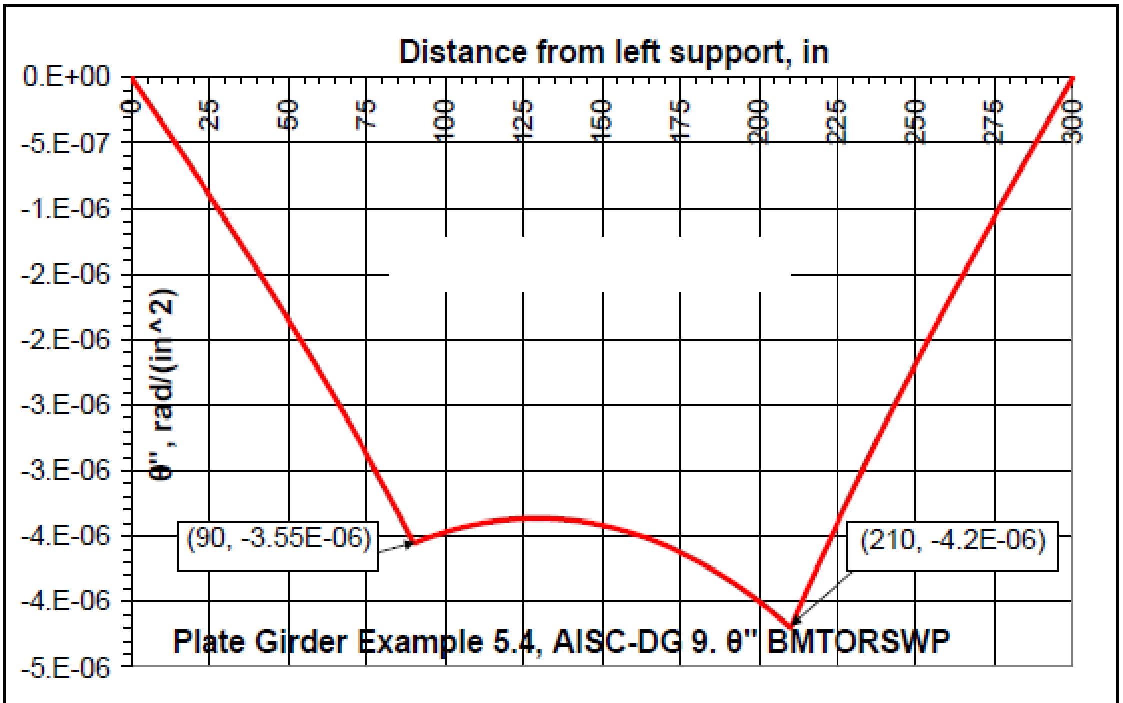

TWB Torque and BC Analogue Shear from BMTORSWP



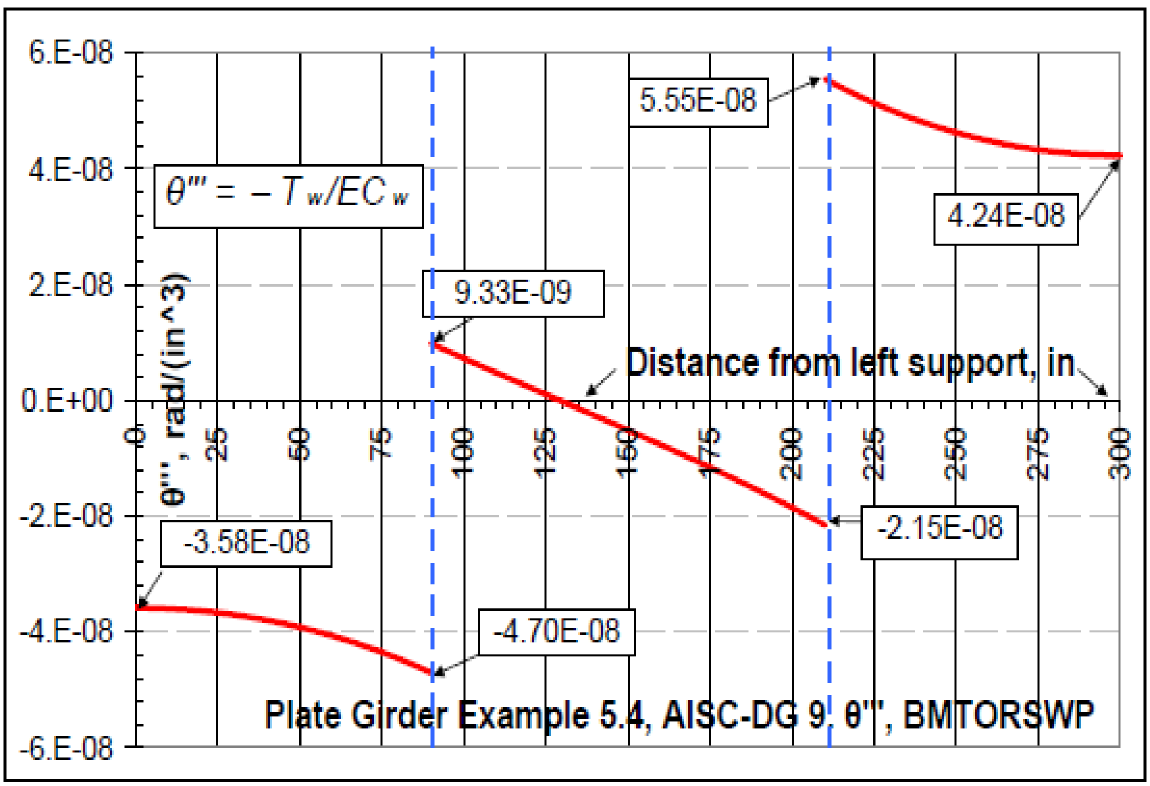

TWB Bimoment and BC Analogue Moment from BMTORSWP



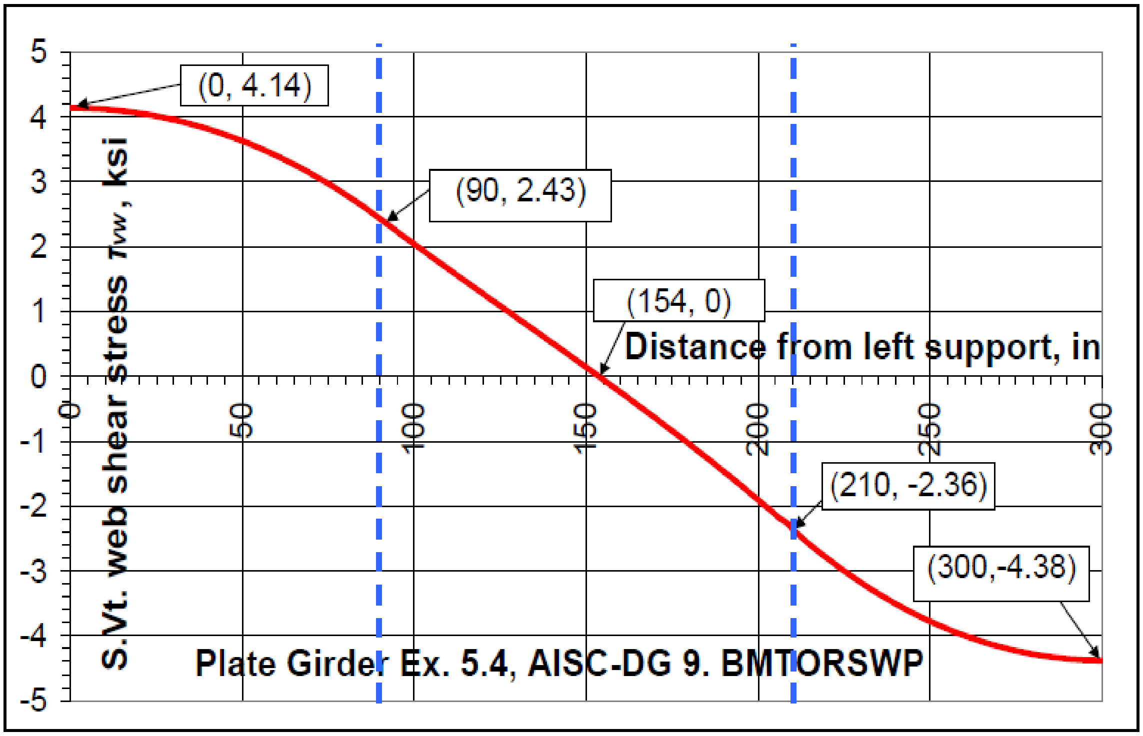

Twist Angle Derivative



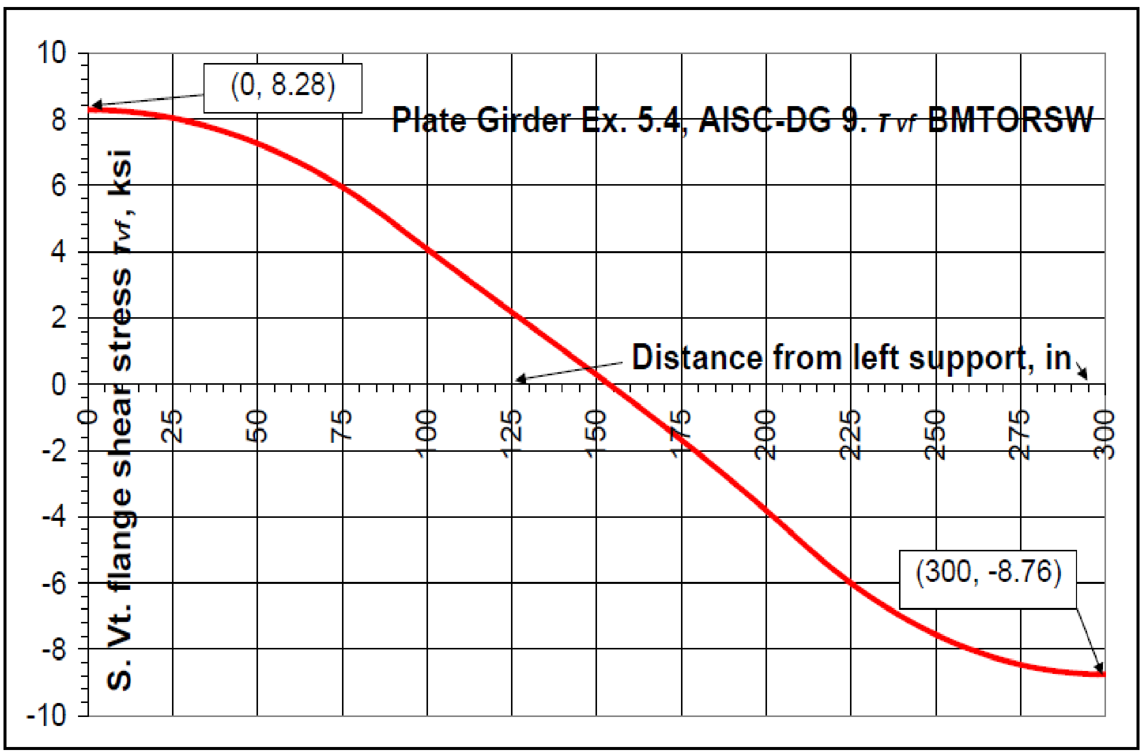

Twist Angle 2nd Derivative



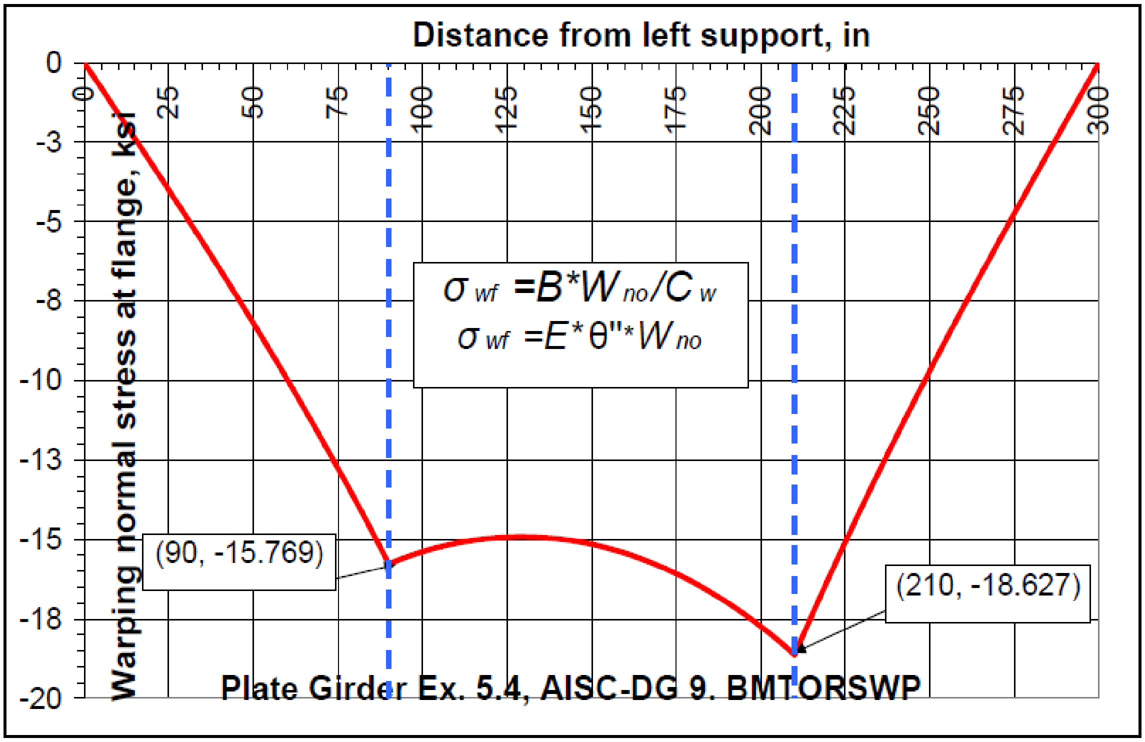

Twist Angle Third Derivative



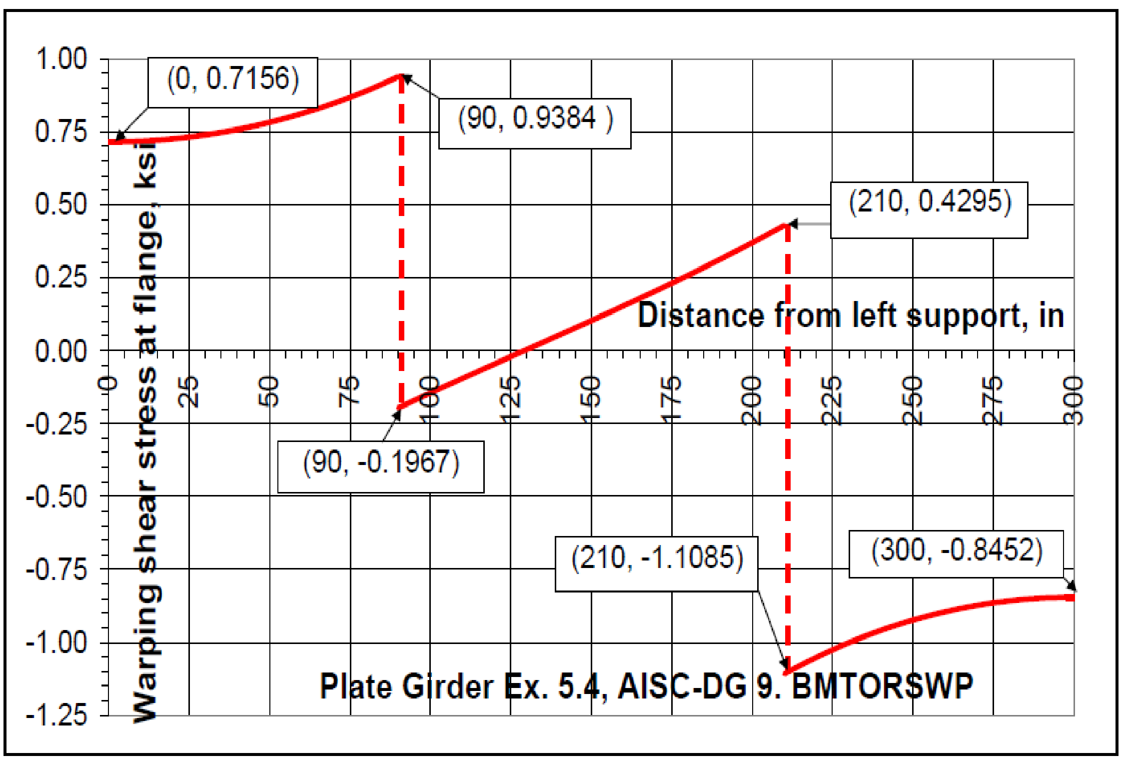

St. Vt. Shear Stress in TWB Web




St. Vt. Shear Stress in TWB Flange




Warping Normal Stresses at TWB Flange



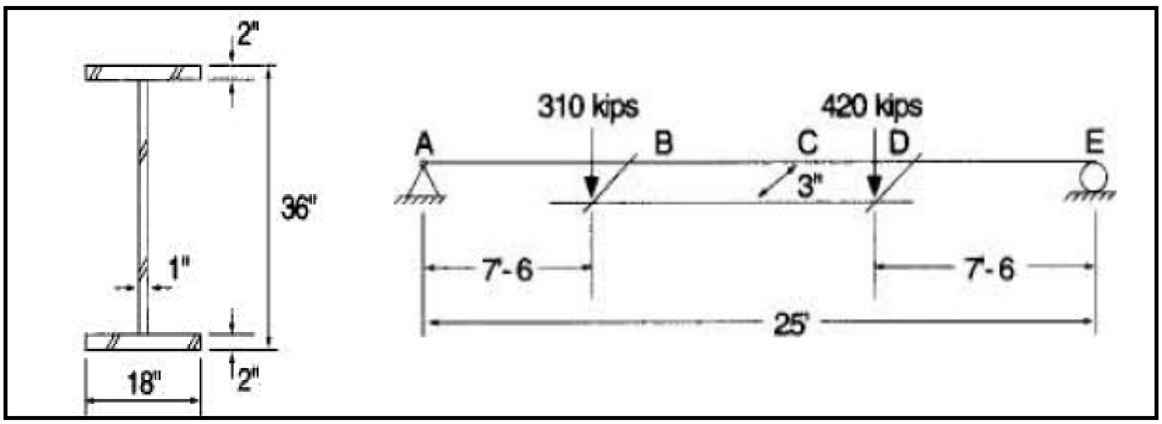

Warping Shear Stresses at TWB Flange

| Location | | $\sigma_w$ | $\sigma_b$ | $f_{un}$ |
|---|---|---|---|---|
| Point D | flange | - 18.63<br>−18.4 | ±26.6 | -45.63<br>−45.0 |
| | web | — | — | — |
| Point E | flange | 0 | 0 | 0 |
| | web | — | — | — |
| Maximum | | | | -45.63<br>−45.0 |

Discrepancy in Total Maximum Normal Stresses in Interest Points

| Location | | $\tau_t$ | $\tau_w$ | $\tau_b$ | $f_{uv}$ |
|---|---|---|---|---|---|
| Point D | flange | - 4.73<br>−4.47 | - 1.11<br>−1.11 | ± 2.37 | - 8.21<br>− 7.95 |
| | web | −2.24 | — | ±12.1 | −14.3 |
| Point E | flange | - 8.75<br>−8.75 | -0.84<br>−0.87 | ± 2.37 | -11.9<br>−12.0 |
| | web | −4.37<br>- 4.38 | — | ±12.1 | −16.5 |
| Maximum | | | | | -16.51<br>−16.5 |

No Discrepancy in Total Maximum Shear Stresses in Interest Points




TWB Cross Section and Interest Points along Beam

From Appendix B, Case 3 with a = 0.3, it is estimated that the maximum rotation will occur at approximately 14½ feet

Exact location for exact "a" 154"=12'-10"

= 0.023 rad. ← 0.024 @12'-10"

Discrepancies on Maximum Twist Angle and Location

Table BMTORSW Output for Graphics

| | BMTORSW P | | | | | | | |
|---|---|---|---|---|---|---|---|---|
| Elem | Z | θ | Z | Moment | Z | θ' | Z | θ" |
| 1 | | | | | | | | |
| | 0 | 0.000E+00 | 0 | 0 | 0 | 3.696E-04 | 0 | 2.940E-13 |
| | 2.25 | 8.000E-04 | 2.25 | -1318.87 | 2.25 | 3.695E-04 | 2.25 | -8.064E-08 |
| | 4.5 | 1.700E-03 | 4.5 | -2638.23 | 4.5 | 3.692E-04 | 4.5 | -1.613E-07 |
| | 6.75 | 2.500E-03 | 6.75 | -3958.57 | 6.75 | 3.687E-04 | 6.75 | -2.420E-07 |
| | 9 | 3.300E-03 | 9 | -5280.37 | 9 | 3.681E-04 | 9 | -3.228E-07 |
| | 11.25 | 4.100E-03 | 11.25 | -6604.14 | 11.25 | 3.673E-04 | 11.25 | -4.038E-07 |
| | 13.5 | 5.000E-03 | 13.5 | -7930.35 | 13.5 | 3.663E-04 | 13.5 | -4.849E-07 |
| | 15.75 | 5.800E-03 | 15.75 | -9259.51 | 15.75 | 3.651E-04 | 15.75 | -5.661E-07 |
| | 18 | 6.600E-03 | 18 | -10592.1 | 18 | 3.637E-04 | 18 | -6.476E-07 |
| | 20.25 | 7.400E-03 | 20.25 | -11928.6 | 20.25 | 3.622E-04 | 20.25 | -7.293E-07 |
| | 22.5 | 8.200E-03 | 22.5 | -13269.6 | 22.5 | 3.605E-04 | 22.5 | -8.113E-07 |
| 2 | | | | | | | | |
| | 22.5 | 8.200E-03 | 22.5 | -13269.6 | 22.5 | 3.605E-04 | 22.5 | -8.113E-07 |
| | 24.75 | 9.100E-03 | 24.75 | -14615.4 | 24.75 | 3.585E-04 | 24.75 | -8.936E-07 |
| | 27 | 9.900E-03 | 27 | -15966.7 | 27 | 3.564E-04 | 27 | -9.762E-07 |
| | 29.25 | 1.070E-02 | 29.25 | -17323.9 | 29.25 | 3.541E-04 | 29.25 | -1.059E-06 |
| | 31.5 | 1.150E-02 | 31.5 | -18687.6 | 31.5 | 3.517E-04 | 31.5 | -1.143E-06 |
| | 33.75 | 1.220E-02 | 33.75 | -20058.1 | 33.75 | 3.490E-04 | 33.75 | -1.226E-06 |
| | 36 | 1.300E-02 | 36 | -21436.2 | 36 | 3.461E-04 | 36 | -1.311E-06 |
| | 38.25 | 1.380E-02 | 38.25 | -22822.1 | 38.25 | 3.431E-04 | 38.25 | -1.395E-06 |
| | 40.5 | 1.460E-02 | 40.5 | -24216.5 | 40.5 | 3.399E-04 | 40.5 | -1.481E-06 |
| | 42.75 | 1.530E-02 | 42.75 | -25620 | 42.75 | 3.364E-04 | 42.75 | -1.566E-06 |
| | 45 | 1.610E-02 | 45 | -27032.9 | 45 | 3.328E-04 | 45 | -1.653E-06 |
| 3 | | | | | | | | |
| | 45 | 1.610E-02 | 45 | -27032.9 | 45 | 3.328E-04 | 45 | -1.653E-06 |
| | 47.25 | 1.680E-02 | 47.25 | -28455.8 | 47.25 | 3.290E-04 | 47.25 | -1.740E-06 |
| | 49.5 | 1.760E-02 | 49.5 | -29889.3 | 49.5 | 3.250E-04 | 49.5 | -1.827E-06 |
| | 51.75 | 1.830E-02 | 51.75 | -31333.9 | 51.75 | 3.208E-04 | 51.75 | -1.916E-06 |
| | 54 | 1.900E-02 | 54 | -32790.1 | 54 | 3.164E-04 | 54 | -2.005E-06 |
| | 56.25 | 1.970E-02 | 56.25 | -34258.5 | 56.25 | 3.118E-04 | 56.25 | -2.095E-06 |
| | 58.5 | 2.040E-02 | 58.5 | -35739.5 | 58.5 | 3.069E-04 | 58.5 | -2.185E-06 |

(continued) Table BMTORSW Output for Graphics

| | | | | | | | | |
|---|---|---|---|---|---|---|---|---|
| | 60.75 | 2.110E-02 | 60.75 | -37233.9 | 60.75 | 3.019E-04 | 60.75 | -2.276E-06 |
| | 63 | 2.180E-02 | 63 | -38742 | 63 | 2.967E-04 | 63 | -2.369E-06 |
| | 65.25 | 2.240E-02 | 65.25 | -40264.5 | 65.25 | 2.913E-04 | 65.25 | -2.462E-06 |
| | 67.5 | 2.310E-02 | 67.5 | -41802 | 67.5 | 2.856E-04 | 67.5 | -2.556E-06 |
| 4 | | | | | | | | |
| | 67.5 | 2.310E-02 | 67.5 | -41802 | 67.5 | 2.856E-04 | 67.5 | -2.556E-06 |
| | 69.75 | 2.370E-02 | 69.75 | -43354.9 | 69.75 | 2.798E-04 | 69.75 | -2.651E-06 |
| | 72 | 2.430E-02 | 72 | -44924 | 72 | 2.737E-04 | 72 | -2.747E-06 |
| | 74.25 | 2.490E-02 | 74.25 | -46509.7 | 74.25 | 2.674E-04 | 74.25 | -2.844E-06 |
| | 76.5 | 2.550E-02 | 76.5 | -48112.6 | 76.5 | 2.609E-04 | 76.5 | -2.942E-06 |
| | 78.75 | 2.610E-02 | 78.75 | -49733.5 | 78.75 | 2.542E-04 | 78.75 | -3.041E-06 |
| | 81 | 2.670E-02 | 81 | -51372.7 | 81 | 2.472E-04 | 81 | -3.141E-06 |
| | 83.25 | 2.720E-02 | 83.25 | -53031 | 83.25 | 2.400E-04 | 83.25 | -3.242E-06 |
| | 85.5 | 2.780E-02 | 85.5 | -54709 | 85.5 | 2.326E-04 | 85.5 | -3.345E-06 |
| | 87.75 | 2.830E-02 | 87.75 | -56407.3 | 87.75 | 2.250E-04 | 87.75 | -3.449E-06 |
| | 90 | 2.880E-02 | 90 | -58126.5 | 90 | 2.171E-04 | 90 | -3.554E-06 |
| 5 | | | | | | | | |
| | 90 | 2.880E-02 | 90 | -58126.5 | 90 | 2.171E-04 | 90 | -3.554E-06 |
| | 92 | 2.920E-02 | 92 | -57812.6 | 92 | 2.100E-04 | 92 | -3.535E-06 |
| | 94 | 2.960E-02 | 94 | -57515.7 | 94 | 2.030E-04 | 94 | -3.516E-06 |
| | 96 | 3.000E-02 | 96 | -57235.7 | 96 | 1.959E-04 | 96 | -3.499E-06 |
| | 98 | 3.040E-02 | 98 | -56972.4 | 98 | 1.890E-04 | 98 | -3.483E-06 |
| | 100 | 3.080E-02 | 100 | -56725.9 | 100 | 1.820E-04 | 100 | -3.468E-06 |
| | 102 | 3.110E-02 | 102 | -56496 | 102 | 1.751E-04 | 102 | -3.454E-06 |
| | 104 | 3.150E-02 | 104 | -56282.6 | 104 | 1.682E-04 | 104 | -3.441E-06 |
| | 106 | 3.180E-02 | 106 | -56085.7 | 106 | 1.613E-04 | 106 | -3.429E-06 |
| | 108 | 3.210E-02 | 108 | -55905.2 | 108 | 1.545E-04 | 108 | -3.418E-06 |
| | 110 | 3.240E-02 | 110 | -55741.2 | 110 | 1.476E-04 | 110 | -3.408E-06 |
| 6 | | | | | | | | |
| | 110 | 3.240E-02 | 110 | -55741.2 | 110 | 1.476E-04 | 110 | -3.408E-06 |
| | 112 | 3.270E-02 | 112 | -55593.5 | 112 | 1.408E-04 | 112 | -3.399E-06 |
| | 114 | 3.300E-02 | 114 | -55462 | 114 | 1.341E-04 | 114 | -3.391E-06 |
| | 116 | 3.320E-02 | 116 | -55346.9 | 116 | 1.273E-04 | 116 | -3.384E-06 |
| | 118 | 3.350E-02 | 118 | -55247.9 | 118 | 1.205E-04 | 118 | -3.378E-06 |

(continued) Table BMTORSW Output for Graphics

| | | | | | | | | |
|---|---|---|---|---|---|---|---|---|
| | 120 | 3.370E-02 | 120 | -55165.2 | 120 | 1.138E-04 | 120 | -3.373E-06 |
| | 122 | 3.390E-02 | 122 | -55098.6 | 122 | 1.070E-04 | 122 | -3.369E-06 |
| | 124 | 3.420E-02 | 124 | -55048.1 | 124 | 1.003E-04 | 124 | -3.366E-06 |
| | 126 | 3.430E-02 | 126 | -55013.8 | 126 | 9.356E-05 | 126 | -3.364E-06 |
| | 128 | 3.450E-02 | 128 | -54995.6 | 128 | 8.683E-05 | 128 | -3.362E-06 |
| | 130 | 3.470E-02 | 130 | -54993.6 | 130 | 8.011E-05 | 130 | -3.362E-06 |
| 7 | | | | | | | | |
| | 130 | 3.470E-02 | 130 | -54993.6 | 130 | 8.011E-05 | 130 | -3.362E-06 |
| | 132 | 3.480E-02 | 132 | -55007.6 | 132 | 7.338E-05 | 132 | -3.363E-06 |
| | 134 | 3.500E-02 | 134 | -55037.8 | 134 | 6.666E-05 | 134 | -3.365E-06 |
| | 136 | 3.510E-02 | 136 | -55084.1 | 136 | 5.992E-05 | 136 | -3.368E-06 |
| | 138 | 3.520E-02 | 138 | -55146.5 | 138 | 5.318E-05 | 138 | -3.372E-06 |
| | 140 | 3.530E-02 | 140 | -55225.2 | 140 | 4.644E-05 | 140 | -3.376E-06 |
| | 142 | 3.540E-02 | 142 | -55320 | 142 | 3.968E-05 | 142 | -3.382E-06 |
| | 144 | 3.550E-02 | 144 | -55431 | 144 | 3.291E-05 | 144 | -3.389E-06 |
| | 146 | 3.550E-02 | 146 | -55558.2 | 146 | 2.612E-05 | 146 | -3.397E-06 |
| | 148 | 3.560E-02 | 148 | -55701.8 | 148 | 1.932E-05 | 148 | -3.406E-06 |
| | 150 | 3.560E-02 | 150 | -55861.6 | 150 | 1.250E-05 | 150 | -3.415E-06 |
| 8 | | | | | | | | |
| | 150 | 3.560E-02 | 150 | -55861.6 | 150 | 1.250E-05 | 150 | -3.415E-06 |
| | 152 | 3.560E-02 | 152 | -56037.9 | 152 | 5.657E-06 | 152 | -3.426E-06 |
| | 154 | 3.560E-02 | 154 | -56230.5 | 154 | -1.207E-06 | 154 | -3.438E-06 |
| | 156 | 3.560E-02 | 156 | -56439.7 | 156 | -8.095E-06 | 156 | -3.451E-06 |
| | 158 | 3.560E-02 | 158 | -56665.4 | 158 | -1.501E-05 | 158 | -3.464E-06 |
| | 160 | 3.560E-02 | 160 | -56907.6 | 160 | -2.195E-05 | 160 | -3.479E-06 |
| | 162 | 3.550E-02 | 162 | -57166.6 | 162 | -2.893E-05 | 162 | -3.495E-06 |
| | 164 | 3.550E-02 | 164 | -57442.3 | 164 | -3.594E-05 | 164 | -3.512E-06 |
| | 166 | 3.540E-02 | 166 | -57734.9 | 166 | -4.298E-05 | 166 | -3.530E-06 |
| | 168 | 3.530E-02 | 168 | -58044.4 | 168 | -5.006E-05 | 168 | -3.549E-06 |
| | 170 | 3.520E-02 | 170 | -58370.9 | 170 | -5.717E-05 | 170 | -3.569E-06 |
| 9 | | | | | | | | |
| | 170 | 3.520E-02 | 170 | -58370.9 | 170 | -5.717E-05 | 170 | -3.569E-06 |
| | 172 | 3.510E-02 | 172 | -58714.5 | 172 | -6.433E-05 | 172 | -3.590E-06 |
| | 174 | 3.490E-02 | 174 | -59075.3 | 174 | -7.153E-05 | 174 | -3.612E-06 |

(continued) Table BMTORSW Output for Graphics

| | | | | | | | | |
|---|---|---|---|---|---|---|---|---|
| | 176 | 3.480E-02 | 176 | -59453.4 | 176 | -7.878E-05 | 176 | -3.635E-06 |
| | 178 | 3.460E-02 | 178 | -59849 | 178 | -8.607E-05 | 178 | -3.659E-06 |
| | 180 | 3.440E-02 | 180 | -60262.1 | 180 | -9.342E-05 | 180 | -3.684E-06 |
| | 182 | 3.420E-02 | 182 | -60692.8 | 182 | -1.008E-04 | 182 | -3.711E-06 |
| | 184 | 3.400E-02 | 184 | -61141.4 | 184 | -1.083E-04 | 184 | -3.738E-06 |
| | 186 | 3.380E-02 | 186 | -61607.8 | 186 | -1.158E-04 | 186 | -3.767E-06 |
| | 188 | 3.360E-02 | 188 | -62092.3 | 188 | -1.233E-04 | 188 | -3.796E-06 |
| | 190 | 3.330E-02 | 190 | -62595 | 190 | -1.310E-04 | 190 | -3.827E-06 |
| 10 | | | | | | | | |
| | 190 | 3.330E-02 | 190 | -62595 | 190 | -1.310E-04 | 190 | -3.827E-06 |
| | 192 | 3.300E-02 | 192 | -63116.1 | 192 | -1.386E-04 | 192 | -3.859E-06 |
| | 194 | 3.280E-02 | 194 | -63655.7 | 194 | -1.464E-04 | 194 | -3.892E-06 |
| | 196 | 3.250E-02 | 196 | -64213.9 | 196 | -1.542E-04 | 196 | -3.926E-06 |
| | 198 | 3.210E-02 | 198 | -64790.9 | 198 | -1.621E-04 | 198 | -3.961E-06 |
| | 200 | 3.180E-02 | 200 | -65386.9 | 200 | -1.701E-04 | 200 | -3.998E-06 |
| | 202 | 3.150E-02 | 202 | -66002.1 | 202 | -1.781E-04 | 202 | -4.035E-06 |
| | 204 | 3.110E-02 | 204 | -66636.7 | 204 | -1.862E-04 | 204 | -4.074E-06 |
| | 206 | 3.070E-02 | 206 | -67290.7 | 206 | -1.944E-04 | 206 | -4.114E-06 |
| | 208 | 3.030E-02 | 208 | -67964.5 | 208 | -2.027E-04 | 208 | -4.155E-06 |
| | 210 | 2.990E-02 | 210 | -68658.2 | 210 | -2.110E-04 | 210 | -4.198E-06 |
| 11 | | | | | | | | |
| | 210 | 2.990E-02 | 210 | -68658.2 | 210 | -2.110E-04 | 210 | -4.198E-06 |
| | 212.25 | 2.940E-02 | 212.25 | -66627.5 | 212.25 | -2.203E-04 | 212.25 | -4.074E-06 |
| | 214.5 | 2.890E-02 | 214.5 | -64621.6 | 214.5 | -2.293E-04 | 214.5 | -3.951E-06 |
| | 216.75 | 2.840E-02 | 216.75 | -62639.5 | 216.75 | -2.381E-04 | 216.75 | -3.830E-06 |
| | 219 | 2.780E-02 | 219 | -60680.8 | 219 | -2.466E-04 | 219 | -3.710E-06 |
| | 221.25 | 2.730E-02 | 221.25 | -58744.5 | 221.25 | -2.548E-04 | 221.25 | -3.592E-06 |
| | 223.5 | 2.670E-02 | 223.5 | -56830 | 223.5 | -2.627E-04 | 223.5 | -3.475E-06 |
| | 225.75 | 2.610E-02 | 225.75 | -54936.6 | 225.75 | -2.704E-04 | 225.75 | -3.359E-06 |
| | 228 | 2.550E-02 | 228 | -53063.6 | 228 | -2.778E-04 | 228 | -3.244E-06 |
| | 230.25 | 2.480E-02 | 230.25 | -51210.3 | 230.25 | -2.850E-04 | 230.25 | -3.131E-06 |
| | 232.5 | 2.420E-02 | 232.5 | -49376 | 232.5 | -2.919E-04 | 232.5 | -3.019E-06 |
| 12 | | | | | | | | |
| | 232.5 | 2.420E-02 | 232.5 | -49376 | 232.5 | -2.919E-04 | 232.5 | -3.019E-06 |

(continued) Table BMTORSW Output for Graphics

| | | | | | | | | |
|---|---|---|---|---|---|---|---|---|
| | 234.75 | 2.350E-02 | 234.75 | -47559.9 | 234.75 | -2.986E-04 | 234.75 | -2.908E-06 |
| | 237 | 2.280E-02 | 237 | -45761.6 | 237 | -3.050E-04 | 237 | -2.798E-06 |
| | 239.25 | 2.220E-02 | 239.25 | -43980.2 | 239.25 | -3.112E-04 | 239.25 | -2.689E-06 |
| | 241.5 | 2.140E-02 | 241.5 | -42215.1 | 241.5 | -3.171E-04 | 241.5 | -2.581E-06 |
| | 243.75 | 2.070E-02 | 243.75 | -40465.6 | 243.75 | -3.228E-04 | 243.75 | -2.474E-06 |
| | 246 | 2.000E-02 | 246 | -38731.2 | 246 | -3.283E-04 | 246 | -2.368E-06 |
| | 248.25 | 1.930E-02 | 248.25 | -37011.2 | 248.25 | -3.335E-04 | 248.25 | -2.263E-06 |
| | 250.5 | 1.850E-02 | 250.5 | -35304.9 | 250.5 | -3.384E-04 | 250.5 | -2.159E-06 |
| | 252.75 | 1.770E-02 | 252.75 | -33611.6 | 252.75 | -3.432E-04 | 252.75 | -2.055E-06 |
| | 255 | 1.700E-02 | 255 | -31930.9 | 255 | -3.477E-04 | 255 | -1.952E-06 |
| 13 | | | | | | | | |
| | 255 | 1.700E-02 | 255 | -31930.9 | 255 | -3.477E-04 | 255 | -1.952E-06 |
| | 257.25 | 1.620E-02 | 257.25 | -30262 | 257.25 | -3.520E-04 | 257.25 | -1.850E-06 |
| | 259.5 | 1.540E-02 | 259.5 | -28604.3 | 259.5 | -3.560E-04 | 259.5 | -1.749E-06 |
| | 261.75 | 1.460E-02 | 261.75 | -26957.2 | 261.75 | -3.598E-04 | 261.75 | -1.648E-06 |
| | 264 | 1.370E-02 | 264 | -25320.1 | 264 | -3.634E-04 | 264 | -1.548E-06 |
| | 266.25 | 1.290E-02 | 266.25 | -23692.4 | 266.25 | -3.668E-04 | 266.25 | -1.449E-06 |
| | 268.5 | 1.210E-02 | 268.5 | -22073.5 | 268.5 | -3.700E-04 | 268.5 | -1.350E-06 |
| | 270.75 | 1.130E-02 | 270.75 | -20462.8 | 270.75 | -3.729E-04 | 270.75 | -1.251E-06 |
| | 273 | 1.040E-02 | 273 | -18859.7 | 273 | -3.756E-04 | 273 | -1.153E-06 |
| | 275.25 | 9.600E-03 | 275.25 | -17263.6 | 275.25 | -3.781E-04 | 275.25 | -1.055E-06 |
| | 277.5 | 8.700E-03 | 277.5 | -15673.8 | 277.5 | -3.803E-04 | 277.5 | -9.583E-07 |
| 14 | | | | | | | | |
| | 277.5 | 8.700E-03 | 277.5 | -15673.8 | 277.5 | -3.803E-04 | 277.5 | -9.583E-07 |
| | 279.75 | 7.900E-03 | 279.75 | -14089.9 | 279.75 | -3.824E-04 | 279.75 | -8.615E-07 |
| | 282 | 7.000E-03 | 282 | -12511.3 | 282 | -3.842E-04 | 282 | -7.649E-07 |
| | 284.25 | 6.100E-03 | 284.25 | -10937.2 | 284.25 | -3.858E-04 | 284.25 | -6.687E-07 |
| | 286.5 | 5.300E-03 | 286.5 | -9367.23 | 286.5 | -3.872E-04 | 286.5 | -5.727E-07 |
| | 288.75 | 4.400E-03 | 288.75 | -7800.72 | 288.75 | -3.884E-04 | 288.75 | -4.769E-07 |
| | 291 | 3.500E-03 | 291 | -6237.11 | 291 | -3.894E-04 | 291 | -3.813E-07 |
| | 293.25 | 2.600E-03 | 293.25 | -4675.81 | 293.25 | -3.901E-04 | 293.25 | -2.859E-07 |
| | 295.5 | 1.800E-03 | 295.5 | -3116.24 | 295.5 | -3.907E-04 | 295.5 | -1.905E-07 |
| | 297.75 | 9.000E-04 | 297.75 | -1557.83 | 297.75 | -3.910E-04 | 297.75 | -9.525E-08 |
| | 300 | 0.000E+00 | 300 | 0 | 300 | -3.911E-04 | 300 | 3.473E-13 |

Table 3rd Derivative and Stresses Processed

| | | | | Pure torsion shear | | | | Warping stresses | | |
|---|---|---|---|---|---|---|---|---|---|---|
| | | | | stress | | stress | | axial | | shear |
| Elem | Z | θ''' | Z | web | Z | flange | Z | flange | Z | flange |
| 1 | | | | | | | | | | |
| | 0 | -3.584E-08 | 0 | 4.14 | 0 | 8.28 | 0 | 0.00 | 0 | 0.72 |
| | 2.25 | -3.584E-08 | 2.25 | 4.14 | 2.25 | 8.28 | 2.25 | -0.36 | 2.25 | 0.72 |
| | 4.5 | -3.586E-08 | 4.5 | 4.14 | 4.5 | 8.27 | 4.5 | -0.72 | 4.5 | 0.72 |
| | 6.75 | -3.590E-08 | 6.75 | 4.13 | 6.75 | 8.26 | 6.75 | -1.07 | 6.75 | 0.72 |
| | 9 | -3.594E-08 | 9 | 4.12 | 9 | 8.25 | 9 | -1.43 | 9 | 0.72 |
| | 11.25 | -3.600E-08 | 11.25 | 4.11 | 11.25 | 8.23 | 11.25 | -1.79 | 11.25 | 0.72 |
| | 13.5 | -3.608E-08 | 13.5 | 4.10 | 13.5 | 8.21 | 13.5 | -2.15 | 13.5 | 0.72 |
| | 15.75 | -3.616E-08 | 15.75 | 4.09 | 15.75 | 8.18 | 15.75 | -2.51 | 15.75 | 0.72 |
| | 18 | -3.626E-08 | 18 | 4.07 | 18 | 8.15 | 18 | -2.87 | 18 | 0.72 |
| | 20.25 | -3.638E-08 | 20.25 | 4.06 | 20.25 | 8.11 | 20.25 | -3.24 | 20.25 | 0.73 |
| | 22.5 | -3.650E-08 | 22.5 | 4.04 | 22.5 | 8.08 | 22.5 | -3.60 | 22.5 | 0.73 |
| 2 | | | | 0.00 | | 0.00 | | 0.00 | | 0.00 |
| | 22.5 | -3.650E-08 | 22.5 | 4.04 | 22.5 | 8.08 | 22.5 | -3.60 | 22.5 | 0.73 |
| | 24.75 | -3.664E-08 | 24.75 | 4.02 | 24.75 | 8.03 | 24.75 | -3.96 | 24.75 | 0.73 |
| | 27 | -3.680E-08 | 27 | 3.99 | 27 | 7.98 | 27 | -4.33 | 27 | 0.73 |
| | 29.25 | -3.696E-08 | 29.25 | 3.97 | 29.25 | 7.93 | 29.25 | -4.70 | 29.25 | 0.74 |
| | 31.5 | -3.715E-08 | 31.5 | 3.94 | 31.5 | 7.88 | 31.5 | -5.07 | 31.5 | 0.74 |
| | 33.75 | -3.734E-08 | 33.75 | 3.91 | 33.75 | 7.82 | 33.75 | -5.44 | 33.75 | 0.75 |
| | 36 | -3.755E-08 | 36 | 3.88 | 36 | 7.75 | 36 | -5.82 | 36 | 0.75 |
| | 38.25 | -3.777E-08 | 38.25 | 3.84 | 38.25 | 7.69 | 38.25 | -6.19 | 38.25 | 0.75 |
| | 40.5 | -3.801E-08 | 40.5 | 3.81 | 40.5 | 7.61 | 40.5 | -6.57 | 40.5 | 0.76 |
| | 42.75 | -3.826E-08 | 42.75 | 3.77 | 42.75 | 7.54 | 42.75 | -6.95 | 42.75 | 0.76 |
| | 45 | -3.853E-08 | 45 | 3.73 | 45 | 7.45 | 45 | -7.33 | 45 | 0.77 |
| 3 | | | | 0.00 | | 0.00 | | 0.00 | | 0.00 |
| | 45 | -3.853E-08 | 45 | 3.73 | 45 | 7.45 | 45 | -7.33 | 45 | 0.77 |
| | 47.25 | -3.881E-08 | 47.25 | 3.68 | 47.25 | 7.37 | 47.25 | -7.72 | 47.25 | 0.77 |
| | 49.5 | -3.910E-08 | 49.5 | 3.64 | 49.5 | 7.28 | 49.5 | -8.11 | 49.5 | 0.78 |
| | 51.75 | -3.941E-08 | 51.75 | 3.59 | 51.75 | 7.19 | 51.75 | -8.50 | 51.75 | 0.79 |
| | 54 | -3.973E-08 | 54 | 3.54 | 54 | 7.09 | 54 | -8.90 | 54 | 0.79 |
| | 56.25 | -4.007E-08 | 56.25 | 3.49 | 56.25 | 6.98 | 56.25 | -9.30 | 56.25 | 0.80 |

(continued) Table 3rd Derivative and Stresses Processed

| | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|
| | 58.5 | -4.042E-08 | 58.5 | 3.44 | 58.5 | 6.87 | 58.5 | -9.69 | 58.5 | 0.81 |
| | 60.75 | -4.079E-08 | 60.75 | 3.38 | 60.75 | 6.76 | 60.75 | -10.10 | 60.75 | 0.81 |
| | 63 | -4.117E-08 | 63 | 3.32 | 63 | 6.65 | 63 | -10.51 | 63 | 0.82 |
| | 65.25 | -4.157E-08 | 65.25 | 3.26 | 65.25 | 6.53 | 65.25 | -10.92 | 65.25 | 0.83 |
| | 67.5 | -4.198E-08 | 67.5 | 3.20 | 67.5 | 6.40 | 67.5 | -11.34 | 67.5 | 0.84 |
| 4 | | | | 0.00 | | 0.00 | | 0.00 | | 0.00 |
| | 67.5 | -4.199E-08 | 67.5 | 3.20 | 67.5 | 6.40 | 67.5 | -11.34 | 67.5 | 0.84 |
| | 69.75 | -4.242E-08 | 69.75 | 3.13 | 69.75 | 6.27 | 69.75 | -11.76 | 69.75 | 0.85 |
| | 72 | -4.286E-08 | 72 | 3.07 | 72 | 6.13 | 72 | -12.19 | 72 | 0.86 |
| | 74.25 | -4.332E-08 | 74.25 | 2.99 | 74.25 | 5.99 | 74.25 | -12.62 | 74.25 | 0.86 |
| | 76.5 | -4.380E-08 | 76.5 | 2.92 | 76.5 | 5.84 | 76.5 | -13.05 | 76.5 | 0.87 |
| | 78.75 | -4.429E-08 | 78.75 | 2.85 | 78.75 | 5.69 | 78.75 | -13.49 | 78.75 | 0.88 |
| | 81 | -4.480E-08 | 81 | 2.77 | 81 | 5.54 | 81 | -13.94 | 81 | 0.89 |
| | 83.25 | -4.533E-08 | 83.25 | 2.69 | 83.25 | 5.38 | 83.25 | -14.38 | 83.25 | 0.91 |
| | 85.5 | -4.587E-08 | 85.5 | 2.61 | 85.5 | 5.21 | 85.5 | -14.84 | 85.5 | 0.92 |
| | 87.75 | -4.643E-08 | 87.75 | 2.52 | 87.75 | 5.04 | 87.75 | -15.30 | 87.75 | 0.93 |
| | 90 | -4.700E-08 | 90 | 2.43 | 90 | 4.86 | 90 | -15.77 | 90 | 0.94 |
| 5 | | | | 0.00 | | 0.00 | | 0.00 | | 0.00 |
| | 90 | 9.852E-09 | 90 | 2.43 | 90 | 4.86 | 90 | -15.77 | 90 | -0.20 |
| | 92 | 9.334E-09 | 92 | 2.35 | 92 | 4.70 | 92 | -15.68 | 92 | -0.19 |
| | 94 | 8.818E-09 | 94 | 2.27 | 94 | 4.55 | 94 | -15.60 | 94 | -0.18 |
| | 96 | 8.304E-09 | 96 | 2.19 | 96 | 4.39 | 96 | -15.53 | 96 | -0.17 |
| | 98 | 7.792E-09 | 98 | 2.12 | 98 | 4.23 | 98 | -15.45 | 98 | -0.16 |
| | 100 | 7.283E-09 | 100 | 2.04 | 100 | 4.08 | 100 | -15.39 | 100 | -0.15 |
| | 102 | 6.775E-09 | 102 | 1.96 | 102 | 3.92 | 102 | -15.33 | 102 | -0.14 |
| | 104 | 6.270E-09 | 104 | 1.88 | 104 | 3.77 | 104 | -15.27 | 104 | -0.13 |
| | 106 | 5.767E-09 | 106 | 1.81 | 106 | 3.61 | 106 | -15.21 | 106 | -0.12 |
| | 108 | 5.266E-09 | 108 | 1.73 | 108 | 3.46 | 108 | -15.17 | 108 | -0.11 |
| | 110 | 4.767E-09 | 110 | 1.65 | 110 | 3.31 | 110 | -15.12 | 110 | -0.10 |
| 6 | | | | 0.00 | | 0.00 | | 0.00 | | 0.00 |
| | 110 | 4.764E-09 | 110 | 1.65 | 110 | 3.31 | 110 | -15.12 | 110 | -0.10 |
| | 112 | 4.266E-09 | 112 | 1.58 | 112 | 3.15 | 112 | -15.08 | 112 | -0.09 |
| | 114 | 3.769E-09 | 114 | 1.50 | 114 | 3.00 | 114 | -15.05 | 114 | -0.08 |
| | 116 | 3.273E-09 | 116 | 1.43 | 116 | 2.85 | 116 | -15.01 | 116 | -0.07 |

(continued) Table 3rd Derivative and Stresses Processed

| | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|
| | 118 | 2.778E-09 | 118 | 1.35 | 118 | 2.70 | 118 | -14.99 | 118 | -0.06 |
| | 120 | 2.283E-09 | 120 | 1.27 | 120 | 2.55 | 120 | -14.97 | 120 | -0.05 |
| | 122 | 1.789E-09 | 122 | 1.20 | 122 | 2.40 | 122 | -14.95 | 122 | -0.04 |
| | 124 | 1.295E-09 | 124 | 1.12 | 124 | 2.25 | 124 | -14.93 | 124 | -0.03 |
| | 126 | 8.022E-10 | 126 | 1.05 | 126 | 2.10 | 126 | -14.93 | 126 | -0.02 |
| | 128 | 3.100E-10 | 128 | 0.97 | 128 | 1.94 | 128 | -14.92 | 128 | -0.01 |
| | 130 | -1.815E-10 | 130 | 0.90 | 130 | 1.79 | 130 | -14.92 | 130 | 0.00 |
| 7 | | | | 0.00 | | 0.00 | | 0.00 | | 0.00 |
| | 130 | -1.849E-10 | 130 | 0.90 | 130 | 1.79 | 130 | -14.92 | 130 | 0.00 |
| | 132 | -6.765E-10 | 132 | 0.82 | 132 | 1.64 | 132 | -14.92 | 132 | 0.01 |
| | 134 | -1.169E-09 | 134 | 0.75 | 134 | 1.49 | 134 | -14.93 | 134 | 0.02 |
| | 136 | -1.662E-09 | 136 | 0.67 | 136 | 1.34 | 136 | -14.94 | 136 | 0.03 |
| | 138 | -2.156E-09 | 138 | 0.60 | 138 | 1.19 | 138 | -14.96 | 138 | 0.04 |
| | 140 | -2.650E-09 | 140 | 0.52 | 140 | 1.04 | 140 | -14.98 | 140 | 0.05 |
| | 142 | -3.146E-09 | 142 | 0.44 | 142 | 0.89 | 142 | -15.01 | 142 | 0.06 |
| | 144 | -3.642E-09 | 144 | 0.37 | 144 | 0.74 | 144 | -15.04 | 144 | 0.07 |
| | 146 | -4.139E-09 | 146 | 0.29 | 146 | 0.59 | 146 | -15.07 | 146 | 0.08 |
| | 148 | -4.637E-09 | 148 | 0.22 | 148 | 0.43 | 148 | -15.11 | 148 | 0.09 |
| | 150 | -5.135E-09 | 150 | 0.14 | 150 | 0.28 | 150 | -15.15 | 150 | 0.10 |
| 8 | | | | 0.00 | | 0.00 | | 0.00 | | 0.00 |
| | 150 | -5.139E-09 | 150 | 0.14 | 150 | 0.28 | 150 | -15.15 | 150 | 0.10 |
| | 152 | -5.639E-09 | 152 | 0.06 | 152 | 0.13 | 152 | -15.20 | 152 | 0.11 |
| | 154 | -6.141E-09 | 154 | -0.01 | 154 | -0.03 | 154 | -15.25 | 154 | 0.12 |
| | 156 | -6.646E-09 | 156 | -0.09 | 156 | -0.18 | 156 | -15.31 | 156 | 0.13 |
| | 158 | -7.152E-09 | 158 | -0.17 | 158 | -0.34 | 158 | -15.37 | 158 | 0.14 |
| | 160 | -7.661E-09 | 160 | -0.25 | 160 | -0.49 | 160 | -15.44 | 160 | 0.15 |
| | 162 | -8.173E-09 | 162 | -0.32 | 162 | -0.65 | 162 | -15.51 | 162 | 0.16 |
| | 164 | -8.686E-09 | 164 | -0.40 | 164 | -0.81 | 164 | -15.58 | 164 | 0.17 |
| | 166 | -9.202E-09 | 166 | -0.48 | 166 | -0.96 | 166 | -15.66 | 166 | 0.18 |
| | 168 | -9.720E-09 | 168 | -0.56 | 168 | -1.12 | 168 | -15.75 | 168 | 0.19 |
| | 170 | -1.024E-08 | 170 | -0.64 | 170 | -1.28 | 170 | -15.84 | 170 | 0.20 |
| 9 | | | | 0.00 | | 0.00 | | 0.00 | | 0.00 |
| | 170 | -1.024E-08 | 170 | -0.64 | 170 | -1.28 | 170 | -15.84 | 170 | 0.20 |
| | 172 | -1.077E-08 | 172 | -0.72 | 172 | -1.44 | 172 | -15.93 | 172 | 0.22 |

(continued) Table 3rd Derivative and Stresses Processed

| | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|
| | 174 | -1.129E-08 | 174 | -0.80 | 174 | -1.60 | 174 | -16.03 | 174 | 0.23 |
| | 176 | -1.182E-08 | 176 | -0.88 | 176 | -1.76 | 176 | -16.13 | 176 | 0.24 |
| | 178 | -1.236E-08 | 178 | -0.96 | 178 | -1.93 | 178 | -16.23 | 178 | 0.25 |
| | 180 | -1.290E-08 | 180 | -1.05 | 180 | -2.09 | 180 | -16.35 | 180 | 0.26 |
| | 182 | -1.344E-08 | 182 | -1.13 | 182 | -2.26 | 182 | -16.47 | 182 | 0.27 |
| | 184 | -1.399E-08 | 184 | -1.21 | 184 | -2.43 | 184 | -16.59 | 184 | 0.28 |
| | 186 | -1.454E-08 | 186 | -1.30 | 186 | -2.59 | 186 | -16.71 | 186 | 0.29 |
| | 188 | -1.509E-08 | 188 | -1.38 | 188 | -2.76 | 188 | -16.84 | 188 | 0.30 |
| | 190 | -1.565E-08 | 190 | -1.47 | 190 | -2.93 | 190 | -16.98 | 190 | 0.31 |
| 10 | | | | 0.00 | | 0.00 | | 0.00 | | 0.00 |
| | 190 | -1.565E-08 | 190 | -1.47 | 190 | -2.93 | 190 | -16.98 | 190 | 0.31 |
| | 192 | -1.621E-08 | 192 | -1.55 | 192 | -3.10 | 192 | -17.12 | 192 | 0.32 |
| | 194 | -1.678E-08 | 194 | -1.64 | 194 | -3.28 | 194 | -17.27 | 194 | 0.34 |
| | 196 | -1.735E-08 | 196 | -1.73 | 196 | -3.45 | 196 | -17.42 | 196 | 0.35 |
| | 198 | -1.793E-08 | 198 | -1.82 | 198 | -3.63 | 198 | -17.57 | 198 | 0.36 |
| | 200 | -1.851E-08 | 200 | -1.91 | 200 | -3.81 | 200 | -17.74 | 200 | 0.37 |
| | 202 | -1.910E-08 | 202 | -1.99 | 202 | -3.99 | 202 | -17.90 | 202 | 0.38 |
| | 204 | -1.970E-08 | 204 | -2.09 | 204 | -4.17 | 204 | -18.08 | 204 | 0.39 |
| | 206 | -2.030E-08 | 206 | -2.18 | 206 | -4.35 | 206 | -18.25 | 206 | 0.41 |
| | 208 | -2.090E-08 | 208.6 | -2.27 | 208 | -4.54 | 208 | -18.44 | 208 | 0.42 |
| | 210 | -2.151E-08 | 210 | -2.36 | 210 | -4.73 | 210 | -18.63 | 210 | 0.43 |
| 11 | | | | 0.00 | | 0.00 | | 0.00 | | 0.00 |
| | 210 | 5.552E-08 | 210 | -2.36 | 210 | -4.73 | 210 | -18.63 | 210 | -1.11 |
| | 212.25 | 5.484E-08 | 212.25 | -2.47 | 212.25 | -4.93 | 212.3 | -18.08 | 212.25 | -1.09 |
| | 214.5 | 5.418E-08 | 214.5 | -2.57 | 214.5 | -5.14 | 214.5 | -17.53 | 214.5 | -1.08 |
| | 216.75 | 5.354E-08 | 216.75 | -2.67 | 216.75 | -5.33 | 216.8 | -16.99 | 216.75 | -1.07 |
| | 219 | 5.292E-08 | 219 | -2.76 | 219 | -5.52 | 219 | -16.46 | 219 | -1.06 |
| | 221.25 | 5.232E-08 | 221.25 | -2.85 | 221.25 | -5.71 | 221.3 | -15.94 | 221.25 | -1.04 |
| | 223.5 | 5.173E-08 | 223.5 | -2.94 | 223.5 | -5.88 | 223.5 | -15.42 | 223.5 | -1.03 |
| | 225.75 | 5.117E-08 | 225.75 | -3.03 | 225.75 | -6.06 | 225.8 | -14.90 | 225.75 | -1.02 |
| | 228 | 5.062E-08 | 228 | -3.11 | 228 | -6.22 | 228 | -14.39 | 228 | -1.01 |
| | 230.25 | 5.010E-08 | 230.25 | -3.19 | 230.25 | -6.38 | 230.3 | -13.89 | 230.25 | -1.00 |
| | 232.5 | 4.960E-08 | 232.5 | -3.27 | 232.5 | -6.54 | 232.5 | -13.40 | 232.5 | -0.99 |
| 12 | | | | 0.00 | | 0.00 | | 0.00 | | 0.00 |

(continued) Table 3rd Derivative and Stresses Processed

| | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|
| | 232.5 | 4.959E-08 | 232.5 | -3.27 | 232.5 | -6.54 | 232.5 | -13.40 | 232.5 | -0.99 |
| | 234.75 | 4.910E-08 | 234.75 | -3.34 | 234.75 | -6.69 | 234.8 | -12.90 | 234.75 | -0.98 |
| | 237 | 4.863E-08 | 237 | -3.42 | 237 | -6.83 | 237 | -12.41 | 237 | -0.97 |
| | 239.25 | 4.818E-08 | 239.25 | -3.49 | 239.25 | -6.97 | 239.3 | -11.93 | 239.25 | -0.96 |
| | 241.5 | 4.775E-08 | 241.5 | -3.55 | 241.5 | -7.10 | 241.5 | -11.45 | 241.5 | -0.95 |
| | 243.75 | 4.733E-08 | 243.75 | -3.62 | 243.75 | -7.23 | 243.8 | -10.98 | 243.75 | -0.95 |
| | 246 | 4.693E-08 | 246 | -3.68 | 246 | -7.35 | 246 | -10.51 | 246 | -0.94 |
| | 248.25 | 4.655E-08 | 248.25 | -3.74 | 248.25 | -7.47 | 248.3 | -10.04 | 248.25 | -0.93 |
| | 250.5 | 4.619E-08 | 250.5 | -3.79 | 250.5 | -7.58 | 250.5 | -9.58 | 250.5 | -0.92 |
| | 252.75 | 4.584E-08 | 252.75 | -3.84 | 252.75 | -7.69 | 252.8 | -9.12 | 252.75 | -0.92 |
| 13 | 255 | 4.551E-08 | 255 | -3.89 | 255 | -7.79 | 255 | -8.66 | 255 | -0.91 |
| | 257.25 | 4.519E-08 | 257.25 | -3.94 | 257.25 | -7.88 | 257.3 | -8.21 | 257.25 | -0.90 |
| | 259.5 | 4.490E-08 | 259.5 | -3.99 | 259.5 | -7.97 | 259.5 | -7.76 | 259.5 | -0.90 |
| | 261.75 | 4.462E-08 | 261.75 | -4.03 | 261.75 | -8.06 | 261.8 | -7.31 | 261.75 | -0.89 |
| | 264 | 4.435E-08 | 264 | -4.07 | 264 | -8.14 | 264 | -6.87 | 264 | -0.89 |
| | 266.25 | 4.411E-08 | 266.25 | -4.11 | 266.25 | -8.22 | 266.3 | -6.43 | 266.25 | -0.88 |
| | 268.5 | 4.388E-08 | 268.5 | -4.14 | 268.5 | -8.29 | 268.5 | -5.99 | 268.5 | -0.88 |
| | 270.75 | 4.366E-08 | 270.75 | -4.18 | 270.75 | -8.35 | 270.8 | -5.55 | 270.75 | -0.87 |
| | 273 | 4.346E-08 | 273 | -4.21 | 273 | -8.41 | 273 | -5.12 | 273 | -0.87 |
| | 275.25 | 4.328E-08 | 275.25 | -4.23 | 275.25 | -8.47 | 275.3 | -4.68 | 275.25 | -0.86 |
| 14 | 277.5 | 4.312E-08 | 277.5 | -4.26 | 277.5 | -8.52 | 277.5 | -4.25 | 277.5 | -0.86 |
| | 279.75 | 4.297E-08 | 279.75 | -4.28 | 279.75 | -8.57 | 279.8 | -3.82 | 279.75 | -0.86 |
| | 282 | 4.283E-08 | 282 | -4.30 | 282 | -8.61 | 282 | -3.39 | 282 | -0.86 |
| | 284.25 | 4.271E-08 | 284.25 | -4.32 | 284.25 | -8.64 | 284.3 | -2.97 | 284.25 | -0.85 |
| | 286.5 | 4.261E-08 | 286.5 | -4.34 | 286.5 | -8.67 | 286.5 | -2.54 | 286.5 | -0.85 |
| | 288.75 | 4.253E-08 | 288.75 | -4.35 | 288.75 | -8.70 | 288.8 | -2.12 | 288.75 | -0.85 |
| | 291 | 4.245E-08 | 291 | -4.36 | 291 | -8.72 | 291 | -1.69 | 291 | -0.85 |
| | 293.25 | 4.240E-08 | 293.25 | -4.37 | 293.25 | -8.74 | 293.3 | -1.27 | 293.25 | -0.85 |
| | 295.5 | 4.236E-08 | 295.5 | -4.38 | 295.5 | -8.75 | 295.5 | -0.85 | 295.5 | -0.85 |
| | 297.75 | 4.234E-08 | 297.75 | -4.38 | 297.75 | -8.76 | 297.8 | -0.42 | 297.75 | -0.85 |
| | 300 | 4.233E-08 | 300 | -4.38 | 300 | -8.76 | 300 | 0.00 | 300 | -0.85 |

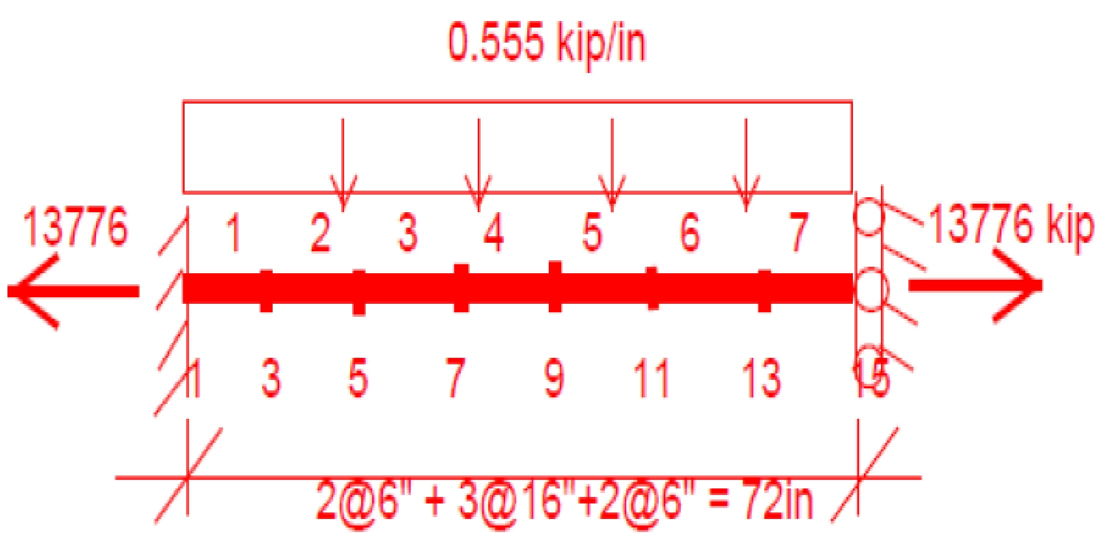
**APPENDIX F**

**AISC-DG9-EXAMPLE 5.5**

In the EBC of case study four, this material corresponds to the input model, input data, input forms, figures of output data with checks in notepad version, and excel processed output data and charts.

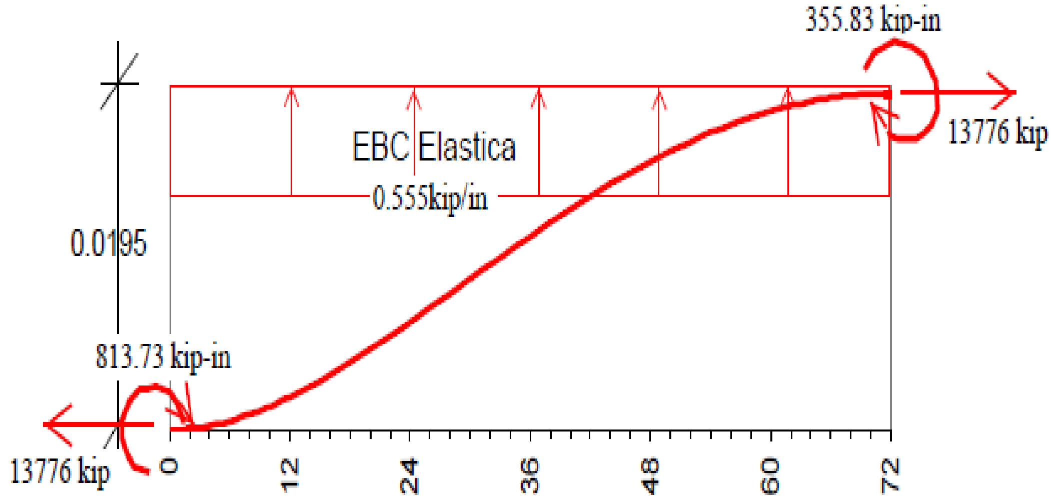
Charts are presented containing both partial and combined tresses along interest points of the beam and cross section profile. Again, the asymptotic behavior of the thin-walled beam elastic line is successfully shown in the charts, which evidences the efficiency of the high order finite element used by BMTORSWP.

It is important to notice that the exact location of the smeared DOF representing the distributed load is unknown. Nevertheless, it is assumed to be at midspan of the element for the purpose of checking summation of moments. This criterion has provided good results as it could be seen in pages 2 and four of the output notepads.




Analog EBC Model

From summation of moments in the EBC $\Delta = (B1 + B2 - wL^2/2)/ GJ = (813.73 + 353.83 - 0.555 \times 72^2/2)/ 13776 = 0.01967 \sim 0.0195$ (Channel MC18x47. BMTORSWP).




Elastic Line and BMTORSWP Model for DG9-Ex. 5.5

```
                                     5-5
AISC Design Guide 9, Example 5.5, MC18x42.7 channel
  7 15  2  0  9  0  0  0 29000.
  1  1  2  3 852.   12.   6.                          -13776. 0.555 0.555
  2  3  4  5 852.   12.   6.                          -13776. 0.555 0.555
  3  5  6  7 852.   12.  16.                          -13776. 0.555 0.555
  4  7  8  9 852.   12.  16.                          -13776. 0.555 0.555
  5  9 10 11 852.   12.  16.                          -13776. 0.555 0.555
  6 11 12 13 852.   12.   6.                          -13776. 0.555 0.555
  7 13 14 15 852.   12.   6.                          -13776. 0.555 0.555
 2 1 1 1 115 0 0 1
```

KX=KY=0 & KZ=1

KX=KY=KZ=1: Fixed end

Notepad Input Data for BMTORSWP.

## PROGRAM BMTORSW INPUT FORM

ALPHAMERIC DESCRIPTION OF THE JOB — FILE NAME:

AISC DESIGN GUIDE 9 EXAMPLE 5.5 ON WARPING

| NEL | NOD | NSUP | NSPD | JBW | NFX | NFY | NFZ | ELASTICITY (E11.4) |
|---|---|---|---|---|---|---|---|---|
| 7 | 15 | 2 | 0 | 9 | 0 | 0 | 0 | 29000. |

| NE | NI | NC | NJ | Cw | AREA | LENGTH | WTANG | SOIL NI | SOI |
|---|---|---|---|---|---|---|---|---|---|
| 1 | 1 | 2 | 3 | 852. | 12. | 6. | | | |
| 2 | 3 | 4 | 5 | 852. | 12. | 6. | | | |
| 3 | 5 | 6 | 7 | 852. | 12. | 16. | | | |
| 4 | 7 | 8 | 9 | 852. | 12. | 16. | | | |
| 5 | 9 | 10 | 11 | 852. | 12. | 16. | | | |
| 6 | 11 | 12 | 13 | 852. | 12. | 6. | | | |
| 7 | 13 | 14 | 15 | 852. | 12. | 6. | | | |

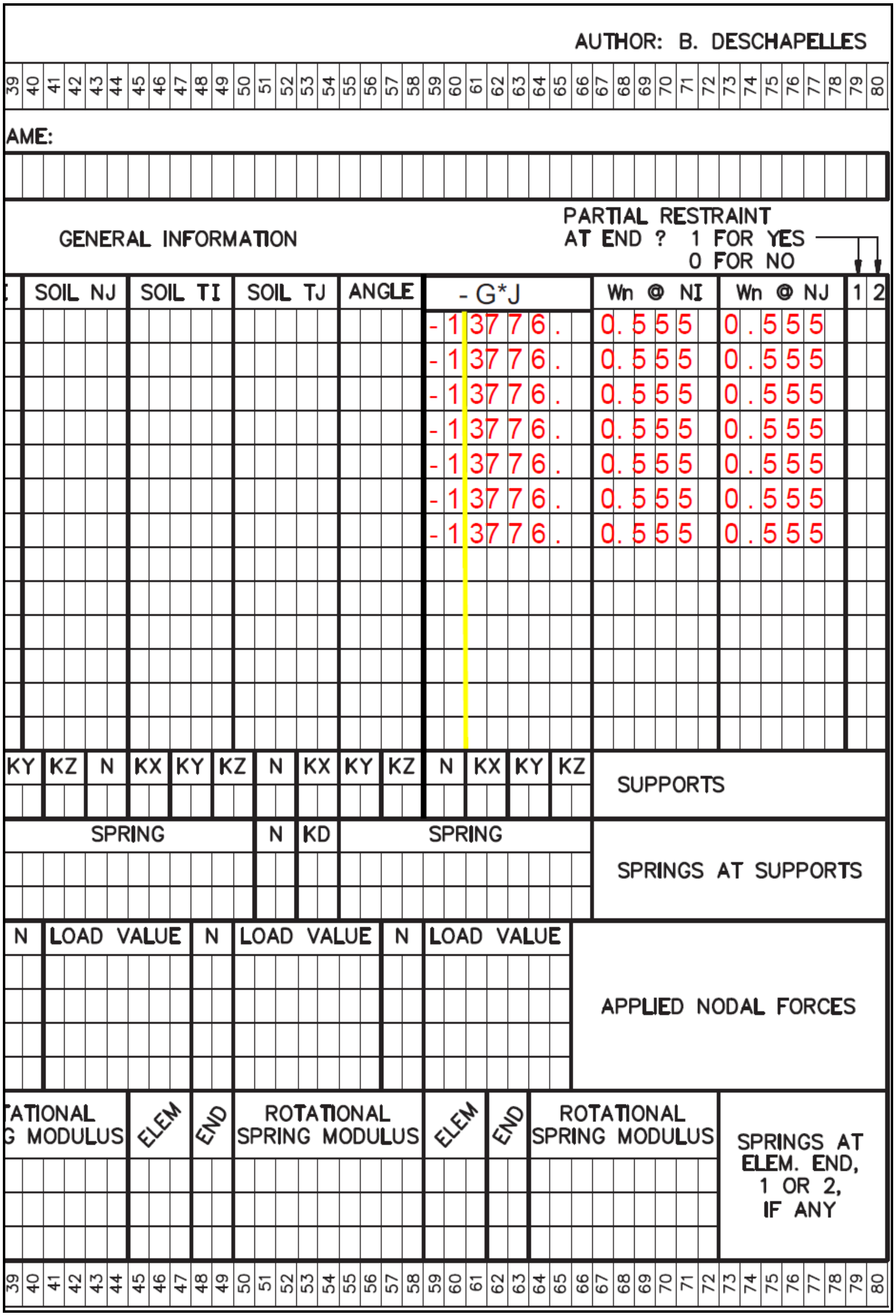
| NF | N | KX | KY | KZ | N | KX | KY | KZ | N | KX | KY | KZ | N | KX | KY | KZ | N | KX | KY | K |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| 2 | 1 | 1 | 1 | 1 | 15 | 0 | 0 | 1 | | | | | | | | | | | | |

| NF | N | KD | SPRING | N | KD | SPRING | N | KD |
|---|---|---|---|---|---|---|---|---|
| | | | | | | | | |

| NF | N | LOAD VALUE | N | LOAD VALUE | N | LOAD VALUE | N | LOAD VALUE | N | L |
|---|---|---|---|---|---|---|---|---|---|---|
| | | | | | | | | | | |

| NF | ELEM | END | ROTATIONAL SPRING MODULUS | ELEM | END | ROTATIONAL SPRING MODULUS | ELEM | END | ROTATION SPRING MOD |
|---|---|---|---|---|---|---|---|---|---|
| | | | | | | | | | |

Input Form Left Side



Input Form Right Side

```
                                            5-5o
 eleme-6ft
------------------------------------------------------------------------
YOU ARE USING COMPUTER PROGRAM BMTORSW,DEVELOPED BY DR. BERNARDO DESCHAPELLES

        INPUT DATA FILE NAME IS = 5-5.txt

                OUTPUT FILE NAME IS = 5-5o.txt

          STORAGE FILE FOR POST-PROCESSING WITH EXCEL = 5-5grf.grf


------------------------------------------------------------------------
AISC Design Guide 9, Example 5.5, MC18x42.7 channel

modulus of elasticity of the material=   29000. k/ft2

ELEM nodes inertia length distrib. load  AXIAL  SOIL NORMAL MODULUS,Ksf  angle
    i    j  ft.4     ft    at i   at j   LOAD    1st END      2nd END    rad
 1   1   3********  6.00  0.555  0.555********     0.0          0.0    0.000 00
 2   3   5********  6.00  0.555  0.555********     0.0          0.0    0.000 00
 3   5   7******** 16.00  0.555  0.555********     0.0          0.0    0.000 00
 4   7   9******** 16.00  0.555  0.555********     0.0          0.0    0.000 00
 5   9  11******** 16.00  0.555  0.555********     0.0          0.0    0.000 00
 6  11  13********  6.00  0.555  0.555********     0.0          0.0    0.000 00
 7  13  15********  6.00  0.555  0.555********     0.0          0.0    0.000 00

    INPUT DATA RELATED TO THE  2 SUPPORTS
 2 1 1 1 115 0 0 1

      FINAL  SOLUTION   FOUND   AFTER    1   ITERATIONS

     Output of nodal displacements in reference to  global axes

node  displ.      displ.      displ.     node  displ.      displ.      displ.
    along  x    along  y    around  z        along  x    along  y    around  z
    or nonn1    or nonn2    or nonn 3        or nonn1    or nonn2    or nonn 3

  1 0.0000E+00  0.0000E+00  0.0000E+00
                                           2 0.0000E+00  0.1835E-03  0.4518E-04

  3 0.0000E+00  0.5367E-03  0.1699E-03
                                           4 0.0000E+00  0.1178E-02  0.1172E-03

  5 0.0000E+00  0.1939E-02  0.2898E-03
                                           6 0.0000E+00  0.4763E-02  0.5041E-03

  7 0.0000E+00  0.7923E-02  0.4174E-03
                                           8 0.0000E+00  0.1118E-01  0.5311E-03
                              Local extreme d2y/dx:2 = 0 = bimoment
  9 0.0000E+00  0.1426E-01  0.3493E-03
                                          10 0.0000E+00  0.1662E-01  0.3555E-03

 11 0.0000E+00  0.1850E-01  0.1686E-03
                                          12 0.0000E+00  0.1893E-01  0.6387E-04

 13 0.0000E+00  0.1927E-01  0.8589E-04
                                          14 0.0000E+00  0.1944E-01  0.2155E-04

 15 0.0000E+00  0.1953E-01  0.0000E+00
                                 Local extreme twist and zero St. Vt. Torque
-----------------------------------------------------------------------
     OUTPUT OF SOIL REACTIONS,STRESSES AND TRANSVERSE DISPLACEMENTS

                                           Page 1
```

Output Data Page 1

```
                                    5-50
----------------------------------------------------------------------
     ELEMENT    1  DISPLACEMENTS IN INCIDENCES    1   2   3
        NODE    1          0.00000E+00          0.00000E+00          0.00000E+00
        NODE    2          0.00000E+00          0.18348E-03          0.45178E-04
        NODE    3          0.00000E+00          0.53673E-03          0.16991E-03
     FORCES ACTING ALONG THE 9 DOF
        NODE    1          0.00000E+00         -0.39960E+02         -0.81373E+03
        NODE    2          0.00000E+00          0.33300E+01         -0.31143E-10
        NODE    3          0.00000E+00          0.36630E+02          0.59135E+03

ELEMENT   1, FROM NODE   1, TO NODE   3 - LENGTH =   6.00 ft

 left half of span,at tenth points of length
             span      span      span      span      span      span
         0.0       0.1       0.2       0.3       0.4       0.5
soil,k/ft   0.000     0.000     0.000     0.000     0.000     0.000
shear,k    -39.96    -39.63    -39.29    -38.96    -38.63    -38.30
bmom,kft   813.73    789.93    766.50    743.41    720.68    698.29
tdisp,ft  0.00000   0.00001   0.00002   0.00005   0.00009   0.00014
axial,k      0.00 AT 1st END and      0.00 AT 2nd END

right half of span,at tenth points of length
             span      span      span      span      span      span
         0.5       0.6       0.7       0.8       0.9       1.0
soil,k/ft   0.000     0.000     0.000     0.000     0.000     0.000
shear,k    -38.30    -37.96    -37.63    -37.30    -36.96    -36.63
bmom,kft   698.29    676.24    654.53    633.14    612.09    591.35
tdisp,ft  0.00014   0.00020   0.00027   0.00035   0.00044   0.00054
axial,k      0.00 AT 1st END and      0.00 AT 2nd END

----------------------------------------------------------------------
     ELEMENT    2  DISPLACEMENTS IN INCIDENCES    3   4   5
        NODE    3          0.00000E+00          0.53673E-03          0.16991E-03
        NODE    4          0.00000E+00          0.11780E-02          0.11724E-03
        NODE    5          0.00000E+00          0.19390E-02          0.28979E-03
     FORCES ACTING ALONG THE 9 DOF
        NODE    3          0.00000E+00         -0.36630E+02         -0.59135E+03
        NODE    4          0.00000E+00          0.33300E+01         -0.15864E-09
        NODE    5          0.00000E+00          0.33300E+02          0.40088E+03

ELEMENT   2, FROM NODE   3, TO NODE   5 - LENGTH =   6.00 ft

 left half of span,at tenth points of length
             span      span      span      span      span      span
         0.0       0.1       0.2       0.3       0.4       0.5
soil,k/ft   0.000     0.000     0.000     0.000     0.000     0.000
shear,k    -36.63    -36.30    -35.96    -35.63    -35.30    -34.97
bmom,kft   591.35    570.94    550.84    531.05    511.56    492.38
tdisp,ft  0.00054   0.00064   0.00076   0.00088   0.00101   0.00115
axial,k      0.00 AT 1st END and      0.00 AT 2nd END

right half of span,at tenth points of length
             span      span      span      span      span      span
         0.5       0.6       0.7       0.8       0.9       1.0
soil,k/ft   0.000     0.000     0.000     0.000     0.000     0.000
shear,k    -34.97    -34.63    -34.30    -33.97    -33.63    -33.30
bmom,kft   492.38    473.50    454.91    436.62    418.61    400.88
tdisp,ft  0.00115   0.00129   0.00144   0.00160   0.00177   0.00194
axial,k      0.00 AT 1st END and      0.00 AT 2nd END

----------------------------------------------------------------------
     ELEMENT    3  DISPLACEMENTS IN INCIDENCES    5   6   7
        NODE    5          0.00000E+00          0.19390E-02          0.28979E-03
                                   Page  2
```

72inx0.555kip/in=39.96kip

(-Di + Dj)* GJ + Vj*L- 3.33xL/2 = Mj-Mi, where GJ=13776

-0.00054+0.00194)13776-33.3*6 -9.99=19.29-199.8 -9.99=-189.98

400.88-591.35=-190.47  OK

Output Data Page 2

```
                                        5-50
        NODE    6          0.00000E+00          0.47633E-02          0.50413E-03
        NODE    7          0.00000E+00          0.79232E-02          0.41740E-03
     FORCES ACTING ALONG THE 9 DOF
        NODE    5          0.00000E+00         -0.33300E+02         -0.40088E+03
        NODE    6          0.00000E+00          0.88800E+01          0.41688E-10
        NODE    7          0.00000E+00          0.24420E+02          0.21560E+02

ELEMENT   3, FROM NODE   5, TO NODE   7 - LENGTH = 16.00 ft

 left half of span,at tenth points of length
              span       span       span       span       span       span
          0.0        0.1        0.2        0.3        0.4        0.5
soil,k/ft    0.000      0.000      0.000      0.000      0.000      0.000
shear,k     -33.30     -32.41     -31.52     -30.64     -29.75     -28.86
bmom,kft    400.88     354.97     310.99     268.88     228.57     190.01
tdisp,ft   0.00194    0.00242    0.00294    0.00350    0.00408    0.00468
axial,k        0.00 AT 1st END and       0.00 AT 2nd END

right half of span,at tenth points of length
              span       span       span       span       span       span
          0.5        0.6        0.7        0.8        0.9        1.0
soil,k/ft    0.000      0.000      0.000      0.000      0.000      0.000
shear,k     -28.86     -27.97     -27.08     -26.20     -25.31     -24.42
bmom,kft    190.01     153.14     117.91      84.26      52.16      21.56
tdisp,ft   0.00468    0.00530    0.00594    0.00660    0.00726    0.00792
axial,k        0.00 AT 1st END and       0.00 AT 2nd END

----------------------------------------------------------------------
     ELEMENT    4  DISPLACEMENTS IN INCIDENCES    7   8   9
        NODE    7          0.00000E+00          0.79232E-02          0.41740E-03
        NODE    8          0.00000E+00          0.11183E-01          0.53113E-03
        NODE    9          0.00000E+00          0.14257E-01          0.34926E-03
     FORCES ACTING ALONG THE 9 DOF
        NODE    7          0.00000E+00         -0.24420E+02         -0.21560E+02
        NODE    8          0.00000E+00          0.88800E+01          0.11555E-09
        NODE    9          0.00000E+00          0.15540E+02         -0.21087E+03

ELEMENT   4, FROM NODE   7, TO NODE   9 - LENGTH = 16.00 ft

 left half of span,at tenth points of length
              span       span       span       span       span       span
          0.0        0.1        0.2        0.3        0.4        0.5
soil,k/ft    0.000      0.000      0.000      0.000      0.000      0.000
shear,k     -24.42     -23.53     -22.64     -21.76     -20.87     -19.98
bmom,kft     21.56      -7.59     -35.34     -61.71     -86.75    -110.49
tdisp,ft   0.00792    0.00859    0.00926    0.00992    0.01058    0.01123
axial,k        0.00 AT 1st END and       0.00 AT 2nd END
                                                Bimoment and 2nd derivative=0
right half of span,at tenth points of length    extreme value for St. Vt. torsion
              span       span       span       span       span       span
          0.5        0.6        0.7        0.8        0.9        1.0
soil,k/ft    0.000      0.000      0.000      0.000      0.000      0.000
shear,k     -19.98     -19.09     -18.20     -17.32     -16.43     -15.54
bmom,kft   -110.49    -132.97    -154.22    -174.27    -193.14    -210.87
tdisp,ft   0.01123    0.01187    0.01249    0.01310    0.01369    0.01426
axial,k        0.00 AT 1st END and       0.00 AT 2nd END

----------------------------------------------------------------------
     ELEMENT    5  DISPLACEMENTS IN INCIDENCES    9  10  11
        NODE    9          0.00000E+00          0.14257E-01          0.34926E-03
        NODE   10          0.00000E+00          0.16622E-01          0.35553E-03
        NODE   11          0.00000E+00          0.18503E-01          0.16864E-03
     FORCES ACTING ALONG THE 9 DOF
                                       Page 3
```

Output Data Page 3

```
                                              5-50
          NODE    9          0.00000E+00         -0.15540E+02          0.21087E+03
          NODE   10          0.00000E+00          0.88800E+01          0.79967E-10
          NODE   11          0.00000E+00          0.66600E+01         -0.32998E+03

ELEMENT   5, FROM NODE   9, TO NODE 11 - LENGTH = 16.00 ft

 left half of span,at tenth points of length
             span      span      span      span      span      span
          0.0       0.1       0.2       0.3       0.4       0.5
soil,k/ft     0.000     0.000     0.000     0.000     0.000     0.000
shear,k      -15.54    -14.65    -13.76    -12.88    -11.99    -11.10
bmom,kft    -210.87   -227.48   -242.99   -257.43   -270.82   -283.17
tdisp,ft    0.01426   0.01480   0.01533   0.01583   0.01630   0.01674
axial,k        0.00 AT 1st END and      0.00 AT 2nd END

right half of span,at tenth points of length
             span      span      span      span      span      span
          0.5       0.6       0.7       0.8       0.9       1.0
soil,k/ft     0.000     0.000     0.000     0.000     0.000     0.000
shear,k      -11.10    -10.21     -9.32     -8.44     -7.55     -6.66
bmom,kft    -283.17   -294.50   -304.84   -314.19   -322.56   -329.98
tdisp,ft    0.01674   0.01716   0.01754   0.01790   0.01822   0.01850
axial,k        0.00 AT 1st END and      0.00 AT 2nd END

----------------------------------------------------------------------
     ELEMENT    6  DISPLACEMENTS IN INCIDENCES  11  12  13
        NODE   11          0.00000E+00          0.18503E-01          0.16864E-03
        NODE   12          0.00000E+00          0.18927E-01          0.63868E-04
        NODE   13          0.00000E+00          0.19268E-01          0.85888E-04
     FORCES ACTING ALONG THE 9 DOF
        NODE   11          0.00000E+00         -0.66600E+01          0.32998E+03
        NODE   12          0.00000E+00          0.33300E+01          0.21120E-08
        NODE   13          0.00000E+00          0.33300E+01         -0.34940E+03

ELEMENT   6, FROM NODE 11, TO NODE 13 - LENGTH =  6.00 ft

 left half of span,at tenth points of length
             span      span      span      span      span      span
          0.0       0.1       0.2       0.3       0.4       0.5
soil,k/ft     0.000     0.000     0.000     0.000     0.000     0.000
shear,k       -6.66     -6.33     -5.99     -5.66     -5.33     -4.99
bmom,kft    -329.98   -332.51   -334.92   -337.19   -339.32   -341.33
tdisp,ft    0.01850   0.01860   0.01870   0.01878   0.01887   0.01895
axial,k        0.00 AT 1st END and      0.00 AT 2nd END

right half of span,at tenth points of length
             span      span      span      span      span      span
          0.5       0.6       0.7       0.8       0.9       1.0
soil,k/ft     0.000     0.000     0.000     0.000     0.000     0.000
shear,k       -4.99     -4.66     -4.33     -4.00     -3.66     -3.33
bmom,kft    -341.33   -343.20   -344.95   -346.56   -348.04   -349.40
tdisp,ft    0.01895   0.01902   0.01909   0.01916   0.01921   0.01927
axial,k        0.00 AT 1st END and      0.00 AT 2nd END

----------------------------------------------------------------------
     ELEMENT    7  DISPLACEMENTS IN INCIDENCES  13  14  15
        NODE   13          0.00000E+00          0.19268E-01          0.85888E-04
        NODE   14          0.00000E+00          0.19441E-01          0.21550E-04
        NODE   15          0.00000E+00          0.19527E-01          0.00000E+00
     FORCES ACTING ALONG THE 9 DOF
        NODE   13          0.00000E+00         -0.33300E+01          0.34940E+03
        NODE   14          0.00000E+00          0.33300E+01         -0.54570E-11
        NODE   15          0.00000E+00         -0.86629E-10         -0.35583E+03
                                            Page  4
```

(-Di + Dj)* GJ +  Vj*L - 3.33L/2 = Mj-Mi, where GJ=13776

-0.01850+0.01927)*13776-3.33*6-9.99=10.6-19.98- 9.99= -19.37

-349.40+329.98=-19.42 OK

Output Data Page 4

```
                                                  5-5o

ELEMENT   7, FROM NODE 13, TO NODE 15 - LENGTH =   6.00 ft

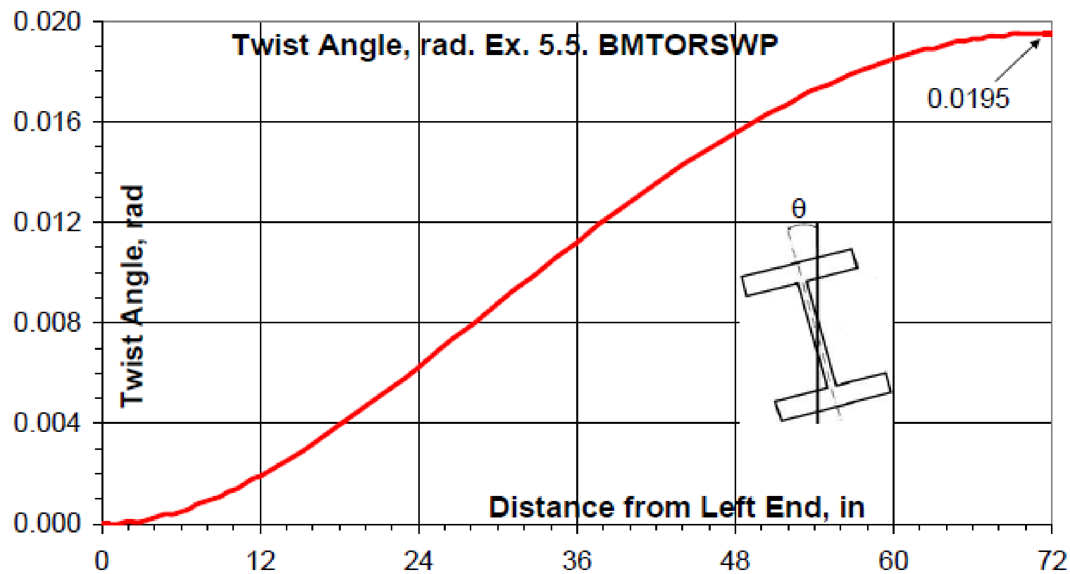
 left half of span,at tenth points of length
             span      span      span      span      span      span
         0.0       0.1       0.2       0.3       0.4       0.5
soil,k/ft   0.000     0.000     0.000     0.000     0.000     0.000
shear,k     -3.33     -3.00     -2.66     -2.33     -2.00     -1.67
bmom,kft  -349.40   -350.62   -351.71   -352.68   -353.51   -354.22
tdisp,ft  0.01927   0.01932   0.01936   0.01940   0.01943   0.01946
axial,k      0.00 AT 1st END and      0.00 AT 2nd END

right half of span,at tenth points of length
             span      span      span      span      span      span
         0.5       0.6       0.7       0.8       0.9       1.0
soil,k/ft   0.000     0.000     0.000     0.000     0.000     0.000
shear,k     -1.67     -1.33     -1.00     -0.67     -0.33      0.00
bmom,kft  -354.22   -354.80   -355.25   -355.57   -355.76   -355.83
tdisp,ft  0.01946   0.01949   0.01950   0.01952   0.01952   0.01953
axial,k      0.00 AT 1st END and      0.00 AT 2nd END
```

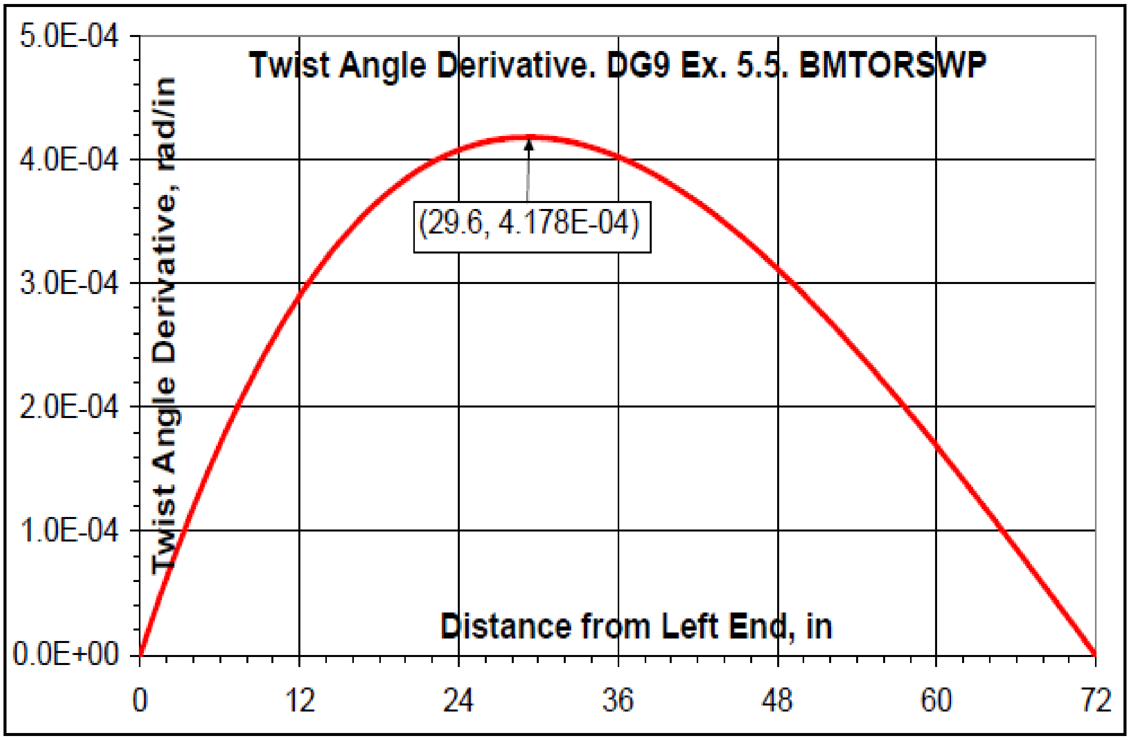
Local extreme twist angle and bi-moment in the TWB. Local extreme analog transverse displacement and moment in the EBC

Output Page 5

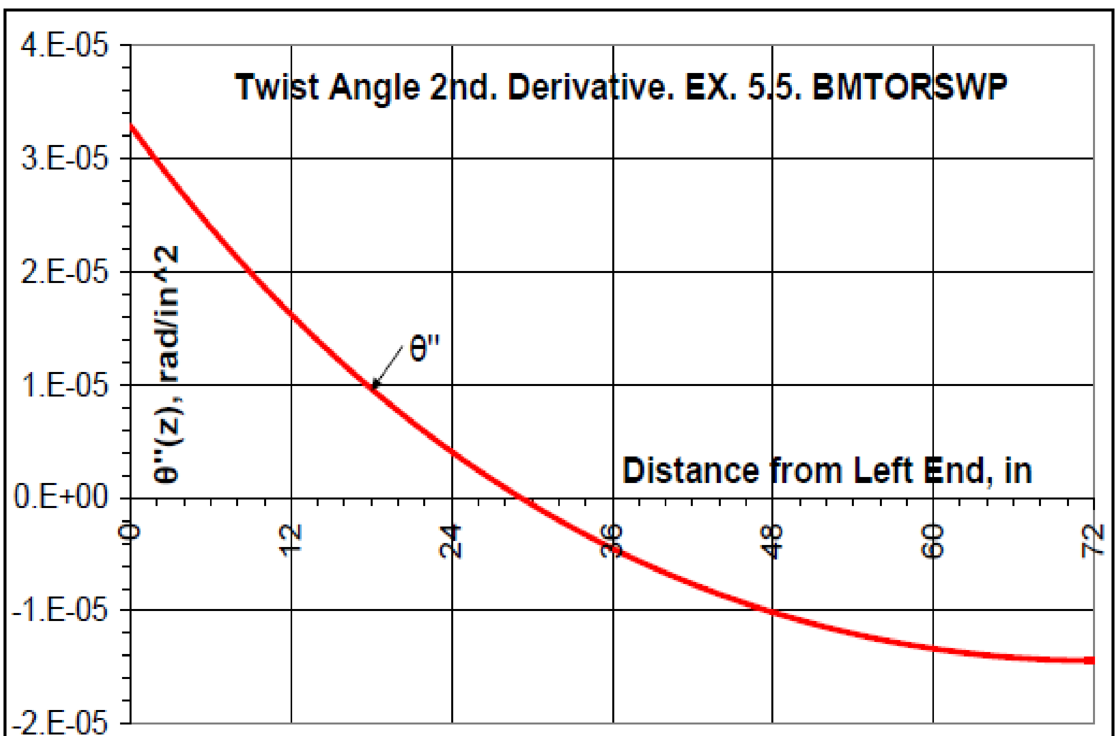
From summation of moments in the EBC $\Delta = (B1 + B2 - wL^2/2)/ GJ =$
$= (813.73 + 353.83 - 0.555 \times 72^2/2)/ 13776 = 0.01967 \sim 0.0195$
(BMTORSWP).



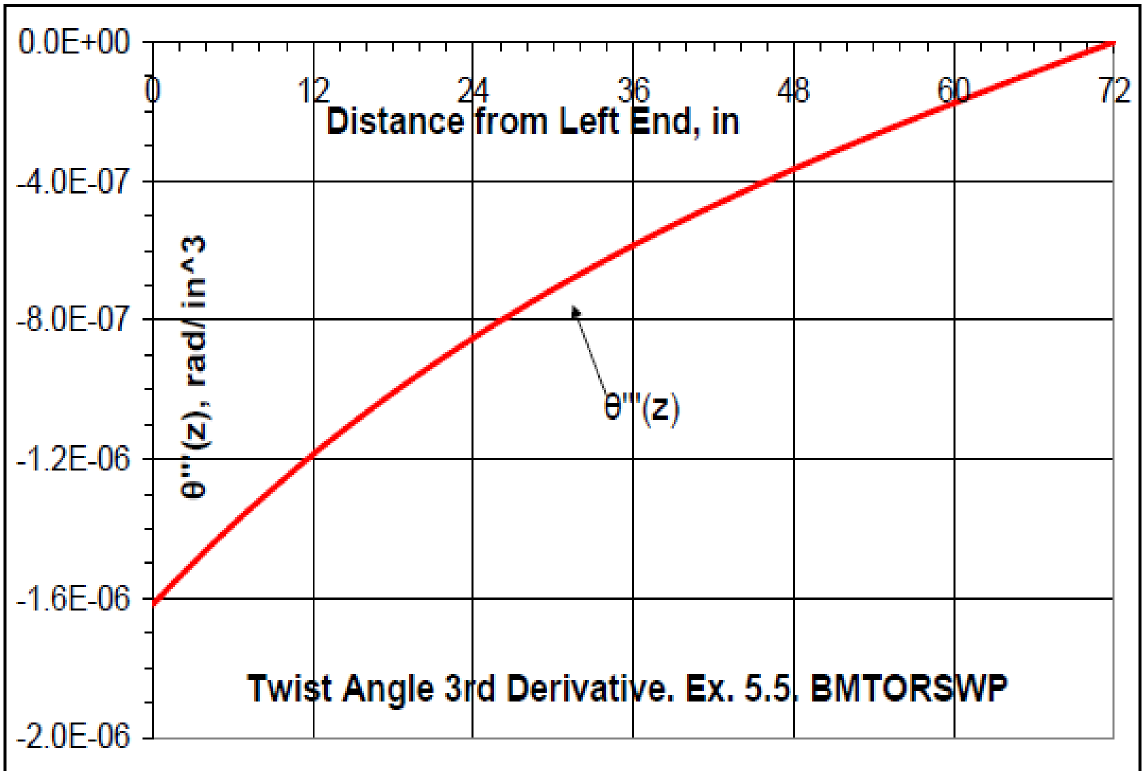

Twist Angle, Ex. 5.5 from BMTORSWP



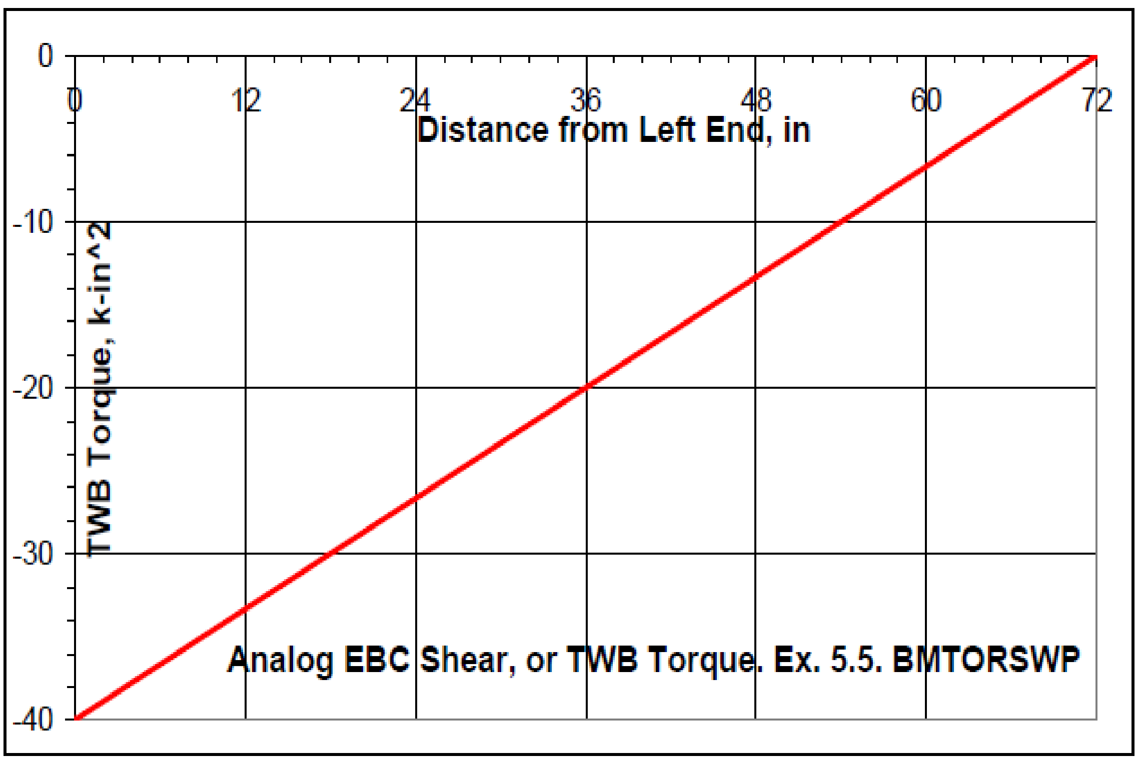

Twist Angle Derivative of DG9 Ex. 5.5 from BMTORSWP




Twist Angle 2nd Derivative of DG9 Ex. 5.5 from BMTORSWP

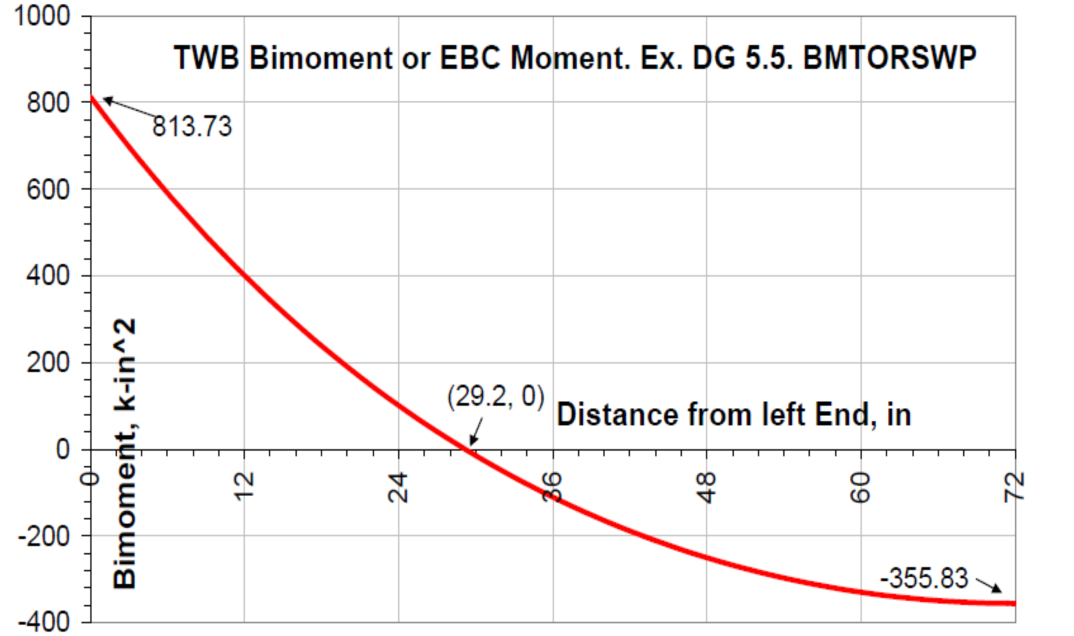



Twist Angle 3rd Derivative of DG9 Ex. 5.5 from BMTORSWP

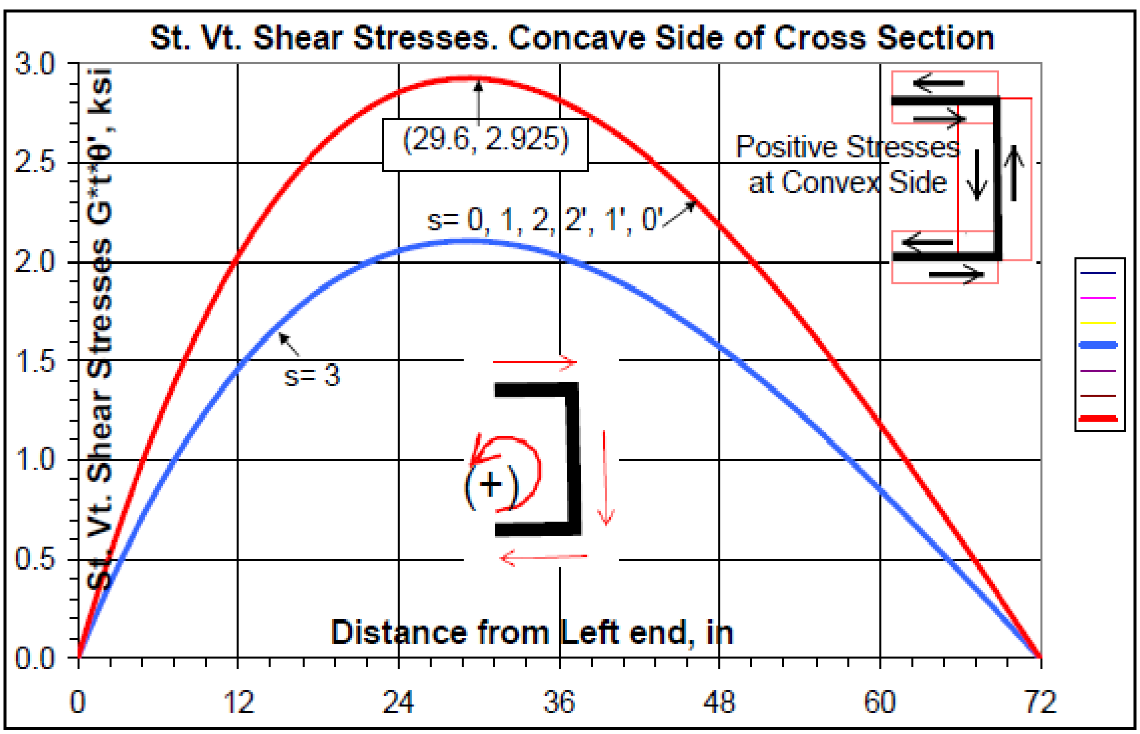



BC Shear and TWB Torque of DG9 Ex. 5.5 from BMTORSWP

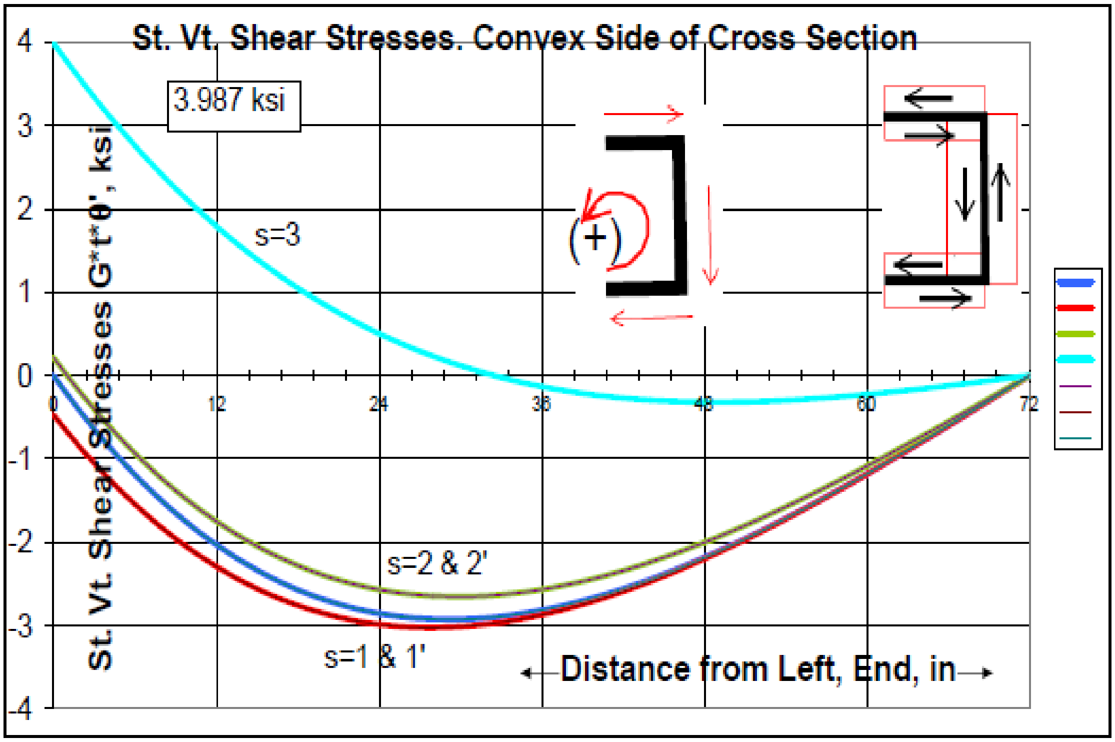
From summation of moments in the EBC:Δ = (B1 + B2 – wL$^2$/2)/ GJ = (813.73 + 353.83 – 0.555 X 722/2)/ 13776 = 0.01967 ~ 0.0195 (BMTORSWP).



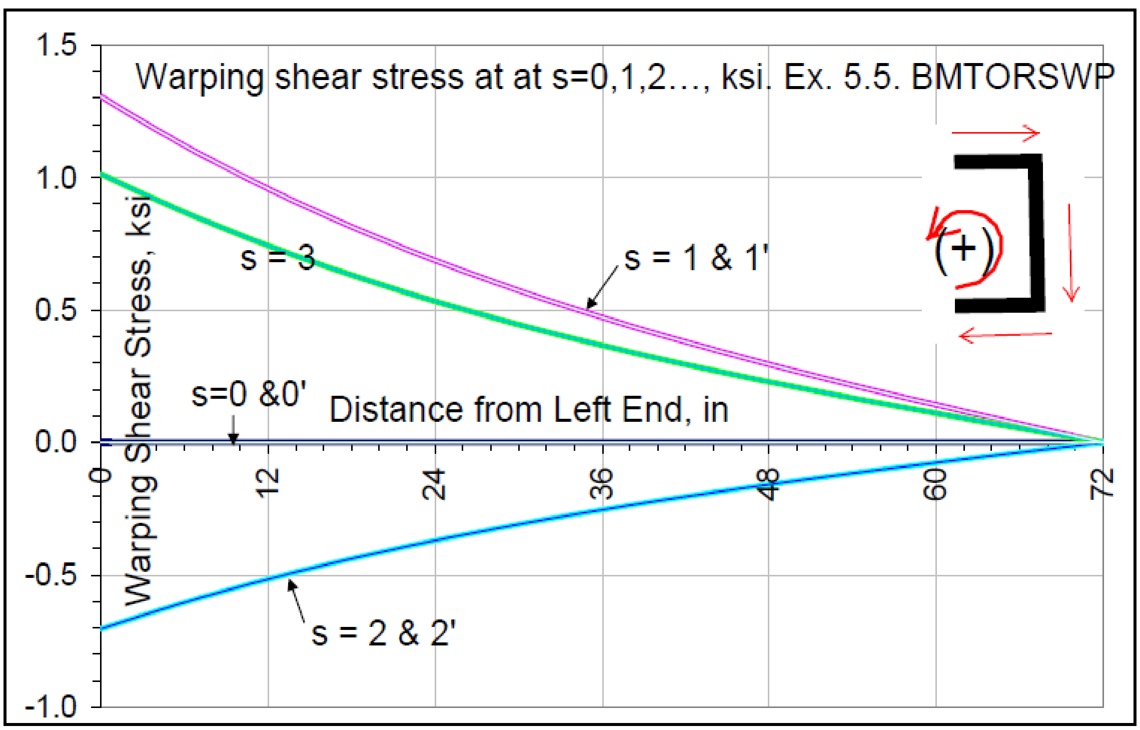

TWB Bimoment and BC Moment of Ex. DG 5.5 from BMTORSWP



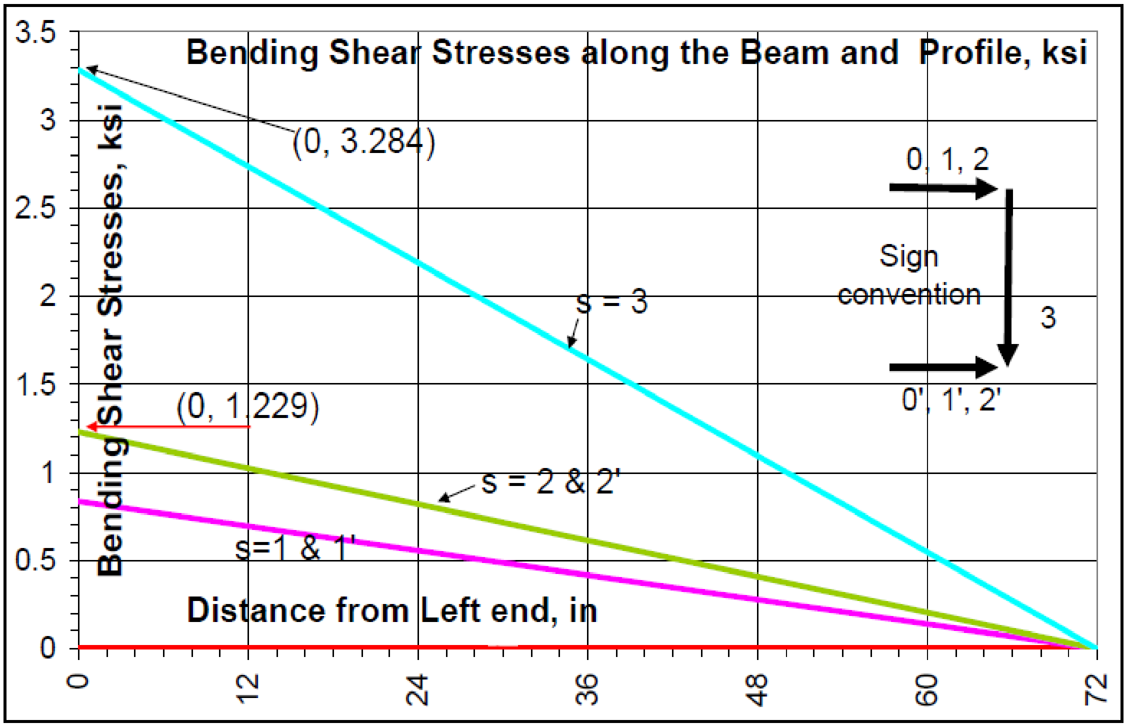

St. Vt. Shear Stresses in Concave Side of Cross Section




St. Vt. Shear Stresses in Convex Side of Cross Section

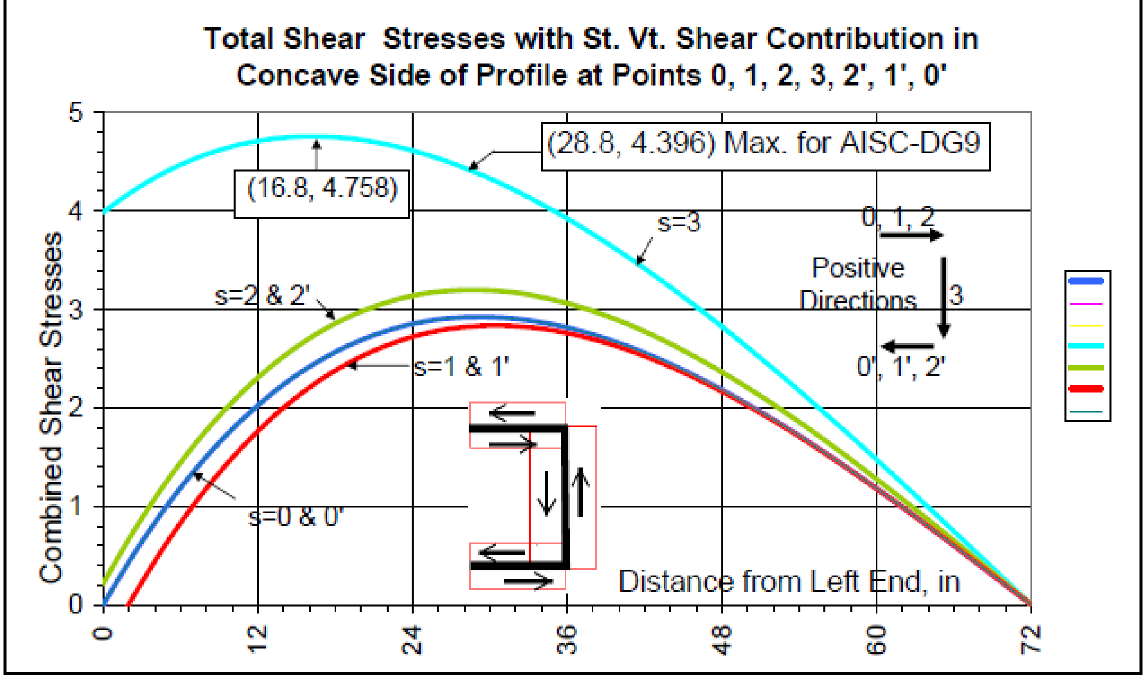



Warping Shear Stress at s = 0,1,2,3 of Ex. 5.5 from BMTORSWP

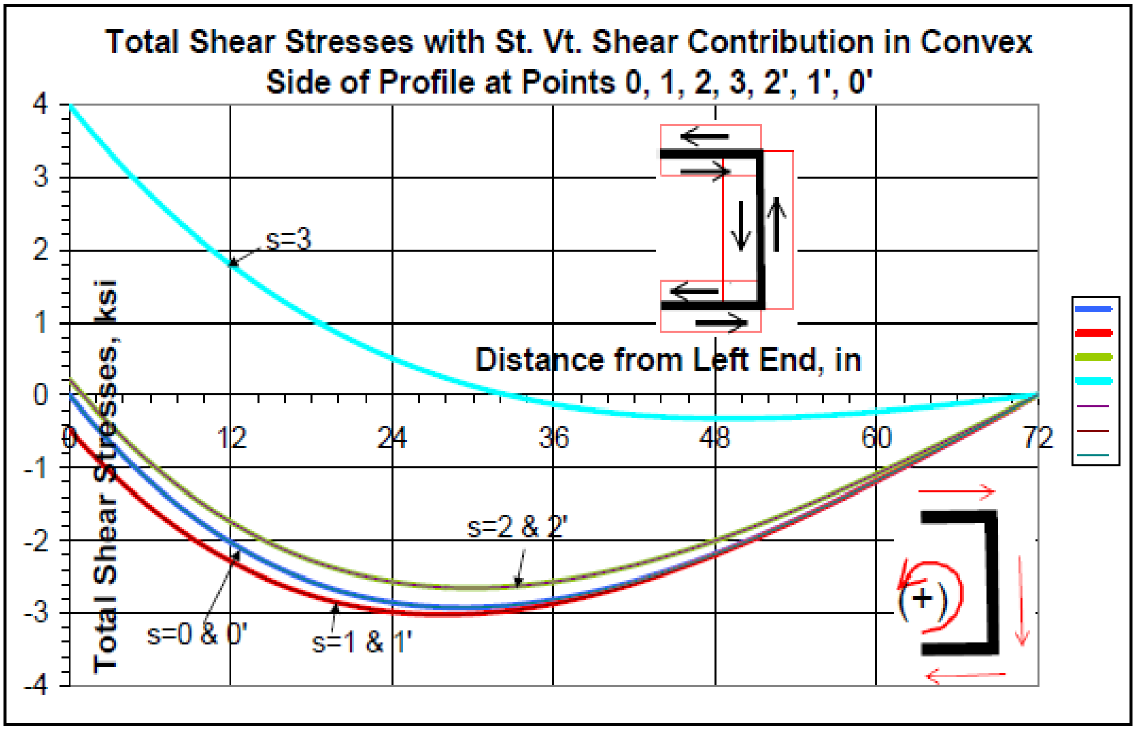



Bending Shear Stresses along the Beam and Profile

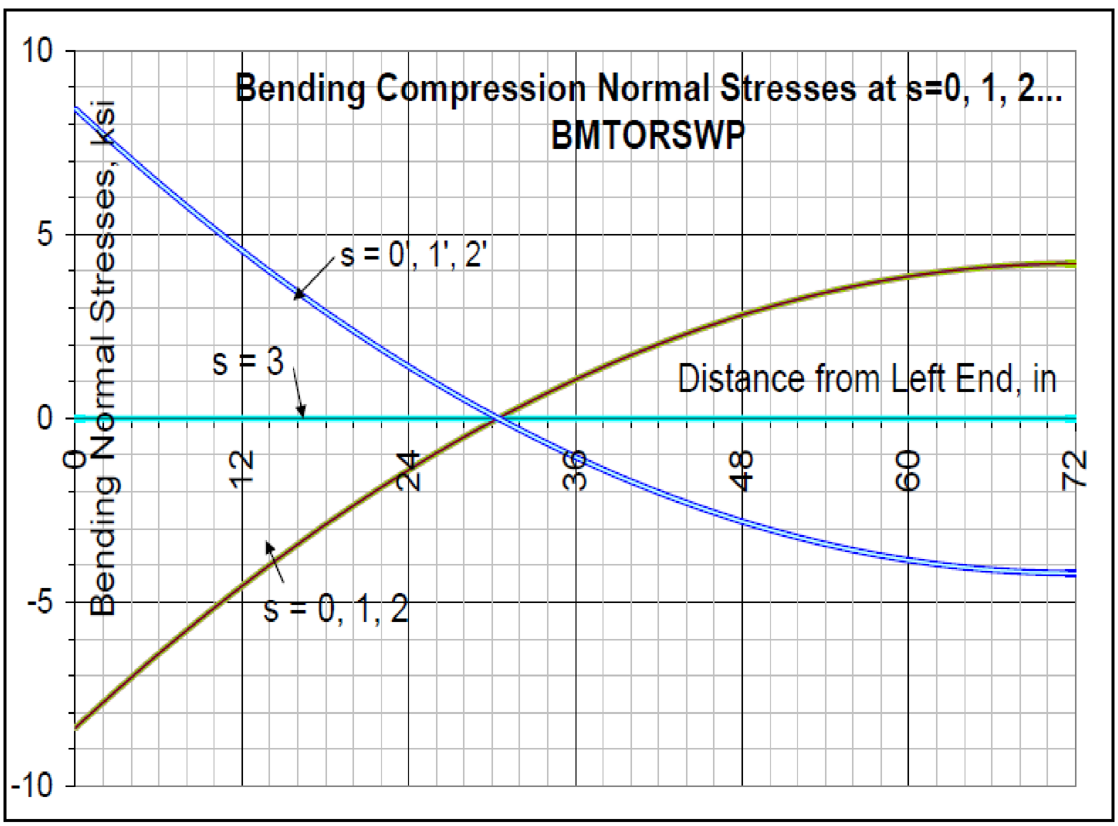
At 23.2in from the left end of the beam at points $s$ = 2 and 2' of the profile occurs the highest combined shear stress unnoticed by the AISC DG9 and shown here:



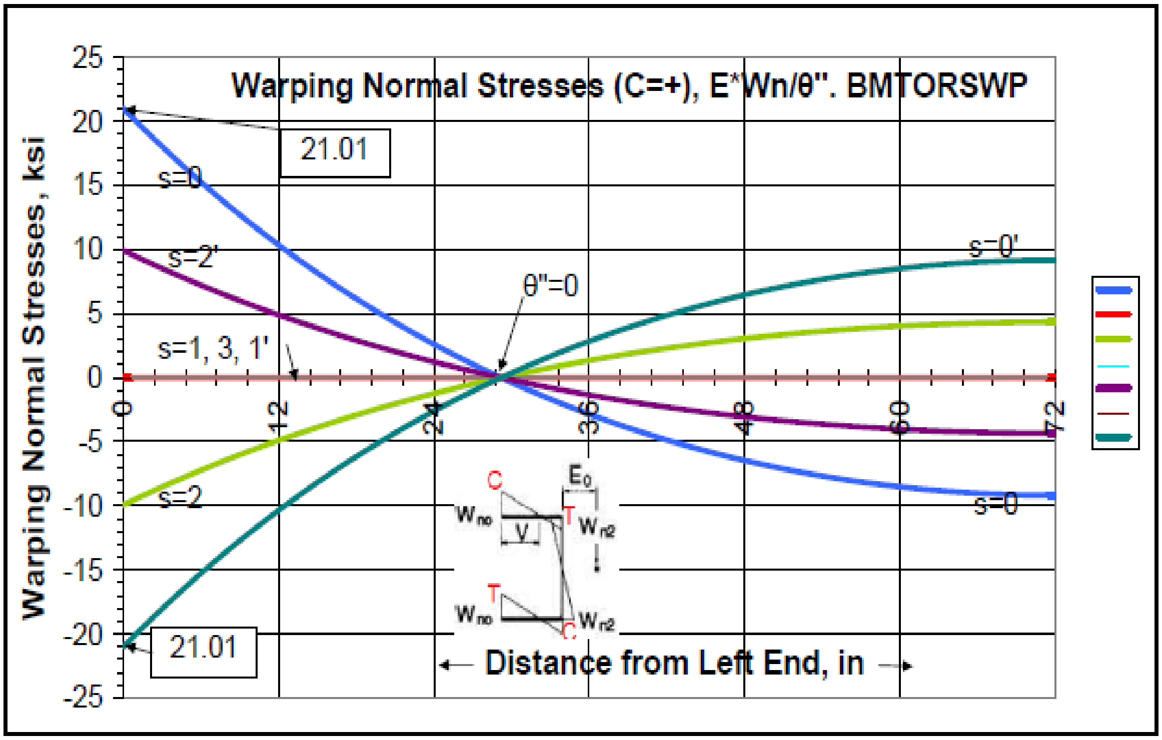

Total Shear Stresses in Concave Side of Profile



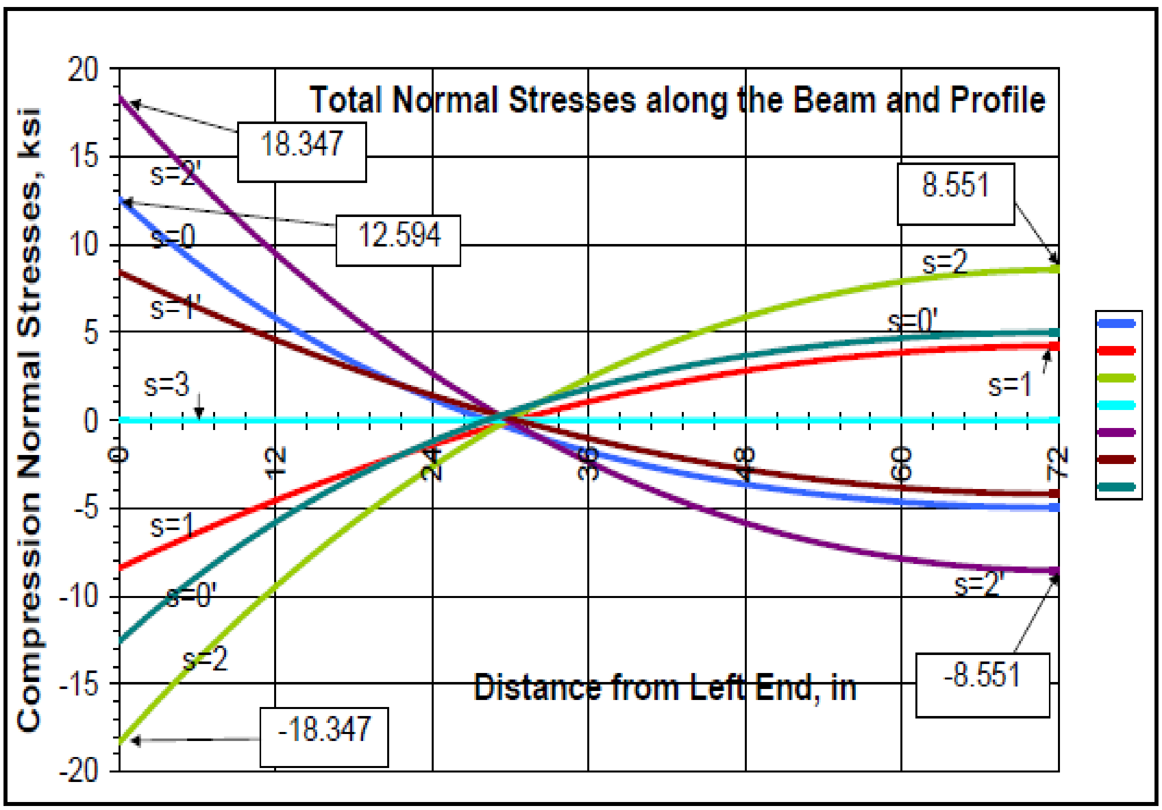

Total Shear Stresses in Convex Side of Profile




Bending Normal Stresses at s =0, 1, 2, 3 from BMTORSWP




Warping Normal Stresses along Beam and Profile




Combined Normal Stresses along the Beam and Profile

**Numeric Checks**

Rows reflect interest points along the axial coordinate, and columns reflect orderly the torque angle and 3 derivatives. Entry arrays begin from zero; thus θ03 in the first row and fourth column is the 3rd derivative at the interest point 0

$$\theta = \begin{pmatrix} 0 & 8.609\times10^{-3} & 0.018 & 0 \\ 0 & 4.061\times10^{-4} & 0 & 0 \\ 3.147\times10^{-5} & 0 & -1.368\times10^{-5} & 3.147\times10^{-5} \\ -1.614\times10^{-6} & -7.422\times10^{-7} & 0 & -1.614\times10^{-6} \end{pmatrix} \quad \text{AISC}$$

$$\theta_{Bmt} := \begin{pmatrix} 0 & 8.3\times10^{-3} & 1.95\times10^{-2} & 0 \\ 0 & 4.176\times10^{-4} & 0 & 0 \\ 3.293\times10^{-5} & 2.825\times10^{-7} & -1.44\times10^{-5} & 3.293\times10^{-5} \\ -1.617\times10^{-6} & -7.372\times10^{-7} & -2.073\times10^{-11} & -1.617\times10^{-6} \end{pmatrix} \quad \text{BMTORSWP}$$

Torque Angles & its 3 Dimensionless Derivatives along the Span

Shear stress due to pure torsion $\tau_t = GJ\theta'$. Pure torsion shear stresses appear in matrix rows containing points of interest 0, 1, 2, 3 of the profile. Each matrix column contains the defined points of interest along the axial coordinate. Points 0, 1 and 2 of the profile have the same pure torsion shear stress because they have the same thickness

$$\tau_{tBmt_{j,i}} := G\cdot t_i\cdot\frac{\theta_{Bmt_{1,j}}}{in} \qquad \tau_{t_{j,i}} := G\cdot t_i\cdot\frac{\theta_{1,j}}{in}$$

$$\tau_{tBmt} = \begin{pmatrix} 0 & 0 & 0 & 0 \\ 2.923 & 2.923 & 2.923 & 2.105 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix}\cdot ksi \quad \begin{matrix}\text{BMTORSW at support} \\ 0.2L \\ 0.5L \\ L\end{matrix}$$

$$\tau_t = \begin{pmatrix} 0 & 0 & 0 & 0 \\ 2.843 & 2.843 & 2.843 & 2.047 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix}\cdot ksi \quad \begin{matrix}\text{AISC-DG9 at support} \\ 0.2L \\ 0.5L \\ L\end{matrix}$$

Shear Stresses due to Pure Torsion along the Span and Profile

Shear stress from warping at point "s" of the profile. Warping shear stresses along one profile appear in matrix rows containing points of interest 0, 1, 2, 3 of the profile. Matrix columns contain the defined points of interest along the axial coordinate

$$\tau_{wBmt_{j,i}} := \frac{-E}{t_i} \cdot S_{w_i} \cdot \left( \theta_{Bmt_{3,j}} \cdot \frac{1}{in^3} \right) \qquad \tau_{w_{j,i}} := \frac{-E}{t_i} \cdot S_{w_i} \cdot \left( \theta_{3,j} \cdot \frac{1}{in^3} \right)$$

$$\tau_{wBmt} = \begin{pmatrix} 0 & 1.306 & -1.013 & 0.703 \\ 0 & 0.595 & -0.462 & 0.321 \\ 0 & 0 & -0 & 0 \\ 0 & 1.306 & -1.013 & 0.703 \end{pmatrix} \cdot ksi$$

BMTORSW at
support
0.2L
0.5L
L

$$\tau_{w} = \begin{pmatrix} 0 & 1.303 & -1.011 & 0.702 \\ 0 & 0.599 & -0.465 & 0.323 \\ 0 & 0 & 0 & 0 \\ 0 & 1.303 & -1.011 & 0.702 \end{pmatrix} \cdot ksi$$

AISC-DG9 at
support
0.2L
0.5L
L

Shear Stresses due to Restrained Warping along the Span and Profile

Warping normal stresses $\sigma ws = E*Wns*\theta$" along one profile appear in a matrix row containing points of interest 0, 1, 2, 3 of the profile. Each matrix column contains the defined points of interest along the axial coordinate.

$$\sigma_{w_{j,i}} := E \cdot W_{n_i} \cdot \left( \theta_{2,j} \cdot \frac{1}{in^2} \right) \qquad \sigma_{wBmt_{j,i}} := E \cdot W_{n_i} \cdot \left( \theta_{Bmt_{2,j}} \cdot \frac{1}{in^2} \right)$$

$$\sigma_{wBmt} = \begin{pmatrix} 21.009 & 0 & -9.932 & 0 \\ 0.18 & 0 & -0.085 & 0 \\ -9.187 & 0 & 4.343 & 0 \\ 21.009 & 0 & -9.932 & 0 \end{pmatrix} \cdot ksi$$

BMTORSW at
support
0.2L
0.5L
L

$$\sigma_{w} = \begin{pmatrix} 20.078 & 0 & -9.491 & 0 \\ 0 & 0 & 0 & 0 \\ -8.729 & 0 & 4.127 & 0 \\ 20.078 & 0 & -9.491 & 0 \end{pmatrix} \cdot ksi$$

AISC-DG9 at
support
0.2L
0.5L
L

Normal Stresses due to Restrained Warping along the Span and Profile service-load torque, T, where T=Tu/LF: and $LF := \frac{w_u}{w} = 1.5$

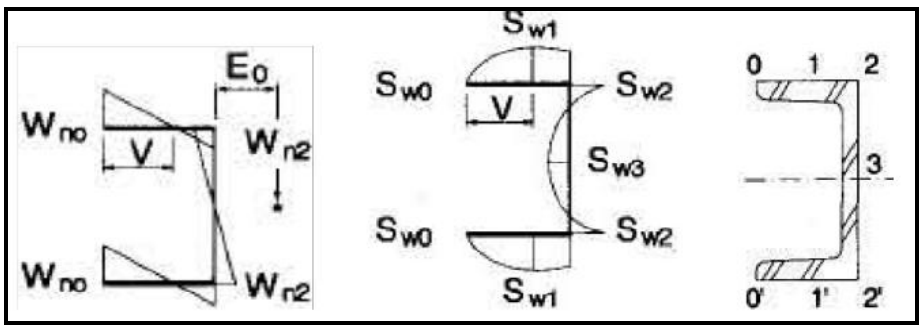
AISC

$$\frac{\max\left[\left(\theta^T\right)^{\langle 0\rangle}\right]}{LF} = 1.230 \times 10^{-2} \cdot rad$$

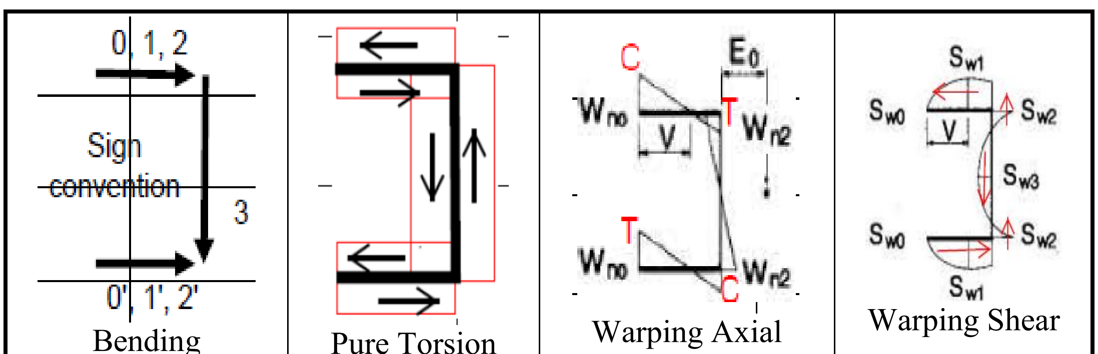
BMTORSW

$$\frac{\max\left[\left(\theta_{Bmt}{}^T\right)^{\langle 0\rangle}\right]}{LF} = 1.300 \times 10^{-2} \cdot rad$$

Maximum Service Load Rotation




Profile Key Points for Stress Computations




Assumed Directions from a Counterclockwise Twist and a Vertical Bending Shear

Positive Directions

| Location | Point | $\sigma_w$ | $\sigma_b$ | $f_{un}$ |
|---|---|---|---|---|
| Support | 0 | 20.1(C) | 8.41(T) | 11.7(C) 12.594 |
| | 1 | 0 | 8.41(T) | 8.41(T) -8.416 |
| | 2 | 9.49(T) | 8.41(T) | 17.9(T) -18.347 |
| | 3 | 0 | 0 | 0 0. |
| Midspan | 0 | 8.73(T) | 4.20(C) | 4.53(T) |
| | 1 | 0 | 4.20(C) | 4.20(C) |
| | 2 | 4.13(C) | 4.20(C) | 8.33(C) |
| | 3 | 0 | 0 | 0 |
| $z/l = 0.20$ | 0 | 0 | — | — |
| | 1 | 0 | — | — |
| | 2 | 0 | — | — |
| | 3 | 0 | — | — |
| Maximum | | | | 17.9(T) -18.347 |

Comparison of Total Normal Stresses along the Span and Profile

$$f_{unBmt} := \sigma_{wBmt} + \sigma\sigma_b \qquad f_{un} := \sigma_w + \sigma\sigma_b$$

$$f_{unBmt} = \begin{pmatrix} 12.594 & -8.416 & -18.347 & 0 \\ -0.156 & -0.337 & -0.422 & 0 \\ -4.979 & 4.208 & 8.551 & 0 \\ 12.594 & -8.416 & -18.347 & 0 \end{pmatrix} \cdot \text{ksi}$$

BMTORSW at
support
0.2L
0.5L
L

$$f_{un} = \begin{pmatrix} 11.662 & -8.416 & -17.907 & 0 \\ -0.337 & -0.337 & -0.337 & 0 \\ -4.522 & 4.208 & 8.334 & 0 \\ 11.662 & -8.416 & -17.907 & 0 \end{pmatrix} \cdot \text{ksi}$$

AISC-DG9 at
support
0.2L
0.5L
L

The maximum normal stress (tension) occurs at the support at point 2 in the flange. A discrepancy of 2.5% is noticed: 17.907 ksi and 18.347 ksi from DG9 and the software respectively. The AISC DG9 data have been recalculated to the third decimal place to be compared with BMTORSWP output calculated to the third decimal place.

Another Comparison of Total Normal Stresses along the Span and Profile

| Location | Point | $\tau_t$ | $\tau_w$ | $\tau_b$ | $f_{uv}$ | |
|---|---|---|---|---|---|---|
| Support | 0 | 0 | 0 | 0 | 0 | |
| | 1 | 0 | 1.30← | — | 1.30← | |
| | 2 | 0 | 1.01← | 1.23→ | 0.22→ | |
| | 3 | 0 | 0.702↓ | 3.28↓ | 3.98↓ | |
| Midspan | 0 | 0 | 0 | 0 | 0 | |
| | 1 | 0 | 0 | 0 | 0 | |
| | 2 | 0 | 0 | 0 | 0 | |
| | 3 | 0 | 0 | 0 | 0 | |
| z/l = 0.20 | 0 | 2.84⇆ | 0 | 0 | 2.84⇆ | 2.923 |
| | 1 | 2.84⇆ | 0.599← | — | 3.44← | 3.158 |
| | 2 | 2.84⇆ | 0.465← | 0.740→ | 3.12→ | 3.199 |
| | 3 | 2.05↓↑ | 0.323↓ | 1.98↓ | 4.35↓ | 4.396 |
| Maximum | | | | | 4.35↓ | 4.396 |

The maximum shear stress occurs at $z/L = 0.20$ at point 3 in the web. A discrepancy of 1.3% is noticed: 4.340 ksi and 4.396 ksi from DG9 and the software respectively. The AISC DG9 data have been recalculated to the third decimal place to be compared with BMTORSWP output calculated to the third decimal place.

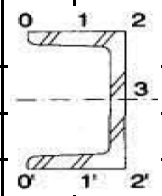
$$f_{uv} := \tau_t + \tau_w + \tau_b \qquad\qquad f_{uvBmt} := \tau_{tBmt} + \tau_{wBmt} + \tau_b$$

$$f_{uv} = \begin{pmatrix} 0 & 1.303 & 0.218 & 3.986 \\ 2.843 & 3.442 & 3.115 & 4.34 \\ 0 & 0 & 0 & 0 \\ 0 & 1.303 & -2.24 & -2.582 \end{pmatrix} \cdot \text{ksi}$$

BMTORSW at
support
0.2L
0.5L
L

Comparison of Total Shear Stresses along the Span and Profile

Torque, Bimoment, Twist Angle and Derivatives




| | | | | | | | | |
|---|---|---|---|---|---|---|---|---|
| | | | | **G =** | 11200 | ksi | | |
| | | 9 | | **E =** | 29000 | ksi | | |
| | | 9.6 | | **Cw=** | 852 | in^6 | | |
| | | 12 | | | | | | |
| | | | | | | | | |
| **Elmn** | **z** | **θ** | **Torq** | **Bmnt** | **z** | **θ'** | **z** | **θ"** |
| 1 | | | | | | | | |
| | 0 | 0 | -39.96 | 813.7299 | 0 | 0.000E+00 | 0 | 3.293E-05 |
| | 0.6 | 0 | -39.63 | 789.9347 | 0.6 | 1.947E-05 | 0.6 | 3.197E-05 |
| | 1.2 | 0 | -39.29 | 766.4978 | 1.2 | 3.837E-05 | 1.2 | 3.102E-05 |
| | 1.8 | 0.0001 | -38.96 | 743.4146 | 1.8 | 5.670E-05 | 1.8 | 3.009E-05 |
| | 2.4 | 0.0001 | -38.63 | 720.6804 | 2.4 | 7.448E-05 | 2.4 | 2.917E-05 |
| | 3 | 0.0001 | -38.3 | 698.2907 | 3 | 9.170E-05 | 3 | 2.826E-05 |
| | 3.6 | 0.0002 | -37.96 | 676.2409 | 3.6 | 1.084E-04 | 3.6 | 2.737E-05 |
| | 4.2 | 0.0003 | -37.63 | 654.5267 | 4.2 | 1.246E-04 | 4.2 | 2.649E-05 |
| | 4.8 | 0.0004 | -37.3 | 633.1436 | 4.8 | 1.402E-04 | 4.8 | 2.563E-05 |
| | 5.4 | 0.0004 | -36.96 | 612.0874 | 5.4 | 1.553E-04 | 5.4 | 2.477E-05 |
| | 6 | 0.0005 | -36.63 | 591.354 | 6 | 1.699E-04 | 6 | 2.393E-05 |
| 2 | | | | | | | | |
| | 6 | 0.0005 | -36.63 | 591.354 | 6 | 1.699E-04 | 6 | 2.393E-05 |
| | 6.6 | 0.0006 | -36.3 | 570.939 | 6.6 | 1.840E-04 | 6.6 | 2.311E-05 |
| | 7.2 | 0.0008 | -35.96 | 550.8384 | 7.2 | 1.976E-04 | 7.2 | 2.229E-05 |
| | 7.8 | 0.0009 | -35.63 | 531.0482 | 7.8 | 2.108E-04 | 7.8 | 2.149E-05 |
| | 8.4 | 0.001 | -35.3 | 511.5643 | 8.4 | 2.234E-04 | 8.4 | 2.070E-05 |
| | 9 | 0.0011 | -34.97 | 492.383 | 9 | 2.356E-04 | 9 | 1.993E-05 |
| | 9.6 | 0.0013 | -34.63 | 473.5003 | 9.6 | 2.474E-04 | 9.6 | 1.916E-05 |
| | 10.2 | 0.0014 | -34.3 | 454.9124 | 10.2 | 2.586E-04 | 10.2 | 1.841E-05 |
| | 10.8 | 0.0016 | -33.97 | 436.6157 | 10.8 | 2.695E-04 | 10.8 | 1.767E-05 |
| | 11.4 | 0.0018 | -33.63 | 418.6064 | 11.4 | 2.798E-04 | 11.4 | 1.694E-05 |
| | 12 | 0.0019 | -33.3 | 400.8809 | 12 | 2.898E-04 | 12 | 1.622E-05 |
| 3 | | | | | | | | |
| | 12 | 0.0019 | -33.3 | 400.8809 | 12 | 2.898E-04 | 12 | 1.622E-05 |
| | 13.6 | 0.0024 | -32.41 | 354.9737 | 13.6 | 3.142E-04 | 13.6 | 1.437E-05 |
| | 15.2 | 0.0029 | -31.52 | 310.9942 | 15.2 | 3.358E-04 | 15.2 | 1.259E-05 |
| | 16.8 | 0.0035 | -30.64 | 268.8796 | 16.8 | 3.546E-04 | 16.8 | 1.088E-05 |
| | 18.4 | 0.0041 | -29.75 | 228.5698 | 18.4 | 3.707E-04 | 18.4 | 9.251E-06 |
| | 20 | 0.0047 | -28.86 | 190.0072 | 20 | 3.842E-04 | 20 | 7.690E-06 |
| | 21.6 | 0.0053 | -27.97 | 153.1369 | 21.6 | 3.953E-04 | 21.6 | 6.198E-06 |
| | 23.2 | 0.0059 | -27.08 | 117.9061 | 23.2 | 4.041E-04 | 23.2 | 4.772E-06 |
| | 24.8 | 0.0066 | -26.2 | 84.2646 | 24.8 | 4.106E-04 | 24.8 | 3.411E-06 |
| | 26.4 | 0.0073 | -25.31 | 52.1643 | 26.4 | 4.150E-04 | 26.4 | 2.111E-06 |

| | | | | | | | | |
|---|---|---|---|---|---|---|---|---|
| | 28 | 0.0079 | -24.42 | 21.5595 | 28 | 4.174E-04 | 28 | 8.720E-07 |
| 4 | | | | | | | | |
| | 28 | 0.0079 | -24.42 | 21.5595 | 28 | 4.174E-04 | 28 | 8.721E-07 |
| | 28.8 | 0.0083 | -23.98 | 6.983 | 28.8 | 4.176E-04 | 28.8 | 2.825E-07 |
| | 29.6 | 0.0086 | -23.53 | -7.5935 | 29.6 | 4.178E-04 | 29.6 | -3.072E-07 |
| | 31.2 | 0.0093 | -22.64 | -35.3365 | 31.2 | 4.164E-04 | 31.2 | -1.430E-06 |
| | 32.8 | 0.0099 | -21.76 | -61.7089 | 32.8 | 4.133E-04 | 32.8 | -2.497E-06 |
| | 34.4 | 0.0106 | -20.87 | -86.7484 | 34.4 | 4.085E-04 | 34.4 | -3.511E-06 |
| | 36 | 0.0112 | -19.98 | -110.491 | 36 | 4.021E-04 | 36 | -4.472E-06 |
| | 37.6 | 0.0119 | -19.09 | -132.97 | 37.6 | 3.942E-04 | 37.6 | -5.382E-06 |
| | 39.2 | 0.0125 | -18.2 | -154.218 | 39.2 | 3.849E-04 | 39.2 | -6.242E-06 |
| | 40.8 | 0.0131 | -17.32 | -174.265 | 40.8 | 3.743E-04 | 40.8 | -7.053E-06 |
| | 42.4 | 0.0137 | -16.43 | -193.14 | 42.4 | 3.624E-04 | 42.4 | -7.817E-06 |
| | 44 | 0.0143 | -15.54 | -210.87 | 44 | 3.493E-04 | 44 | -8.535E-06 |
| 5 | | | | | | | | |
| | 44 | 0.0143 | -15.54 | -210.87 | 44 | 3.493E-04 | 44 | -8.535E-06 |
| | 45.6 | 0.0148 | -14.65 | -227.48 | 45.6 | 3.351E-04 | 45.6 | -9.207E-06 |
| | 47.2 | 0.0153 | -13.76 | -242.993 | 47.2 | 3.198E-04 | 47.2 | -9.834E-06 |
| | 48.8 | 0.0158 | -12.88 | -257.433 | 48.8 | 3.036E-04 | 48.8 | -1.042E-05 |
| | 50.4 | 0.0163 | -11.99 | -270.818 | 50.4 | 2.865E-04 | 50.4 | -1.096E-05 |
| | 52 | 0.0167 | -11.1 | -283.17 | 52 | 2.686E-04 | 52 | -1.146E-05 |
| | 53.6 | 0.0172 | -10.21 | -294.505 | 53.6 | 2.499E-04 | 53.6 | -1.192E-05 |
| | 55.2 | 0.0175 | -9.324 | -304.839 | 55.2 | 2.304E-04 | 55.2 | -1.234E-05 |
| | 56.8 | 0.0179 | -8.436 | -314.188 | 56.8 | 2.104E-04 | 56.8 | -1.272E-05 |
| | 58.4 | 0.0182 | -7.548 | -322.564 | 58.4 | 1.898E-04 | 58.4 | -1.305E-05 |
| | 60 | 0.0185 | -6.66 | -329.979 | 60 | 1.686E-04 | 60 | -1.336E-05 |
| 6 | | | | | | | | |
| | 60 | 0.0185 | -6.66 | -329.979 | 60 | 1.686E-04 | 60 | -1.336E-05 |
| | 60.6 | 0.0186 | -6.327 | -332.514 | 60.6 | 1.606E-04 | 60.6 | -1.346E-05 |
| | 61.2 | 0.0187 | -5.994 | -334.917 | 61.2 | 1.525E-04 | 61.2 | -1.355E-05 |
| | 61.8 | 0.0188 | -5.661 | -337.186 | 61.8 | 1.443E-04 | 61.8 | -1.365E-05 |
| | 62.4 | 0.0189 | -5.328 | -339.324 | 62.4 | 1.361E-04 | 62.4 | -1.373E-05 |
| | 63 | 0.0189 | -4.995 | -341.33 | 63 | 1.279E-04 | 63 | -1.381E-05 |
| | 63.6 | 0.019 | -4.662 | -343.205 | 63.6 | 1.195E-04 | 63.6 | -1.389E-05 |
| | 64.2 | 0.0191 | -4.329 | -344.948 | 64.2 | 1.112E-04 | 64.2 | -1.396E-05 |
| | 64.8 | 0.0192 | -3.996 | -346.561 | 64.8 | 1.028E-04 | 64.8 | -1.403E-05 |
| | 65.4 | 0.0192 | -3.663 | -348.044 | 65.4 | 9.436E-05 | 65.4 | -1.409E-05 |
| | 66 | 0.0193 | -3.33 | -349.397 | 66 | 8.589E-05 | 66 | -1.414E-05 |
| 7 | | | | | | | | |
| | 66 | 0.0193 | -3.33 | -349.397 | 66 | 8.589E-05 | 66 | -1.414E-05 |
| | 66.6 | 0.0193 | -2.997 | -350.621 | 66.6 | 7.739E-05 | 66.6 | -1.419E-05 |

(continued) Torque, Bimoment, Twist Angle and Derivatives

| | | | | | | | |
|---|---|---|---|---|---|---|---|
| 67.2 | 0.0194 | -2.664 | -351.714 | 67.2 | 6.886E-05 | 67.2 | -1.423E-05 |
| 67.8 | 0.0194 | -2.331 | -352.679 | 67.8 | 6.031E-05 | 67.8 | -1.427E-05 |
| 68.4 | 0.0194 | -1.998 | -353.515 | 68.4 | 5.173E-05 | 68.4 | -1.431E-05 |
| 69 | 0.0195 | -1.665 | -354.222 | 69 | 4.314E-05 | 69 | -1.434E-05 |
| 69.6 | 0.0195 | -1.332 | -354.8 | 69.6 | 3.453E-05 | 69.6 | -1.436E-05 |
| 70.2 | 0.0195 | -0.999 | -355.249 | 70.2 | 2.591E-05 | 70.2 | -1.438E-05 |
| 70.8 | 0.0195 | -0.666 | -355.57 | 70.8 | 1.728E-05 | 70.8 | -1.439E-05 |
| 71.4 | 0.0195 | -0.333 | -355.763 | 71.4 | 8.640E-06 | 71.4 | -1.440E-05 |
| 72 | 0.0195 | 0 | -355.827 | 72 | 0.000E+00 | 72 | -1.440E-05 |

Table of Shear Stress due to Pure Torsion

| Pure Torsion Shear Stress G*t*θ' | | | | | | | | |
|---|---|---|---|---|---|---|---|---|
| | point | 0 | 1 | 2 | 3 | 2' | 1' | 0' |
| | t= | 0.625 | 0.625 | 0.625 | 0.450 | 0.625 | 0.625 | 0.625 |
| **θ'''** | **z** | | | | | | | |
| -1.617E-06 | 0 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 |
| -1.593E-06 | 0.6 | 0.136 | 0.136 | 0.136 | 0.098 | 0.136 | 0.136 | 0.136 |
| -1.569E-06 | 1.2 | 0.269 | 0.269 | 0.269 | 0.193 | 0.269 | 0.269 | 0.269 |
| -1.545E-06 | 1.8 | 0.397 | 0.397 | 0.397 | 0.286 | 0.397 | 0.397 | 0.397 |
| -1.522E-06 | 2.4 | 0.521 | 0.521 | 0.521 | 0.375 | 0.521 | 0.521 | 0.521 |
| -1.499E-06 | 3 | 0.642 | 0.642 | 0.642 | 0.462 | 0.642 | 0.642 | 0.642 |
| -1.476E-06 | 3.6 | 0.759 | 0.759 | 0.759 | 0.546 | 0.759 | 0.759 | 0.759 |
| -1.453E-06 | 4.2 | 0.872 | 0.872 | 0.872 | 0.628 | 0.872 | 0.872 | 0.872 |
| -1.431E-06 | 4.8 | 0.981 | 0.981 | 0.981 | 0.707 | 0.981 | 0.981 | 0.981 |
| -1.409E-06 | 5.4 | 1.087 | 1.087 | 1.087 | 0.783 | 1.087 | 1.087 | 1.087 |
| -1.388E-06 | 6 | 1.189 | 1.189 | 1.189 | 0.856 | 1.189 | 1.189 | 1.189 |
| | | | | | | | | |
| -1.388E-06 | 6 | 1.189 | 1.189 | 1.189 | 0.856 | 1.189 | 1.189 | 1.189 |
| -1.366E-06 | 6.6 | 1.288 | 1.288 | 1.288 | 0.927 | 1.288 | 1.288 | 1.288 |
| -1.345E-06 | 7.2 | 1.383 | 1.383 | 1.383 | 0.996 | 1.383 | 1.383 | 1.383 |
| -1.325E-06 | 7.8 | 1.476 | 1.476 | 1.476 | 1.062 | 1.476 | 1.476 | 1.476 |
| -1.304E-06 | 8.4 | 1.564 | 1.564 | 1.564 | 1.126 | 1.564 | 1.564 | 1.564 |
| -1.284E-06 | 9 | 1.649 | 1.649 | 1.649 | 1.187 | 1.649 | 1.649 | 1.649 |
| -1.264E-06 | 9.6 | 1.732 | 1.732 | 1.732 | 1.247 | 1.732 | 1.732 | 1.732 |
| -1.244E-06 | 10.2 | 1.810 | 1.810 | 1.810 | 1.303 | 1.810 | 1.810 | 1.810 |
| -1.224E-06 | 10.8 | 1.887 | 1.887 | 1.887 | 1.358 | 1.887 | 1.887 | 1.887 |
| -1.205E-06 | 11.4 | 1.959 | 1.959 | 1.959 | 1.410 | 1.959 | 1.959 | 1.959 |
| -1.186E-06 | 12 | 2.029 | 2.029 | 2.029 | 1.461 | 2.029 | 2.029 | 2.029 |
| | | | | | | | | |
| -1.185E-06 | 12 | 2.029 | 2.029 | 2.029 | 1.461 | 2.029 | 2.029 | 2.029 |
| -1.136E-06 | 13.6 | 2.199 | 2.199 | 2.199 | 1.584 | 2.199 | 2.199 | 2.199 |

(continued) Table of Shear Stress due to Pure Torsion

| | | | | | | | | |
|---|---|---|---|---|---|---|---|---|
| -1.089E-06 | 15.2 | 2.351 | 2.351 | 2.351 | 1.692 | 2.351 | 2.351 | 2.351 |
| -1.042E-06 | 16.8 | 2.482 | 2.482 | 2.482 | 1.787 | 2.482 | 2.482 | 2.482 |
| -9.974E-07 | 18.4 | 2.595 | 2.595 | 2.595 | 1.868 | 2.595 | 2.595 | 2.595 |
| -9.538E-07 | 20 | 2.689 | 2.689 | 2.689 | 1.936 | 2.689 | 2.689 | 2.689 |
| -9.116E-07 | 21.6 | 2.767 | 2.767 | 2.767 | 1.992 | 2.767 | 2.767 | 2.767 |
| -8.707E-07 | 23.2 | 2.829 | 2.829 | 2.829 | 2.037 | 2.829 | 2.829 | 2.829 |
| -8.312E-07 | 24.8 | 2.874 | 2.874 | 2.874 | 2.069 | 2.874 | 2.874 | 2.874 |
| -7.931E-07 | 26.4 | 2.905 | 2.905 | 2.905 | 2.092 | 2.905 | 2.905 | 2.905 |
| -7.563E-07 | 28 | 2.922 | 2.922 | 2.922 | 2.104 | 2.922 | 2.922 | 2.922 |
| | | | | | | | | |
| -7.551E-07 | 28 | 2.922 | 2.922 | 2.922 | 2.104 | 2.922 | 2.922 | 2.922 |
| -7.37E-07 | 28.8 | 2.923 | 2.923 | 2.923 | 2.105 | 2.923 | 2.923 | 2.923 |
| -7.193E-07 | 29.6 | 2.925 | 2.925 | 2.925 | 2.106 | 2.925 | 2.925 | 2.925 |
| -6.843E-07 | 31.2 | 2.915 | 2.915 | 2.915 | 2.099 | 2.915 | 2.915 | 2.915 |
| -6.502E-07 | 32.8 | 2.893 | 2.893 | 2.893 | 2.083 | 2.893 | 2.893 | 2.893 |
| -6.169E-07 | 34.4 | 2.860 | 2.860 | 2.860 | 2.059 | 2.860 | 2.860 | 2.860 |
| -5.845E-07 | 36 | 2.815 | 2.815 | 2.815 | 2.027 | 2.815 | 2.815 | 2.815 |
| -5.528E-07 | 37.6 | 2.759 | 2.759 | 2.759 | 1.987 | 2.759 | 2.759 | 2.759 |
| -5.221E-07 | 39.2 | 2.694 | 2.694 | 2.694 | 1.940 | 2.694 | 2.694 | 2.694 |
| -4.921E-07 | 40.8 | 2.620 | 2.620 | 2.620 | 1.886 | 2.620 | 2.620 | 2.620 |
| -4.630E-07 | 42.4 | 2.537 | 2.537 | 2.537 | 1.826 | 2.537 | 2.537 | 2.537 |
| -4.347E-07 | 44 | 2.445 | 2.445 | 2.445 | 1.760 | 2.445 | 2.445 | 2.445 |
| | | | | | | | | |
| -4.338E-07 | 44 | 2.445 | 2.445 | 2.445 | 1.760 | 2.445 | 2.445 | 2.445 |
| -4.061E-07 | 45.6 | 2.346 | 2.346 | 2.346 | 1.689 | 2.346 | 2.346 | 2.346 |
| -3.788E-07 | 47.2 | 2.239 | 2.239 | 2.239 | 1.612 | 2.239 | 2.239 | 2.239 |
| -3.519E-07 | 48.8 | 2.125 | 2.125 | 2.125 | 1.530 | 2.125 | 2.125 | 2.125 |
| -3.255E-07 | 50.4 | 2.006 | 2.006 | 2.006 | 1.444 | 2.006 | 2.006 | 2.006 |
| -2.995E-07 | 52 | 1.880 | 1.880 | 1.880 | 1.354 | 1.880 | 1.880 | 1.880 |
| -2.739E-07 | 53.6 | 1.749 | 1.749 | 1.749 | 1.259 | 1.749 | 1.749 | 1.749 |
| -2.488E-07 | 55.2 | 1.613 | 1.613 | 1.613 | 1.161 | 1.613 | 1.613 | 1.613 |
| -2.241E-07 | 56.8 | 1.473 | 1.473 | 1.473 | 1.060 | 1.473 | 1.473 | 1.473 |
| -1.998E-07 | 58.4 | 1.329 | 1.329 | 1.329 | 0.957 | 1.329 | 1.329 | 1.329 |
| -1.760E-07 | 60 | 1.180 | 1.180 | 1.180 | 0.850 | 1.180 | 1.180 | 1.180 |
| | | | | | | | | |
| -1.755E-07 | 60 | 1.180 | 1.180 | 1.180 | 0.850 | 1.180 | 1.180 | 1.180 |
| -1.665E-07 | 60.6 | 1.124 | 1.124 | 1.124 | 0.809 | 1.124 | 1.124 | 1.124 |
| -1.576E-07 | 61.2 | 1.068 | 1.068 | 1.068 | 0.769 | 1.068 | 1.068 | 1.068 |
| -1.486E-07 | 61.8 | 1.010 | 1.010 | 1.010 | 0.727 | 1.010 | 1.010 | 1.010 |
| -1.397E-07 | 62.4 | 0.953 | 0.953 | 0.953 | 0.686 | 0.953 | 0.953 | 0.953 |
| -1.309E-07 | 63 | 0.895 | 0.895 | 0.895 | 0.645 | 0.895 | 0.895 | 0.895 |
| -1.220E-07 | 63.6 | 0.837 | 0.837 | 0.837 | 0.602 | 0.837 | 0.837 | 0.837 |

(continued) Table of Shear Stress due to Pure Torsion

| | | | | | | | | |
|---|---|---|---|---|---|---|---|---|
| -1.132E-07 | 64.2 | 0.778 | 0.778 | 0.778 | 0.560 | 0.778 | 0.778 | 0.778 |
| -1.044E-07 | 64.8 | 0.720 | 0.720 | 0.720 | 0.518 | 0.720 | 0.720 | 0.720 |
| -9.565E-08 | 65.4 | 0.661 | 0.661 | 0.661 | 0.476 | 0.661 | 0.661 | 0.661 |
| -8.691E-08 | 66 | 0.601 | 0.601 | 0.601 | 0.433 | 0.601 | 0.601 | 0.601 |
| | | | | | | | | |
| -8.687E-08 | 66 | 0.601 | 0.601 | 0.601 | 0.433 | 0.601 | 0.601 | 0.601 |
| -7.814E-08 | 66.6 | 0.542 | 0.542 | 0.542 | 0.390 | 0.542 | 0.542 | 0.542 |
| -6.943E-08 | 67.2 | 0.482 | 0.482 | 0.482 | 0.347 | 0.482 | 0.482 | 0.482 |
| -6.072E-08 | 67.8 | 0.422 | 0.422 | 0.422 | 0.304 | 0.422 | 0.422 | 0.422 |
| -5.202E-08 | 68.4 | 0.362 | 0.362 | 0.362 | 0.261 | 0.362 | 0.362 | 0.362 |
| -4.333E-08 | 69 | 0.302 | 0.302 | 0.302 | 0.217 | 0.302 | 0.302 | 0.302 |
| -3.465E-08 | 69.6 | 0.242 | 0.242 | 0.242 | 0.174 | 0.242 | 0.242 | 0.242 |
| -2.598E-08 | 70.2 | 0.181 | 0.181 | 0.181 | 0.131 | 0.181 | 0.181 | 0.181 |
| -1.732E-08 | 70.8 | 0.121 | 0.121 | 0.121 | 0.087 | 0.121 | 0.121 | 0.121 |
| -8.666E-09 | 71.4 | 0.060 | 0.060 | 0.060 | 0.044 | 0.060 | 0.060 | 0.060 |
| -2.073E-11 | 72 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 |

Table of Warping Shear Stress Data

| Warping shear stress at "s", - E*Sw*θ'"/ t | | | | | | | |
|---|---|---|---|---|---|---|---|
| | 0 | 1 | 2 | 3 | 2' | 1' | 0' |
| Sw | 0.00 | -17.40 | -13.50 | 6.75 | -13.50 | -17.40 | 0.00 |
| **z** | | | | | | | |
| 0 | 0 | -1.306 | -1.013 | 0.703 | -1.013 | -1.306 | 0 |
| 0.6 | 0 | -1.286 | -0.998 | 0.693 | -0.998 | -1.286 | 0 |
| 1.2 | 0 | -1.267 | -0.983 | 0.683 | -0.983 | -1.267 | 0 |
| 1.8 | 0 | -1.247 | -0.968 | 0.672 | -0.968 | -1.247 | 0 |
| 2.4 | 0 | -1.229 | -0.953 | 0.662 | -0.953 | -1.229 | 0 |
| 3 | 0 | -1.210 | -0.939 | 0.652 | -0.939 | -1.210 | 0 |
| 3.6 | 0 | -1.192 | -0.925 | 0.642 | -0.925 | -1.192 | 0 |
| 4.2 | 0 | -1.173 | -0.910 | 0.632 | -0.910 | -1.173 | 0 |
| 4.8 | 0 | -1.155 | -0.896 | 0.622 | -0.896 | -1.155 | 0 |
| 5.4 | 0 | -1.138 | -0.883 | 0.613 | -0.883 | -1.138 | 0 |
| 6 | 0 | -1.121 | -0.869 | 0.604 | -0.869 | -1.121 | 0 |
| | | | | | | | |
| 6 | 0 | -1.121 | -0.869 | 0.604 | -0.869 | -1.121 | 0 |
| 6.6 | 0 | -1.103 | -0.856 | 0.594 | -0.856 | -1.103 | 0 |
| 7.2 | 0 | -1.086 | -0.843 | 0.585 | -0.843 | -1.086 | 0 |
| 7.8 | 0 | -1.070 | -0.830 | 0.576 | -0.830 | -1.070 | 0 |
| 8.4 | 0 | -1.053 | -0.817 | 0.567 | -0.817 | -1.053 | 0 |
| 9 | 0 | -1.037 | -0.804 | 0.559 | -0.804 | -1.037 | 0 |
| 9.6 | 0 | -1.021 | -0.792 | 0.550 | -0.792 | -1.021 | 0 |
| 10.2 | 0 | -1.004 | -0.779 | 0.541 | -0.779 | -1.004 | 0 |

(continued) Table of Warping Shear Stress Data

| | | | | | | | |
|---|---|---|---|---|---|---|---|
| 10.8 | 0 | -0.988 | -0.767 | 0.532 | -0.767 | -0.988 | 0 |
| 11.4 | 0 | -0.973 | -0.755 | 0.524 | -0.755 | -0.973 | 0 |
| 12 | 0 | -0.958 | -0.743 | 0.516 | -0.743 | -0.958 | 0 |
| 12 | 0 | -0.957 | -0.742 | 0.515 | -0.742 | -0.957 | 0 |
| 13.6 | 0 | -0.917 | -0.712 | 0.494 | -0.712 | -0.917 | 0 |
| 15.2 | 0 | -0.879 | -0.682 | 0.474 | -0.682 | -0.879 | 0 |
| 16.8 | 0 | -0.841 | -0.653 | 0.453 | -0.653 | -0.841 | 0 |
| 18.4 | 0 | -0.805 | -0.625 | 0.434 | -0.625 | -0.805 | 0 |
| 20 | 0 | -0.770 | -0.597 | 0.415 | -0.597 | -0.770 | 0 |
| 21.6 | 0 | -0.736 | -0.571 | 0.397 | -0.571 | -0.736 | 0 |
| 23.2 | 0 | -0.703 | -0.545 | 0.379 | -0.545 | -0.703 | 0 |
| 24.8 | 0 | -0.671 | -0.521 | 0.362 | -0.521 | -0.671 | 0 |
| 26.4 | 0 | -0.640 | -0.497 | 0.345 | -0.497 | -0.640 | 0 |
| 28 | 0 | -0.611 | -0.474 | 0.329 | -0.474 | -0.611 | 0 |
| | | | | | | | |
| 28 | 0 | -0.610 | -0.473 | 0.328 | -0.473 | -0.610 | 0 |
| 28.8 | 0 | -0.595 | -0.462 | 0.321 | -0.462 | -0.595 | 0 |
| 29.6 | 0 | -0.581 | -0.451 | 0.313 | -0.451 | -0.581 | 0 |
| 31.2 | 0 | -0.552 | -0.429 | 0.298 | -0.429 | -0.552 | 0 |
| 32.8 | 0 | -0.525 | -0.407 | 0.283 | -0.407 | -0.525 | 0 |
| 34.4 | 0 | -0.498 | -0.386 | 0.268 | -0.386 | -0.498 | 0 |
| 36 | 0 | -0.472 | -0.366 | 0.254 | -0.366 | -0.472 | 0 |
| 37.6 | 0 | -0.446 | -0.346 | 0.240 | -0.346 | -0.446 | 0 |
| 39.2 | 0 | -0.422 | -0.327 | 0.227 | -0.327 | -0.422 | 0 |
| 40.8 | 0 | -0.397 | -0.308 | 0.214 | -0.308 | -0.397 | 0 |
| 42.4 | 0 | -0.374 | -0.290 | 0.201 | -0.290 | -0.374 | 0 |
| 44 | 0 | -0.351 | -0.272 | 0.189 | -0.272 | -0.351 | 0 |
| | | | | | | | |
| 44 | 0 | -0.350 | -0.272 | 0.189 | -0.272 | -0.350 | 0 |
| 45.6 | 0 | -0.328 | -0.254 | 0.177 | -0.254 | -0.328 | 0 |
| 47.2 | 0 | -0.306 | -0.237 | 0.165 | -0.237 | -0.306 | 0 |
| 48.8 | 0 | -0.284 | -0.220 | 0.153 | -0.220 | -0.284 | 0 |
| 50.4 | 0 | -0.263 | -0.204 | 0.142 | -0.204 | -0.263 | 0 |
| 52 | 0 | -0.242 | -0.188 | 0.130 | -0.188 | -0.242 | 0 |
| 53.6 | 0 | -0.221 | -0.172 | 0.119 | -0.172 | -0.221 | 0 |
| 55.2 | 0 | -0.201 | -0.156 | 0.108 | -0.156 | -0.201 | 0 |
| 56.8 | 0 | -0.181 | -0.140 | 0.097 | -0.140 | -0.181 | 0 |
| 58.4 | 0 | -0.161 | -0.125 | 0.087 | -0.125 | -0.161 | 0 |
| 60 | 0 | -0.142 | -0.110 | 0.077 | -0.110 | -0.142 | 0 |
| | | | | | | | |
| 60 | 0 | -0.142 | -0.110 | 0.076 | -0.110 | -0.142 | 0 |
| 60.6 | 0 | -0.134 | -0.104 | 0.072 | -0.104 | -0.134 | 0 |

(continued) Table of Warping Shear Stress Data

| | | | | | | | |
|---|---|---|---|---|---|---|---|
| 61.2 | 0 | -0.127 | -0.099 | 0.069 | -0.099 | -0.127 | 0 |
| 61.8 | 0 | -0.120 | -0.093 | 0.065 | -0.093 | -0.120 | 0 |
| 62.4 | 0 | -0.113 | -0.088 | 0.061 | -0.088 | -0.113 | 0 |
| 63 | 0 | -0.106 | -0.082 | 0.057 | -0.082 | -0.106 | 0 |
| 63.6 | 0 | -0.098 | -0.076 | 0.053 | -0.076 | -0.098 | 0 |
| 64.2 | 0 | -0.091 | -0.071 | 0.049 | -0.071 | -0.091 | 0 |
| 64.8 | 0 | -0.084 | -0.065 | 0.045 | -0.065 | -0.084 | 0 |
| 65.4 | 0 | -0.077 | -0.060 | 0.042 | -0.060 | -0.077 | 0 |
| 66 | 0 | -0.070 | -0.054 | 0.038 | -0.054 | -0.070 | 0 |
| | | | | | | | |
| 66 | 0 | -0.070 | -0.054 | 0.038 | -0.054 | -0.070 | 0 |
| 66.6 | 0 | -0.063 | -0.049 | 0.034 | -0.049 | -0.063 | 0 |
| 67.2 | 0 | -0.056 | -0.043 | 0.030 | -0.043 | -0.056 | 0 |
| 67.8 | 0 | -0.049 | -0.038 | 0.026 | -0.038 | -0.049 | 0 |
| 68.4 | 0 | -0.042 | -0.033 | 0.023 | -0.033 | -0.042 | 0 |
| 69 | 0 | -0.035 | -0.027 | 0.019 | -0.027 | -0.035 | 0 |
| 69.6 | 0 | -0.028 | -0.022 | 0.015 | -0.022 | -0.028 | 0 |
| 70.2 | 0 | -0.021 | -0.016 | 0.011 | -0.016 | -0.021 | 0 |
| 70.8 | 0 | -0.014 | -0.011 | 0.008 | -0.011 | -0.014 | 0 |
| 71.4 | 0 | -0.007 | -0.005 | 0.004 | -0.005 | -0.007 | 0 |
| 72 | 0 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0 |

Table of Warping Normal Stresses

| Warping Normal Stresses, E*Wn/θ" | | | | | | | |
|---|---|---|---|---|---|---|---|
| | 0 | 1 | 2 | 3 | 2' | 1' | 0' |
| Wn | 22.00 | 0.00 | -10.4 | 0.00 | 10.40 | 0.00 | -22.00 |
| **z** | | | | | | | |
| 0 | 21.009 | 0 | -9.932 | 0 | 9.932 | 0 | -21.009 |
| 0.6 | 20.397 | 0 | -9.642 | 0 | 9.642 | 0 | -20.397 |
| 1.2 | 19.791 | 0 | -9.356 | 0 | 9.356 | 0 | -19.791 |
| 1.8 | 19.197 | 0 | -9.075 | 0 | 9.075 | 0 | -19.197 |
| 2.4 | 18.610 | 0 | -8.798 | 0 | 8.798 | 0 | -18.610 |
| 3 | 18.030 | 0 | -8.523 | 0 | 8.523 | 0 | -18.030 |
| 3.6 | 17.462 | 0 | -8.255 | 0 | 8.255 | 0 | -17.462 |
| 4.2 | 16.901 | 0 | -7.989 | 0 | 7.989 | 0 | -16.901 |
| 4.8 | 16.352 | 0 | -7.730 | 0 | 7.730 | 0 | -16.352 |
| 5.4 | 15.803 | 0 | -7.471 | 0 | 7.471 | 0 | -15.803 |
| 6 | 15.267 | 0 | -7.217 | 0 | 7.217 | 0 | -15.267 |
| | | | | | | | |
| 6 | 15.267 | 0 | -7.217 | 0 | 7.217 | 0 | -15.267 |
| 6.6 | 14.744 | 0 | -6.970 | 0 | 6.970 | 0 | -14.744 |
| 7.2 | 14.221 | 0 | -6.723 | 0 | 6.723 | 0 | -14.221 |

(continued)Table of Warping Normal Stresses

| | | | | | | | |
|---|---|---|---|---|---|---|---|
| 7.8 | 13.711 | 0 | -6.481 | 0 | 6.481 | 0 | -13.711 |
| 8.4 | 13.207 | 0 | -6.243 | 0 | 6.243 | 0 | -13.207 |
| 9 | 12.715 | 0 | -6.011 | 0 | 6.011 | 0 | -12.715 |
| 9.6 | 12.224 | 0 | -5.779 | 0 | 5.779 | 0 | -12.224 |
| 10.2 | 11.746 | 0 | -5.552 | 0 | 5.552 | 0 | -11.746 |
| 10.8 | 11.273 | 0 | -5.329 | 0 | 5.329 | 0 | -11.273 |
| 11.4 | 10.808 | 0 | -5.109 | 0 | 5.109 | 0 | -10.808 |
| 12 | 10.348 | 0 | -4.892 | 0 | 4.892 | 0 | -10.348 |
| | | | | | | | |
| 12 | 10.348 | 0 | -4.892 | 0 | 4.892 | 0 | -10.348 |
| 13.6 | 9.168 | 0 | -4.334 | 0 | 4.334 | 0 | -9.168 |
| 15.2 | 8.032 | 0 | -3.797 | 0 | 3.797 | 0 | -8.032 |
| 16.8 | 6.941 | 0 | -3.281 | 0 | 3.281 | 0 | -6.941 |
| 18.4 | 5.902 | 0 | -2.790 | 0 | 2.790 | 0 | -5.902 |
| 20 | 4.906 | 0 | -2.319 | 0 | 2.319 | 0 | -4.906 |
| 21.6 | 3.954 | 0 | -1.869 | 0 | 1.869 | 0 | -3.954 |
| 23.2 | 3.045 | 0 | -1.439 | 0 | 1.439 | 0 | -3.045 |
| 24.8 | 2.176 | 0 | -1.029 | 0 | 1.029 | 0 | -2.176 |
| 26.4 | 1.347 | 0 | -0.637 | 0 | 0.637 | 0 | -1.347 |
| 28 | 0.556 | 0 | -0.263 | 0 | 0.263 | 0 | -0.556 |
| | | | | | | | |
| 28 | 0.556 | 0 | -0.263 | 0 | 0.263 | 0 | -0.556 |
| 28.8 | 0.180 | 0 | -0.085 | 0 | 0.085 | 0 | -0.180 |
| 29.6 | -0.196 | 0 | 0.093 | 0 | -0.093 | 0 | 0.196 |
| 31.2 | -0.912 | 0 | 0.431 | 0 | -0.431 | 0 | 0.912 |
| 32.8 | -1.593 | 0 | 0.753 | 0 | -0.753 | 0 | 1.593 |
| 34.4 | -2.240 | 0 | 1.059 | 0 | -1.059 | 0 | 2.240 |
| 36 | -2.853 | 0 | 1.349 | 0 | -1.349 | 0 | 2.853 |
| 37.6 | -3.434 | 0 | 1.623 | 0 | -1.623 | 0 | 3.434 |
| 39.2 | -3.982 | 0 | 1.883 | 0 | -1.883 | 0 | 3.982 |
| 40.8 | -4.500 | 0 | 2.127 | 0 | -2.127 | 0 | 4.500 |
| 42.4 | -4.987 | 0 | 2.358 | 0 | -2.358 | 0 | 4.987 |
| 44 | -5.445 | 0 | 2.574 | 0 | -2.574 | 0 | 5.445 |
| | | | | | | | |
| 44 | -5.445 | 0 | 2.574 | 0 | -2.574 | 0 | 5.445 |
| 45.6 | -5.874 | 0 | 2.777 | 0 | -2.777 | 0 | 5.874 |
| 47.2 | -6.274 | 0 | 2.966 | 0 | -2.966 | 0 | 6.274 |
| 48.8 | -6.648 | 0 | 3.143 | 0 | -3.143 | 0 | 6.648 |
| 50.4 | -6.992 | 0 | 3.306 | 0 | -3.306 | 0 | 6.992 |
| 52 | -7.311 | 0 | 3.456 | 0 | -3.456 | 0 | 7.311 |
| 53.6 | -7.605 | 0 | 3.595 | 0 | -3.595 | 0 | 7.605 |
| 55.2 | -7.873 | 0 | 3.722 | 0 | -3.722 | 0 | 7.873 |

(continued)Table of Warping Normal Stresses

| | | | | | | | |
|---|---|---|---|---|---|---|---|
| 56.8 | -8.115 | 0 | 3.836 | 0 | -3.836 | 0 | 8.115 |
| 58.4 | -8.326 | 0 | 3.936 | 0 | -3.936 | 0 | 8.326 |
| 60 | -8.524 | 0 | 4.029 | 0 | -4.029 | 0 | 8.524 |
| 60 | -8.524 | 0 | 4.029 | 0 | -4.029 | 0 | 8.524 |
| 60.6 | -8.587 | 0 | 4.060 | 0 | -4.060 | 0 | 8.587 |
| 61.2 | -8.645 | 0 | 4.087 | 0 | -4.087 | 0 | 8.645 |
| 61.8 | -8.709 | 0 | 4.117 | 0 | -4.117 | 0 | 8.709 |
| 62.4 | -8.760 | 0 | 4.141 | 0 | -4.141 | 0 | 8.760 |
| 63 | -8.811 | 0 | 4.165 | 0 | -4.165 | 0 | 8.811 |
| 63.6 | -8.862 | 0 | 4.189 | 0 | -4.189 | 0 | 8.862 |
| 64.2 | -8.906 | 0 | 4.210 | 0 | -4.210 | 0 | 8.906 |
| 64.8 | -8.951 | 0 | 4.231 | 0 | -4.231 | 0 | 8.951 |
| 65.4 | -8.989 | 0 | 4.250 | 0 | -4.250 | 0 | 8.989 |
| 66 | -9.021 | 0 | 4.265 | 0 | -4.265 | 0 | 9.021 |
| | | | | | | | |
| 66 | -9.021 | 0 | 4.265 | 0 | -4.265 | 0 | 9.021 |
| 66.6 | -9.053 | 0 | 4.280 | 0 | -4.280 | 0 | 9.053 |
| 67.2 | -9.079 | 0 | 4.292 | 0 | -4.292 | 0 | 9.079 |
| 67.8 | -9.104 | 0 | 4.304 | 0 | -4.304 | 0 | 9.104 |
| 68.4 | -9.130 | 0 | 4.316 | 0 | -4.316 | 0 | 9.130 |
| 69 | -9.149 | 0 | 4.325 | 0 | -4.325 | 0 | 9.149 |
| 69.6 | -9.162 | 0 | 4.331 | 0 | -4.331 | 0 | 9.162 |
| 70.2 | -9.174 | 0 | 4.337 | 0 | -4.337 | 0 | 9.174 |
| 70.8 | -9.181 | 0 | 4.340 | 0 | -4.340 | 0 | 9.181 |
| 71.4 | -9.187 | 0 | 4.343 | 0 | -4.343 | 0 | 9.187 |
| 72 | -9.187 | 0 | 4.343 | 0 | -4.343 | 0 | 9.187 |

Table of Bending Normal Stresses

| Bending Normal Stresses, M/ Sx | | | | | | | | |
|---|---|---|---|---|---|---|---|---|
| | | 1 | 1 | 1 | 0 | -1 | -1 | -1 |
| Mus | **z** | | | | | | | |
| -518.40 | 0 | -8.42 | -8.42 | -8.42 | 0.00 | 8.42 | 8.42 | 8.42 |
| -505.49 | 0.6 | -8.21 | -8.21 | -8.21 | 0 | 8.21 | 8.21 | 8.21 |
| -492.70 | 1.2 | -8.00 | -8.00 | -8.00 | 0 | 8.00 | 8.00 | 8.00 |
| -480.01 | 1.8 | -7.79 | -7.79 | -7.79 | 0 | 7.79 | 7.79 | 7.79 |
| -467.42 | 2.4 | -7.59 | -7.59 | -7.59 | 0 | 7.59 | 7.59 | 7.59 |
| -454.95 | 3 | -7.39 | -7.39 | -7.39 | 0 | 7.39 | 7.39 | 7.39 |
| -442.58 | 3.6 | -7.18 | -7.18 | -7.18 | 0 | 7.18 | 7.18 | 7.18 |
| -430.33 | 4.2 | -6.99 | -6.99 | -6.99 | 0 | 6.99 | 6.99 | 6.99 |
| -418.18 | 4.8 | -6.79 | -6.79 | -6.79 | 0 | 6.79 | 6.79 | 6.79 |
| -406.13 | 5.4 | -6.59 | -6.59 | -6.59 | 0 | 6.59 | 6.59 | 6.59 |
| -394.20 | 6 | -6.40 | -6.40 | -6.40 | 0 | 6.40 | 6.40 | 6.40 |
| | | | | | | | | |
| -394.20 | 6 | -6.40 | -6.40 | -6.40 | 0 | 6.40 | 6.40 | 6.40 |
| -382.37 | 6.6 | -6.21 | -6.21 | -6.21 | 0 | 6.21 | 6.21 | 6.21 |
| -370.66 | 7.2 | -6.02 | -6.02 | -6.02 | 0 | 6.02 | 6.02 | 6.02 |
| -359.05 | 7.8 | -5.83 | -5.83 | -5.83 | 0 | 5.83 | 5.83 | 5.83 |
| -347.54 | 8.4 | -5.64 | -5.64 | -5.64 | 0 | 5.64 | 5.64 | 5.64 |
| -336.15 | 9 | -5.46 | -5.46 | -5.46 | 0 | 5.46 | 5.46 | 5.46 |
| -324.86 | 9.6 | -5.27 | -5.27 | -5.27 | 0 | 5.27 | 5.27 | 5.27 |
| -313.69 | 10.2 | -5.09 | -5.09 | -5.09 | 0 | 5.09 | 5.09 | 5.09 |
| -302.62 | 10.8 | -4.91 | -4.91 | -4.91 | 0 | 4.91 | 4.91 | 4.91 |
| -291.65 | 11.4 | -4.73 | -4.73 | -4.73 | 0 | 4.73 | 4.73 | 4.73 |
| -280.80 | 12 | -4.56 | -4.56 | -4.56 | 0 | 4.56 | 4.56 | 4.56 |
| | | | | | | | | |
| -280.80 | 12 | -4.56 | -4.56 | -4.56 | 0 | 4.56 | 4.56 | 4.56 |
| -252.38 | 13.6 | -4.10 | -4.10 | -4.10 | 0 | 4.10 | 4.10 | 4.10 |
| -224.74 | 15.2 | -3.65 | -3.65 | -3.65 | 0 | 3.65 | 3.65 | 3.65 |
| -197.86 | 16.8 | -3.21 | -3.21 | -3.21 | 0 | 3.21 | 3.21 | 3.21 |
| -171.74 | 18.4 | -2.79 | -2.79 | -2.79 | 0 | 2.79 | 2.79 | 2.79 |
| -146.40 | 20 | -2.38 | -2.38 | -2.38 | 0 | 2.38 | 2.38 | 2.38 |
| -121.82 | 21.6 | -1.98 | -1.98 | -1.98 | 0 | 1.98 | 1.98 | 1.98 |
| -98.02 | 23.2 | -1.59 | -1.59 | -1.59 | 0 | 1.59 | 1.59 | 1.59 |
| -74.98 | 24.8 | -1.22 | -1.22 | -1.22 | 0 | 1.22 | 1.22 | 1.22 |
| -52.70 | 26.4 | -0.86 | -0.86 | -0.86 | 0 | 0.86 | 0.86 | 0.86 |
| -31.20 | 28 | -0.51 | -0.51 | -0.51 | 0 | 0.51 | 0.51 | 0.51 |
| | | | | | | | | |
| -31.20 | 28 | -0.51 | -0.51 | -0.51 | 0 | 0.51 | 0.51 | 0.51 |
| -20.74 | 28.8 | -0.34 | -0.34 | -0.34 | 0 | 0.34 | 0.34 | 0.34 |
| -10.46 | 29.6 | -0.17 | -0.17 | -0.17 | 0 | 0.17 | 0.17 | 0.17 |

(continued) Table of Bending Normal Stresses

| | | | | | | | | |
|---|---|---|---|---|---|---|---|---|
| 9.50 | 31.2 | 0.15 | 0.15 | 0.15 | 0 | -0.15 | -0.15 | -0.15 |
| 28.70 | 32.8 | 0.47 | 0.47 | 0.47 | 0 | -0.47 | -0.47 | -0.47 |
| 47.14 | 34.4 | 0.77 | 0.77 | 0.77 | 0 | -0.77 | -0.77 | -0.77 |
| 64.80 | 36 | 1.05 | 1.05 | 1.05 | 0 | -1.05 | -1.05 | -1.05 |
| 81.70 | 37.6 | 1.33 | 1.33 | 1.33 | 0 | -1.33 | -1.33 | -1.33 |
| 97.82 | 39.2 | 1.59 | 1.59 | 1.59 | 0 | -1.59 | -1.59 | -1.59 |
| 113.18 | 40.8 | 1.84 | 1.84 | 1.84 | 0 | -1.84 | -1.84 | -1.84 |
| 127.78 | 42.4 | 2.07 | 2.07 | 2.07 | 0 | -2.07 | -2.07 | -2.07 |
| 141.60 | 44 | 2.30 | 2.30 | 2.30 | 0 | -2.30 | -2.30 | -2.30 |
| | | | | | | | | |
| 141.60 | 44 | 2.30 | 2.30 | 2.30 | 0 | -2.30 | -2.30 | -2.30 |
| 154.66 | 45.6 | 2.51 | 2.51 | 2.51 | 0 | -2.51 | -2.51 | -2.51 |
| 166.94 | 47.2 | 2.71 | 2.71 | 2.71 | 0 | -2.71 | -2.71 | -2.71 |
| 178.46 | 48.8 | 2.90 | 2.90 | 2.90 | 0 | -2.90 | -2.90 | -2.90 |
| 189.22 | 50.4 | 3.07 | 3.07 | 3.07 | 0 | -3.07 | -3.07 | -3.07 |
| 199.20 | 52 | 3.23 | 3.23 | 3.23 | 0 | -3.23 | -3.23 | -3.23 |
| 208.42 | 53.6 | 3.38 | 3.38 | 3.38 | 0 | -3.38 | -3.38 | -3.38 |
| 216.86 | 55.2 | 3.52 | 3.52 | 3.52 | 0 | -3.52 | -3.52 | -3.52 |
| 224.54 | 56.8 | 3.65 | 3.65 | 3.65 | 0 | -3.65 | -3.65 | -3.65 |
| 231.46 | 58.4 | 3.76 | 3.76 | 3.76 | 0 | -3.76 | -3.76 | -3.76 |
| 237.60 | 60 | 3.86 | 3.86 | 3.86 | 0 | -3.86 | -3.86 | -3.86 |
| | | | | | | | | |
| 237.60 | 60 | 3.86 | 3.86 | 3.86 | 0 | -3.86 | -3.86 | -3.86 |
| 239.71 | 60.6 | 3.89 | 3.89 | 3.89 | 0 | -3.89 | -3.89 | -3.89 |
| 241.70 | 61.2 | 3.92 | 3.92 | 3.92 | 0 | -3.92 | -3.92 | -3.92 |
| 243.59 | 61.8 | 3.95 | 3.95 | 3.95 | 0 | -3.95 | -3.95 | -3.95 |
| 245.38 | 62.4 | 3.98 | 3.98 | 3.98 | 0 | -3.98 | -3.98 | -3.98 |
| 247.05 | 63 | 4.01 | 4.01 | 4.01 | 0 | -4.01 | -4.01 | -4.01 |
| 248.62 | 63.6 | 4.04 | 4.04 | 4.04 | 0 | -4.04 | -4.04 | -4.04 |
| 250.07 | 64.2 | 4.06 | 4.06 | 4.06 | 0 | -4.06 | -4.06 | -4.06 |
| 251.42 | 64.8 | 4.08 | 4.08 | 4.08 | 0 | -4.08 | -4.08 | -4.08 |
| 252.67 | 65.4 | 4.10 | 4.10 | 4.10 | 0 | -4.10 | -4.10 | -4.10 |
| 253.80 | 66 | 4.12 | 4.12 | 4.12 | 0 | -4.12 | -4.12 | -4.12 |
| | | | | | | | | |
| 253.80 | 66 | 4.12 | 4.12 | 4.12 | 0 | -4.12 | -4.12 | -4.12 |
| 254.83 | 66.6 | 4.14 | 4.14 | 4.14 | 0 | -4.14 | -4.14 | -4.14 |
| 255.74 | 67.2 | 4.15 | 4.15 | 4.15 | 0 | -4.15 | -4.15 | -4.15 |
| 256.55 | 67.8 | 4.16 | 4.16 | 4.16 | 0 | -4.16 | -4.16 | -4.16 |
| 257.26 | 68.4 | 4.18 | 4.18 | 4.18 | 0 | -4.18 | -4.18 | -4.18 |
| 257.85 | 69 | 4.19 | 4.19 | 4.19 | 0 | -4.19 | -4.19 | -4.19 |
| 258.34 | 69.60 | 4.19 | 4.19 | 4.19 | 0 | -4.19 | -4.19 | -4.19 |
| 258.71 | 70.2 | 4.20 | 4.20 | 4.20 | 0 | -4.20 | -4.20 | -4.20 |

(continued) Table of Bending Normal Stresses

| | | | | | | | | |
|---|---|---|---|---|---|---|---|---|
| 258.98 | 70.8 | 4.20 | 4.20 | 4.20 | 0 | -4.20 | -4.20 | -4.20 |
| 259.15 | 71.4 | 4.21 | 4.21 | 4.21 | 0 | -4.21 | -4.21 | -4.21 |
| 259.20 | 72 | 4.21 | 4.21 | 4.21 | 0 | -4.21 | -4.21 | -4.21 |

Table of Bending Shear Stresses

| Vu=21.6 | | Bending Shear Stresses, V*Q/ I*t | | | | | | |
|---|---|---|---|---|---|---|---|---|
| | Ix | 554 | | | | | | |
| | t | 0.625 | 0.625 | 0.625 | 0.450 | 0.625 | 0.625 | 1 |
| | Q | 0 | 13.380 | 19.70 | 37.90 | 19.70 | 13.380 | 0.00 |
| Shear | z | | | | | | | |
| | | | | | | | | |
| 21.60 | 0.0 | 0 | 0.835 | 1.229 | 3.284 | 1.229 | 0.835 | 0 |
| 21.42 | 0.6 | 0 | 0.828 | 1.219 | 3.256 | 1.219 | 0.828 | 0 |
| 21.24 | 1.2 | 0 | 0.821 | 1.208 | 3.229 | 1.208 | 0.821 | 0 |
| 21.06 | 1.8 | 0 | 0.814 | 1.198 | 3.202 | 1.198 | 0.814 | 0 |
| 20.88 | 2.4 | 0 | 0.807 | 1.188 | 3.174 | 1.188 | 0.807 | 0 |
| 20.70 | 3.0 | 0 | 0.800 | 1.178 | 3.147 | 1.178 | 0.800 | 0 |
| 20.52 | 3.6 | 0 | 0.793 | 1.167 | 3.120 | 1.167 | 0.793 | 0 |
| 20.34 | 4.2 | 0 | 0.786 | 1.157 | 3.092 | 1.157 | 0.786 | 0 |
| 20.16 | 4.8 | 0 | 0.779 | 1.147 | 3.065 | 1.147 | 0.779 | 0 |
| 19.98 | 5.4 | 0 | 0.772 | 1.137 | 3.037 | 1.137 | 0.772 | 0 |
| 19.80 | 6.0 | 0 | 0.765 | 1.127 | 3.010 | 1.127 | 0.765 | 0 |
| | | | | | | | | |
| 19.80 | 6.0 | 0 | 0.765 | 1.127 | 3.010 | 1.127 | 0.765 | 0 |
| 19.62 | 6.6 | 0 | 0.758 | 1.116 | 2.983 | 1.116 | 0.758 | 0 |
| 19.44 | 7.2 | 0 | 0.751 | 1.106 | 2.955 | 1.106 | 0.751 | 0 |
| 19.26 | 7.8 | 0 | 0.744 | 1.096 | 2.928 | 1.096 | 0.744 | 0 |
| 19.08 | 8.4 | 0 | 0.737 | 1.086 | 2.901 | 1.086 | 0.737 | 0 |
| 18.90 | 9.0 | 0 | 0.730 | 1.075 | 2.873 | 1.075 | 0.730 | 0 |
| 18.72 | 9.6 | 0 | 0.723 | 1.065 | 2.846 | 1.065 | 0.723 | 0 |
| 18.54 | 10.2 | 0 | 0.716 | 1.055 | 2.819 | 1.055 | 0.716 | 0 |
| 18.36 | 10.8 | 0 | 0.709 | 1.045 | 2.791 | 1.045 | 0.709 | 0 |
| 18.18 | 11.4 | 0 | 0.703 | 1.034 | 2.764 | 1.034 | 0.703 | 0 |
| 18.00 | 12.0 | 0 | 0.696 | 1.024 | 2.736 | 1.024 | 0.696 | 0 |
| | | | | | | | | |
| 18.00 | 12.0 | 0 | 0.696 | 1.024 | 2.736 | 1.024 | 0.696 | 0 |
| 17.52 | 13.6 | 0 | 0.677 | 0.997 | 2.663 | 0.997 | 0.677 | 0 |
| 17.04 | 15.2 | 0 | 0.658 | 0.969 | 2.591 | 0.969 | 0.658 | 0 |
| 16.56 | 16.8 | 0 | 0.640 | 0.942 | 2.518 | 0.942 | 0.640 | 0 |
| 16.08 | 18.4 | 0 | 0.621 | 0.915 | 2.445 | 0.915 | 0.621 | 0 |
| 15.60 | 20.0 | 0 | 0.603 | 0.888 | 2.372 | 0.888 | 0.603 | 0 |
| 15.12 | 21.6 | 0 | 0.584 | 0.860 | 2.299 | 0.860 | 0.584 | 0 |

(continued) Table of Bending Shear Stresses

| | | | | | | | | |
|---|---|---|---|---|---|---|---|---|
| 14.64 | 23.2 | 0 | 0.566 | 0.833 | 2.226 | 0.833 | 0.566 | 0 |
| 14.16 | 24.8 | 0 | 0.547 | 0.806 | 2.153 | 0.806 | 0.547 | 0 |
| 13.68 | 26.4 | 0 | 0.529 | 0.778 | 2.080 | 0.778 | 0.529 | 0 |
| 13.20 | 28.0 | 0 | 0.510 | 0.751 | 2.007 | 0.751 | 0.510 | 0 |
| | | | | | | | | |
| 13.20 | 28.0 | 0 | 0.510 | 0.751 | 2.007 | 0.751 | 0.510 | 0 |
| 12.96 | 28.8 | 0 | 0.501 | 0.737 | 1.970 | 0.737 | 0.501 | 0 |
| 12.72 | 29.6 | 0 | 0.492 | 0.724 | 1.934 | 0.724 | 0.492 | 0 |
| 12.24 | 31.2 | 0 | 0.473 | 0.696 | 1.861 | 0.696 | 0.473 | 0 |
| 11.76 | 32.8 | 0 | 0.454 | 0.669 | 1.788 | 0.669 | 0.454 | 0 |
| 11.28 | 34.4 | 0 | 0.436 | 0.642 | 1.715 | 0.642 | 0.436 | 0 |
| 10.80 | 36.0 | 0 | 0.417 | 0.614 | 1.642 | 0.614 | 0.417 | 0 |
| 10.32 | 37.6 | 0 | 0.399 | 0.587 | 1.569 | 0.587 | 0.399 | 0 |
| 9.84 | 39.2 | 0 | 0.380 | 0.560 | 1.496 | 0.560 | 0.380 | 0 |
| 9.36 | 40.8 | 0 | 0.362 | 0.533 | 1.423 | 0.533 | 0.362 | 0 |
| 8.88 | 42.4 | 0 | 0.343 | 0.505 | 1.350 | 0.505 | 0.343 | 0 |
| 8.40 | 44.0 | 0 | 0.325 | 0.478 | 1.277 | 0.478 | 0.325 | 0 |
| | | | | | | | | |
| 8.40 | 44.0 | 0 | 0.325 | 0.478 | 1.277 | 0.478 | 0.325 | 0 |
| 7.92 | 45.6 | 0 | 0.306 | 0.451 | 1.204 | 0.451 | 0.306 | 0 |
| 7.44 | 47.2 | 0 | 0.288 | 0.423 | 1.131 | 0.423 | 0.288 | 0 |
| 6.96 | 48.8 | 0 | 0.269 | 0.396 | 1.058 | 0.396 | 0.269 | 0 |
| 6.48 | 50.4 | 0 | 0.250 | 0.369 | 0.985 | 0.369 | 0.250 | 0 |
| 6.00 | 52.0 | 0 | 0.232 | 0.341 | 0.912 | 0.341 | 0.232 | 0 |
| 5.52 | 53.6 | 0 | 0.213 | 0.314 | 0.839 | 0.314 | 0.213 | 0 |
| 5.04 | 55.2 | 0 | 0.195 | 0.287 | 0.766 | 0.287 | 0.195 | 0 |
| 4.56 | 56.8 | 0 | 0.176 | 0.259 | 0.693 | 0.259 | 0.176 | 0 |
| 4.08 | 58.4 | 0 | 0.158 | 0.232 | 0.620 | 0.232 | 0.158 | 0 |
| 3.60 | 60.0 | 0 | 0.139 | 0.205 | 0.547 | 0.205 | 0.139 | 0 |
| | | | | | | | | |
| 3.60 | 60.0 | 0 | 0.139 | 0.205 | 0.547 | 0.205 | 0.139 | 0 |
| 3.42 | 60.6 | 0 | 0.132 | 0.195 | 0.520 | 0.195 | 0.132 | 0 |
| 3.24 | 61.2 | 0 | 0.125 | 0.184 | 0.493 | 0.184 | 0.125 | 0 |
| 3.06 | 61.8 | 0 | 0.118 | 0.174 | 0.465 | 0.174 | 0.118 | 0 |
| 2.88 | 62.4 | 0 | 0.111 | 0.164 | 0.438 | 0.164 | 0.111 | 0 |
| 2.70 | 63.0 | 0 | 0.104 | 0.154 | 0.410 | 0.154 | 0.104 | 0 |
| 2.52 | 63.6 | 0 | 0.097 | 0.143 | 0.383 | 0.143 | 0.097 | 0 |
| 2.34 | 64.2 | 0 | 0.090 | 0.133 | 0.356 | 0.133 | 0.090 | 0 |
| 2.16 | 64.8 | 0 | 0.083 | 0.123 | 0.328 | 0.123 | 0.083 | 0 |
| 1.98 | 65.4 | 0 | 0.077 | 0.113 | 0.301 | 0.113 | 0.077 | 0 |
| 1.80 | 66.0 | 0 | 0.070 | 0.102 | 0.274 | 0.102 | 0.070 | 0 |
| | | | | | | | | |

(continued) Table of Bending Shear Stresses

| | | | | | | | | |
|---|---|---|---|---|---|---|---|---|
| 1.80 | 66.0 | 0 | 0.070 | 0.102 | 0.274 | 0.102 | 0.070 | 0 |
| 1.62 | 66.6 | 0 | 0.063 | 0.092 | 0.246 | 0.092 | 0.063 | 0 |
| 1.44 | 67.2 | 0 | 0.056 | 0.082 | 0.219 | 0.082 | 0.056 | 0 |
| 1.26 | 67.8 | 0 | 0.049 | 0.072 | 0.192 | 0.072 | 0.049 | 0 |
| 1.08 | 68.4 | 0 | 0.042 | 0.061 | 0.164 | 0.061 | 0.042 | 0 |
| 0.90 | 69.0 | 0 | 0.035 | 0.051 | 0.137 | 0.051 | 0.035 | 0 |
| 0.72 | 69.6 | 0 | 0.028 | 0.041 | 0.109 | 0.041 | 0.028 | 0 |
| 0.54 | 70.2 | 0 | 0.021 | 0.031 | 0.082 | 0.031 | 0.021 | 0 |
| 0.36 | 70.8 | 0 | 0.014 | 0.020 | 0.055 | 0.020 | 0.014 | 0 |
| 0.18 | 71.4 | 0 | 0.007 | 0.010 | 0.027 | 0.010 | 0.007 | 0 |
| 0.00 | 72.0 | 0 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0 |

Table of Total Normal Stresses

| | Combined Normal Stresses | | | | | | |
|---|---|---|---|---|---|---|---|
| | 0 | 1 | 2 | 3 | 2' | 1' | 0' |
| **z** | | | | | | | |
| 0 | 12.594 | -8.416 | -18.347 | 0.0 | 18.347 | 8.416 | -12.594 |
| 0.6 | 12.191 | -8.206 | -17.848 | 0 | 17.848 | 8.206 | -12.191 |
| 1.2 | 11.792 | -7.998 | -17.354 | 0 | 17.354 | 7.998 | -11.792 |
| 1.8 | 11.405 | -7.792 | -16.867 | 0 | 16.867 | 7.792 | -11.405 |
| 2.4 | 11.022 | -7.588 | -16.386 | 0 | 16.386 | 7.588 | -11.022 |
| 3 | 10.644 | -7.386 | -15.909 | 0 | 15.909 | 7.386 | -10.644 |
| 3.6 | 10.277 | -7.185 | -15.440 | 0 | 15.440 | 7.185 | -10.277 |
| 4.2 | 9.915 | -6.986 | -14.975 | 0 | 14.975 | 6.986 | -9.915 |
| 4.8 | 9.563 | -6.789 | -14.519 | 0 | 14.519 | 6.789 | -9.563 |
| 5.4 | 9.210 | -6.593 | -14.064 | 0 | 14.064 | 6.593 | -9.210 |
| 6 | 8.868 | -6.399 | -13.617 | 0 | 13.617 | 6.399 | -8.868 |
| | 0.000 | 0.000 | 0.000 | 0 | 0.000 | 0.000 | 0.000 |
| 6 | 8.868 | -6.399 | -13.617 | 0 | 13.617 | 6.399 | -8.868 |
| 6.6 | 8.537 | -6.207 | -13.177 | 0 | 13.177 | 6.207 | -8.537 |
| 7.2 | 8.204 | -6.017 | -12.740 | 0 | 12.740 | 6.017 | -8.204 |
| 7.8 | 7.882 | -5.829 | -12.310 | 0 | 12.310 | 5.829 | -7.882 |
| 8.4 | 7.565 | -5.642 | -11.885 | 0 | 11.885 | 5.642 | -7.565 |
| 9 | 7.258 | -5.457 | -11.468 | 0 | 11.468 | 5.457 | -7.258 |
| 9.6 | 6.950 | -5.274 | -11.052 | 0 | 11.052 | 5.274 | -6.950 |
| 10.2 | 6.653 | -5.092 | -10.645 | 0 | 10.645 | 5.092 | -6.653 |
| 10.8 | 6.361 | -4.913 | -10.242 | 0 | 10.242 | 4.913 | -6.361 |
| 11.4 | 6.073 | -4.735 | -9.844 | 0 | 9.844 | 4.735 | -6.073 |
| 12 | 5.790 | -4.558 | -9.450 | 0 | 9.450 | 4.558 | -5.790 |
| | 0.000 | 0.000 | 0.000 | 0 | 0.000 | 0.000 | 0.000 |
| 12 | 5.790 | -4.558 | -9.450 | 0 | 9.450 | 4.558 | -5.790 |
| 13.6 | 5.071 | -4.097 | -8.431 | 0 | 8.431 | 4.097 | -5.071 |

(continued) Table of Total Normal Stresses

| | | | | | | | |
|---|---|---|---|---|---|---|---|
| 15.2 | 4.384 | -3.648 | -7.445 | 0 | 7.445 | 3.648 | -4.384 |
| 16.8 | 3.729 | -3.212 | -6.493 | 0 | 6.493 | 3.212 | -3.729 |
| 18.4 | 3.114 | -2.788 | -5.578 | 0 | 5.578 | 2.788 | -3.114 |
| 20 | 2.530 | -2.377 | -4.696 | 0 | 4.696 | 2.377 | -2.530 |
| 21.6 | 1.977 | -1.978 | -3.847 | 0 | 3.847 | 1.978 | -1.977 |
| 23.2 | 1.453 | -1.591 | -3.030 | 0 | 3.030 | 1.591 | -1.453 |
| 24.8 | 0.959 | -1.217 | -2.246 | 0 | 2.246 | 1.217 | -0.959 |
| 26.4 | 0.491 | -0.856 | -1.492 | 0 | 1.492 | 0.856 | -0.491 |
| 28 | 0.050 | -0.506 | -0.769 | 0 | 0.769 | 0.506 | -0.050 |
| | 0.000 | 0.000 | 0.000 | 0 | 0.000 | 0.000 | 0.000 |
| 28 | 0.050 | -0.506 | -0.770 | 0 | 0.770 | 0.506 | -0.050 |
| 28.8 | -0.156 | -0.337 | -0.422 | 0 | 0.422 | 0.337 | 0.156 |
| 29.6 | -0.366 | -0.170 | -0.077 | 0 | 0.077 | 0.170 | 0.366 |
| 31.2 | -0.758 | 0.154 | 0.586 | 0 | -0.586 | -0.154 | 0.758 |
| 32.8 | -1.127 | 0.466 | 1.219 | 0 | -1.219 | -0.466 | 1.127 |
| 34.4 | -1.475 | 0.765 | 1.824 | 0 | -1.824 | -0.765 | 1.475 |
| 36 | -1.801 | 1.052 | 2.401 | 0 | -2.401 | -1.052 | 1.801 |
| 37.6 | -2.107 | 1.326 | 2.949 | 0 | -2.949 | -1.326 | 2.107 |
| 39.2 | -2.394 | 1.588 | 3.471 | 0 | -3.471 | -1.588 | 2.394 |
| 40.8 | -2.662 | 1.837 | 3.965 | 0 | -3.965 | -1.837 | 2.662 |
| 42.4 | -2.913 | 2.074 | 4.432 | 0 | -4.432 | -2.074 | 2.913 |
| 44 | -3.147 | 2.299 | 4.873 | 0 | -4.873 | -2.299 | 3.147 |
| | 0.000 | 0.000 | 0.000 | 0 | 0.000 | 0.000 | 0.000 |
| 44 | -3.147 | 2.299 | 4.873 | 0 | -4.873 | -2.299 | 3.147 |
| 45.6 | -3.363 | 2.511 | 5.287 | 0 | -5.287 | -2.511 | 3.363 |
| 47.2 | -3.564 | 2.710 | 5.676 | 0 | -5.676 | -2.710 | 3.564 |
| 48.8 | -3.751 | 2.897 | 6.040 | 0 | -6.040 | -2.897 | 3.751 |
| 50.4 | -3.921 | 3.072 | 6.377 | 0 | -6.377 | -3.072 | 3.921 |
| 52 | -4.078 | 3.234 | 6.690 | 0 | -6.690 | -3.234 | 4.078 |
| 53.6 | -4.222 | 3.383 | 6.978 | 0 | -6.978 | -3.383 | 4.222 |
| 55.2 | -4.352 | 3.521 | 7.242 | 0 | -7.242 | -3.521 | 4.352 |
| 56.8 | -4.470 | 3.645 | 7.482 | 0 | -7.482 | -3.645 | 4.470 |
| 58.4 | -4.568 | 3.757 | 7.693 | 0 | -7.693 | -3.757 | 4.568 |
| 60 | -4.667 | 3.857 | 7.887 | 0 | -7.887 | -3.857 | 4.667 |
| | 0.000 | 0.000 | 0.000 | 0 | 0.000 | 0.000 | 0.000 |
| 60 | -4.667 | 3.857 | 7.887 | 0 | -7.887 | -3.857 | 4.667 |
| 60.6 | -4.696 | 3.891 | 7.951 | 0 | -7.951 | -3.891 | 4.696 |
| 61.2 | -4.721 | 3.924 | 8.010 | 0 | -8.010 | -3.924 | 4.721 |
| 61.8 | -4.754 | 3.954 | 8.071 | 0 | -8.071 | -3.954 | 4.754 |
| 62.4 | -4.776 | 3.983 | 8.124 | 0 | -8.124 | -3.983 | 4.776 |
| 63 | -4.800 | 4.011 | 8.176 | 0 | -8.176 | -4.011 | 4.800 |
| 63.6 | -4.826 | 4.036 | 8.225 | 0 | -8.225 | -4.036 | 4.826 |

Table of Total Normal Stresses

| | | | | | | | |
|---|---|---|---|---|---|---|---|
| 64.2 | -4.847 | 4.060 | 8.270 | 0 | -8.270 | -4.060 | 4.847 |
| 64.8 | -4.870 | 4.082 | 8.313 | 0 | -8.313 | -4.082 | 4.870 |
| 65.4 | -4.888 | 4.102 | 8.351 | 0 | -8.351 | -4.102 | 4.888 |
| 66 | -4.901 | 4.120 | 8.385 | 0 | -8.385 | -4.120 | 4.901 |
| | 0.000 | 0.000 | 0.000 | 0 | 0.000 | 0.000 | 0.000 |
| 66 | -4.901 | 4.120 | 8.385 | 0 | -8.385 | -4.120 | 4.901 |
| 66.6 | -4.916 | 4.137 | 8.416 | 0 | -8.416 | -4.137 | 4.916 |
| 67.2 | -4.927 | 4.152 | 8.443 | 0 | -8.443 | -4.152 | 4.927 |
| 67.8 | -4.939 | 4.165 | 8.469 | 0 | -8.469 | -4.165 | 4.939 |
| 68.4 | -4.954 | 4.176 | 8.492 | 0 | -8.492 | -4.176 | 4.954 |
| 69 | -4.963 | 4.186 | 8.511 | 0 | -8.511 | -4.186 | 4.963 |
| 69.6 | -4.968 | 4.194 | 8.525 | 0 | -8.525 | -4.194 | 4.968 |
| 70.2 | -4.975 | 4.200 | 8.537 | 0 | -8.537 | -4.200 | 4.975 |
| 70.8 | -4.977 | 4.204 | 8.544 | 0 | -8.544 | -4.204 | 4.977 |
| 71.4 | -4.980 | 4.207 | 8.550 | 0 | -8.550 | -4.207 | 4.980 |
| 72 | -4.979 | 4.208 | 8.551 | 0 | -8.551 | -4.208 | 4.979 |

Table of Combined Shear St. Vt. τ positive

| S= | 0 | 1 | 2 | 3 | 2' | 1' | 0' |
|---|---|---|---|---|---|---|---|
| z | | | | | | | |
| 0 | 0.000 | -0.471 | 0.216 | 3.987 | 0.216 | -0.471 | 0.000 |
| 0.6 | 0.136 | -0.322 | 0.357 | 4.047 | 0.357 | -0.322 | 0.136 |
| 1.2 | 0.269 | -0.177 | 0.494 | 4.105 | 0.494 | -0.177 | 0.269 |
| 1.8 | 0.397 | -0.037 | 0.627 | 4.160 | 0.627 | -0.037 | 0.397 |
| 2.4 | 0.521 | 0.099 | 0.756 | 4.212 | 0.756 | 0.099 | 0.521 |
| 3 | 0.642 | 0.232 | 0.881 | 4.261 | 0.881 | 0.232 | 0.642 |
| 3.6 | 0.759 | 0.360 | 1.002 | 4.308 | 1.002 | 0.360 | 0.759 |
| 4.2 | 0.872 | 0.485 | 1.119 | 4.352 | 1.119 | 0.485 | 0.872 |
| 4.8 | 0.981 | 0.605 | 1.232 | 4.394 | 1.232 | 0.605 | 0.981 |
| 5.4 | 1.087 | 0.722 | 1.341 | 4.433 | 1.341 | 0.722 | 1.087 |
| 6 | 1.189 | 0.834 | 1.446 | 4.470 | 1.446 | 0.834 | 1.189 |
| | | | | | | | |
| 6 | 1.189 | 0.834 | 1.446 | 4.470 | 1.446 | 0.834 | 1.189 |
| 6.6 | 1.288 | 0.943 | 1.549 | 4.504 | 1.549 | 0.943 | 1.288 |
| 7.2 | 1.383 | 1.049 | 1.647 | 4.536 | 1.647 | 1.049 | 1.383 |
| 7.8 | 1.476 | 1.150 | 1.741 | 4.567 | 1.741 | 1.150 | 1.476 |
| 8.4 | 1.564 | 1.248 | 1.833 | 4.594 | 1.833 | 1.248 | 1.564 |
| 9 | 1.649 | 1.343 | 1.920 | 4.619 | 1.920 | 1.343 | 1.649 |
| 9.6 | 1.732 | 1.435 | 2.005 | 4.643 | 2.005 | 1.435 | 1.732 |
| 10.2 | 1.810 | 1.522 | 2.086 | 4.663 | 2.086 | 1.522 | 1.810 |
| 10.8 | 1.887 | 1.608 | 2.164 | 4.682 | 2.164 | 1.608 | 1.887 |
| 11.4 | 1.959 | 1.688 | 2.238 | 4.698 | 2.238 | 1.688 | 1.959 |

(continued) Table of Combined Shear St. Vt. τ positive

| | | | | | | | |
|---|---|---|---|---|---|---|---|
| 12 | 2.029 | 1.767 | 2.310 | 4.713 | 2.310 | 1.767 | 2.029 |
| | | | | | | | |
| 12 | 2.029 | 1.767 | 2.310 | 4.713 | 2.310 | 1.767 | 2.029 |
| 13.6 | 2.199 | 1.959 | 2.485 | 4.741 | 2.485 | 1.959 | 2.199 |
| 15.2 | 2.351 | 2.130 | 2.638 | 4.757 | 2.638 | 2.130 | 2.351 |
| 16.8 | 2.482 | 2.281 | 2.772 | 4.758 | 2.772 | 2.281 | 2.482 |
| 18.4 | 2.595 | 2.411 | 2.885 | 4.747 | 2.885 | 2.411 | 2.595 |
| 20 | 2.689 | 2.522 | 2.980 | 4.723 | 2.980 | 2.522 | 2.689 |
| 21.6 | 2.767 | 2.615 | 3.056 | 4.687 | 3.056 | 2.615 | 2.767 |
| 23.2 | 2.829 | 2.691 | 3.116 | 4.641 | 3.116 | 2.691 | 2.829 |
| 24.8 | 2.874 | 2.750 | 3.159 | 4.584 | 3.159 | 2.750 | 2.874 |
| 26.4 | 2.905 | 2.793 | 3.187 | 4.516 | 3.187 | 2.793 | 2.905 |
| 28 | 2.922 | 2.821 | 3.199 | 4.439 | 3.199 | 2.821 | 2.922 |
| | | | | | | | |
| 28 | 2.922 | 2.822 | 3.200 | 4.439 | 3.200 | 2.822 | 2.922 |
| 28.8 | 2.923 | 2.829 | 3.199 | 4.396 | 3.199 | 2.829 | 2.923 |
| 29.6 | 2.925 | 2.835 | 3.198 | 4.352 | 3.198 | 2.835 | 2.925 |
| 31.2 | 2.915 | 2.835 | 3.183 | 4.257 | 3.183 | 2.835 | 2.915 |
| 32.8 | 2.893 | 2.823 | 3.155 | 4.154 | 3.155 | 2.823 | 2.893 |
| 34.4 | 2.860 | 2.797 | 3.115 | 4.042 | 3.115 | 2.797 | 2.860 |
| 36 | 2.815 | 2.760 | 3.063 | 3.923 | 3.063 | 2.760 | 2.815 |
| 37.6 | 2.759 | 2.712 | 3.000 | 3.796 | 3.000 | 2.712 | 2.759 |
| 39.2 | 2.694 | 2.653 | 2.927 | 3.663 | 2.927 | 2.653 | 2.694 |
| 40.8 | 2.620 | 2.584 | 2.844 | 3.523 | 2.844 | 2.584 | 2.620 |
| 42.4 | 2.537 | 2.506 | 2.752 | 3.378 | 2.752 | 2.506 | 2.537 |
| 44 | 2.445 | 2.419 | 2.651 | 3.227 | 2.651 | 2.419 | 2.445 |
| | | | | | | | |
| 44 | 2.445 | 2.419 | 2.651 | 3.226 | 2.651 | 2.419 | 2.445 |
| 45.6 | 2.346 | 2.324 | 2.542 | 3.070 | 2.542 | 2.324 | 2.346 |
| 47.2 | 2.239 | 2.220 | 2.425 | 2.908 | 2.425 | 2.220 | 2.239 |
| 48.8 | 2.125 | 2.110 | 2.301 | 2.741 | 2.301 | 2.110 | 2.125 |
| 50.4 | 2.006 | 1.993 | 2.170 | 2.571 | 2.170 | 1.993 | 2.006 |
| 52 | 1.880 | 1.870 | 2.034 | 2.396 | 2.034 | 1.870 | 1.880 |
| 53.6 | 1.749 | 1.741 | 1.892 | 2.218 | 1.892 | 1.741 | 1.749 |
| 55.2 | 1.613 | 1.607 | 1.744 | 2.036 | 1.744 | 1.607 | 1.613 |
| 56.8 | 1.473 | 1.468 | 1.592 | 1.851 | 1.592 | 1.468 | 1.473 |
| 58.4 | 1.329 | 1.325 | 1.436 | 1.664 | 1.436 | 1.325 | 1.329 |
| 60 | 1.180 | 1.177 | 1.275 | 1.474 | 1.275 | 1.177 | 1.180 |
| | | | | | | | |
| 60 | 1.180 | 1.178 | 1.275 | 1.473 | 1.275 | 1.178 | 1.180 |
| 60.6 | 1.124 | 1.122 | 1.214 | 1.402 | 1.214 | 1.122 | 1.124 |
| 61.2 | 1.068 | 1.065 | 1.153 | 1.330 | 1.153 | 1.065 | 1.068 |

(continued) Table of Combined Shear St. Vt. τ positive

| | | | | | | | |
|---|---|---|---|---|---|---|---|
| 61.8 | 1.010 | 1.008 | 1.091 | 1.257 | 1.091 | 1.008 | 1.010 |
| 62.4 | 0.953 | 0.951 | 1.029 | 1.185 | 1.029 | 0.951 | 0.953 |
| 63 | 0.895 | 0.894 | 0.967 | 1.112 | 0.967 | 0.894 | 0.895 |
| 63.6 | 0.837 | 0.835 | 0.903 | 1.038 | 0.903 | 0.835 | 0.837 |
| 64.2 | 0.778 | 0.777 | 0.841 | 0.965 | 0.841 | 0.777 | 0.778 |
| 64.8 | 0.720 | 0.719 | 0.777 | 0.892 | 0.777 | 0.719 | 0.720 |
| 65.4 | 0.661 | 0.660 | 0.713 | 0.818 | 0.713 | 0.660 | 0.661 |
| 66 | 0.601 | 0.601 | 0.649 | 0.744 | 0.649 | 0.601 | 0.601 |
| 66.6 | 0.542 | 0.541 | 0.585 | 0.670 | 0.585 | 0.541 | 0.542 |
| 67.2 | 0.482 | 0.482 | 0.520 | 0.596 | 0.520 | 0.482 | 0.482 |
| 67.8 | 0.422 | 0.422 | 0.456 | 0.522 | 0.456 | 0.422 | 0.422 |
| 68.4 | 0.362 | 0.362 | 0.391 | 0.448 | 0.391 | 0.362 | 0.362 |
| 69 | 0.302 | 0.302 | 0.326 | 0.373 | 0.326 | 0.302 | 0.302 |
| 69.6 | 0.242 | 0.242 | 0.261 | 0.299 | 0.261 | 0.242 | 0.242 |
| 70.2 | 0.181 | 0.181 | 0.196 | 0.224 | 0.196 | 0.181 | 0.181 |
| 70.8 | 0.121 | 0.121 | 0.131 | 0.149 | 0.131 | 0.121 | 0.121 |
| 71.4 | 0.060 | 0.060 | 0.065 | 0.075 | 0.065 | 0.060 | 0.060 |
| 72 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 |

Table of Total Shear Stress with Negative St. Vt.

| S= | 0 | 1 | 2 | 3 | 2' | 1' | 0' |
|---|---|---|---|---|---|---|---|
| z | | | | | | | |
| 0 | 0.000 | -0.471 | 0.216 | 3.987 | 0.216 | -0.471 | 0.000 |
| 0.6 | -0.136 | -0.595 | 0.085 | 3.851 | 0.085 | -0.595 | -0.136 |
| 1.2 | -0.269 | -0.715 | -0.043 | 3.718 | -0.043 | -0.715 | -0.269 |
| 1.8 | -0.397 | -0.830 | -0.166 | 3.588 | -0.166 | -0.830 | -0.397 |
| 2.4 | -0.521 | -0.943 | -0.287 | 3.461 | -0.287 | -0.943 | -0.521 |
| 3 | -0.642 | -1.052 | -0.403 | 3.337 | -0.403 | -1.052 | -0.642 |
| 3.6 | -0.759 | -1.158 | -0.516 | 3.215 | -0.516 | -1.158 | -0.759 |
| 4.2 | -0.872 | -1.259 | -0.625 | 3.096 | -0.625 | -1.259 | -0.872 |
| 4.8 | -0.981 | -1.358 | -0.731 | 2.981 | -0.731 | -1.358 | -0.981 |
| 5.4 | -1.087 | -1.453 | -0.833 | 2.868 | -0.833 | -1.453 | -1.087 |
| 6 | -1.189 | -1.545 | -0.932 | 2.758 | -0.932 | -1.545 | -1.189 |
| | | | | | | | |
| 6 | -1.189 | -1.545 | -0.932 | 2.758 | -0.932 | -1.545 | -1.189 |
| 6.6 | -1.288 | -1.633 | -1.027 | 2.650 | -1.027 | -1.633 | -1.288 |
| 7.2 | -1.383 | -1.718 | -1.120 | 2.545 | -1.120 | -1.718 | -1.383 |
| 7.8 | -1.476 | -1.801 | -1.210 | 2.442 | -1.210 | -1.801 | -1.476 |
| 8.4 | -1.564 | -1.879 | -1.295 | 2.342 | -1.295 | -1.879 | -1.564 |
| 9 | -1.649 | -1.956 | -1.378 | 2.244 | -1.378 | -1.956 | -1.649 |
| 9.6 | -1.732 | -2.029 | -1.458 | 2.149 | -1.458 | -2.029 | -1.732 |
| 10.2 | -1.810 | -2.098 | -1.535 | 2.056 | -1.535 | -2.098 | -1.810 |

(continued) Table of Total Shear Stress with Negative St. Vt.

| | | | | | | | |
|---|---|---|---|---|---|---|---|
| 10.8 | -1.887 | -2.165 | -1.609 | 1.965 | -1.609 | -2.165 | -1.887 |
| 11.4 | -1.959 | -2.229 | -1.679 | 1.878 | -1.679 | -2.229 | -1.959 |
| 12 | -2.029 | -2.291 | -1.747 | 1.792 | -1.747 | -2.291 | -2.029 |
| | | | | | | | |
| 12 | -2.029 | -2.290 | -1.747 | 1.791 | -1.747 | -2.290 | -2.029 |
| 13.6 | -2.199 | -2.440 | -1.914 | 1.574 | -1.914 | -2.440 | -2.199 |
| 15.2 | -2.351 | -2.571 | -2.063 | 1.372 | -2.063 | -2.571 | -2.351 |
| 16.8 | -2.482 | -2.684 | -2.193 | 1.184 | -2.193 | -2.684 | -2.482 |
| 18.4 | -2.595 | -2.779 | -2.305 | 1.010 | -2.305 | -2.779 | -2.595 |
| 20 | -2.689 | -2.857 | -2.399 | 0.850 | -2.399 | -2.857 | -2.689 |
| 21.6 | -2.767 | -2.919 | -2.478 | 0.703 | -2.478 | -2.919 | -2.767 |
| 23.2 | -2.829 | -2.966 | -2.541 | 0.568 | -2.541 | -2.966 | -2.829 |
| 24.8 | -2.874 | -2.998 | -2.589 | 0.445 | -2.589 | -2.998 | -2.874 |
| 26.4 | -2.905 | -3.017 | -2.623 | 0.333 | -2.623 | -3.017 | -2.905 |
| 28 | -2.922 | -3.022 | -2.645 | 0.232 | -2.645 | -3.022 | -2.922 |
| | | | | | | | |
| 28 | -2.922 | -3.021 | -2.644 | 0.232 | -2.644 | -3.021 | -2.922 |
| 28.8 | -2.923 | -3.018 | -2.648 | 0.186 | -2.648 | -3.018 | -2.923 |
| 29.6 | -2.925 | -3.014 | -2.651 | 0.141 | -2.651 | -3.014 | -2.925 |
| 31.2 | -2.915 | -2.994 | -2.647 | 0.060 | -2.647 | -2.994 | -2.915 |
| 32.8 | -2.893 | -2.964 | -2.631 | -0.012 | -2.631 | -2.964 | -2.893 |
| 34.4 | -2.860 | -2.922 | -2.604 | -0.076 | -2.604 | -2.922 | -2.860 |
| 36 | -2.815 | -2.869 | -2.566 | -0.130 | -2.566 | -2.869 | -2.815 |
| 37.6 | -2.759 | -2.807 | -2.519 | -0.177 | -2.519 | -2.807 | -2.759 |
| 39.2 | -2.694 | -2.736 | -2.461 | -0.217 | -2.461 | -2.736 | -2.694 |
| 40.8 | -2.620 | -2.656 | -2.396 | -0.249 | -2.396 | -2.656 | -2.620 |
| 42.4 | -2.537 | -2.567 | -2.322 | -0.275 | -2.322 | -2.567 | -2.537 |
| 44 | -2.445 | -2.471 | -2.239 | -0.294 | -2.239 | -2.471 | -2.445 |
| 44 | -2.445 | -2.471 | -2.239 | -0.295 | -2.239 | -2.471 | -2.445 |
| 45.6 | -2.346 | -2.368 | -2.149 | -0.308 | -2.149 | -2.368 | -2.346 |
| 47.2 | -2.239 | -2.257 | -2.053 | -0.316 | -2.053 | -2.257 | -2.239 |
| 48.8 | -2.125 | -2.140 | -1.950 | -0.319 | -1.950 | -2.140 | -2.125 |
| 50.4 | -2.006 | -2.018 | -1.841 | -0.317 | -1.841 | -2.018 | -2.006 |
| 52 | -1.880 | -1.890 | -1.726 | -0.311 | -1.726 | -1.890 | -1.880 |
| 53.6 | -1.749 | -1.757 | -1.607 | -0.301 | -1.607 | -1.757 | -1.749 |
| 55.2 | -1.613 | -1.619 | -1.482 | -0.287 | -1.482 | -1.619 | -1.613 |
| 56.8 | -1.473 | -1.478 | -1.354 | -0.270 | -1.354 | -1.478 | -1.473 |
| 58.4 | -1.329 | -1.332 | -1.222 | -0.249 | -1.222 | -1.332 | -1.329 |
| 60 | -1.180 | -1.183 | -1.086 | -0.226 | -1.086 | -1.183 | -1.180 |
| 60 | -1.180 | -1.183 | -1.085 | -0.226 | -1.085 | -1.183 | -1.180 |
| 60.6 | -1.124 | -1.126 | -1.034 | -0.217 | -1.034 | -1.126 | -1.124 |
| 61.2 | -1.068 | -1.070 | -0.982 | -0.207 | -0.982 | -1.070 | -1.068 |

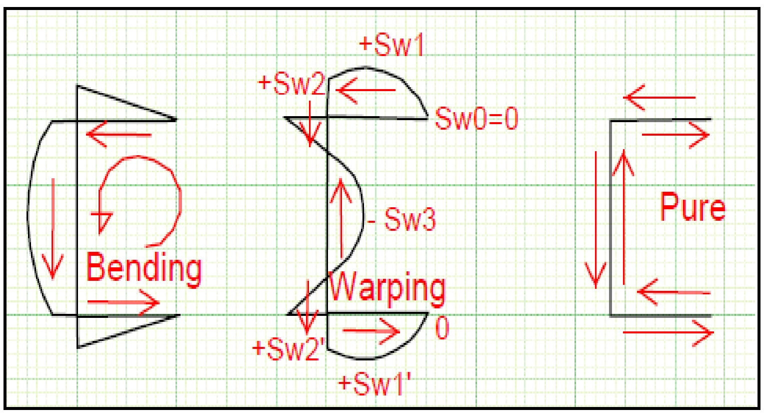
(continued) Table of Total Shear Stress with Negative St. Vt.

| | | | | | | | |
|---|---|---|---|---|---|---|---|
| 61.8 | -1.010 | -1.012 | -0.929 | -0.197 | -0.929 | -1.012 | -1.010 |
| 62.4 | -0.953 | -0.954 | -0.876 | -0.187 | -0.876 | -0.954 | -0.953 |
| 63 | -0.895 | -0.897 | -0.824 | -0.177 | -0.824 | -0.897 | -0.895 |
| 63.6 | -0.837 | -0.838 | -0.770 | -0.166 | -0.770 | -0.838 | -0.837 |
| 64.2 | -0.778 | -0.779 | -0.716 | -0.155 | -0.716 | -0.779 | -0.778 |
| 64.8 | -0.720 | -0.720 | -0.662 | -0.144 | -0.662 | -0.720 | -0.720 |
| 65.4 | -0.661 | -0.661 | -0.608 | -0.133 | -0.608 | -0.661 | -0.661 |
| 66 | -0.601 | -0.602 | -0.553 | -0.121 | -0.553 | -0.602 | -0.601 |
| 66 | -0.601 | -0.602 | -0.553 | -0.121 | -0.553 | -0.602 | -0.601 |
| 66.6 | -0.542 | -0.542 | -0.499 | -0.110 | -0.499 | -0.542 | -0.542 |
| 67.2 | -0.482 | -0.482 | -0.444 | -0.098 | -0.444 | -0.482 | -0.482 |
| 67.8 | -0.422 | -0.423 | -0.389 | -0.086 | -0.389 | -0.423 | -0.422 |
| 68.4 | -0.362 | -0.362 | -0.333 | -0.074 | -0.333 | -0.362 | -0.362 |
| 69 | -0.302 | -0.302 | -0.278 | -0.062 | -0.278 | -0.302 | -0.302 |
| 69.6 | -0.242 | -0.242 | -0.222 | -0.049 | -0.222 | -0.242 | -0.242 |
| 70.2 | -0.181 | -0.181 | -0.167 | -0.037 | -0.167 | -0.181 | -0.181 |
| 70.8 | -0.121 | -0.121 | -0.111 | -0.025 | -0.111 | -0.121 | -0.121 |
| 71.4 | -0.060 | -0.061 | -0.056 | -0.012 | -0.056 | -0.061 | -0.060 |
| 72 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 |

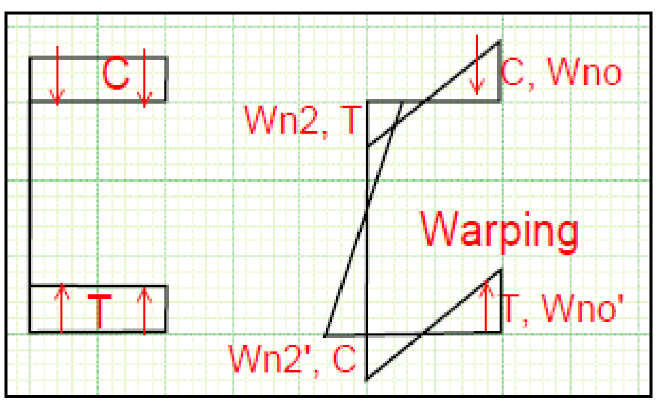
## APPENDIX G

## BOOTHBY'S TORSION PROBLEM

In the EBC of case study four, this material corresponds to the input model, input data, input forms, figures of output data with checks in notepad version, and excel processed output data and charts. Charts are presented containing both partial and combined tresses along interest points of the beam and cross section profile. Again, the asymptotic behavior of the thin-walled beam elastic line is successfully shown in the charts, which evidences the efficiency of the high order finite element used by BMTORSWP. Positive shear and axial stresses are assumed similar to those occurring at the cross section flanges and web when the beam undergoes bending as shown below.

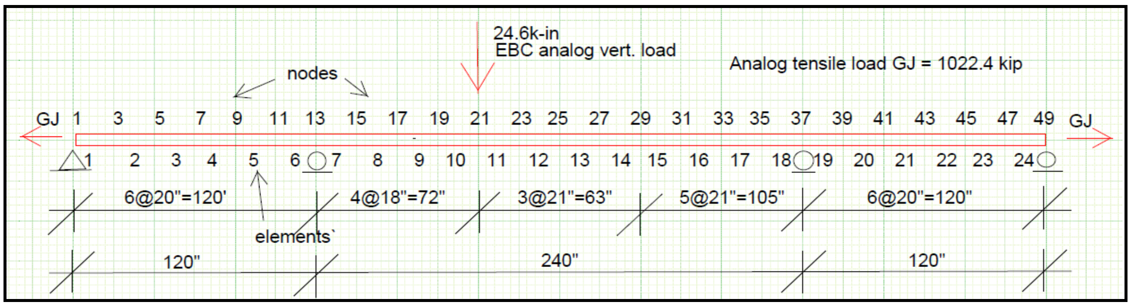


Convention for Shear Contributions

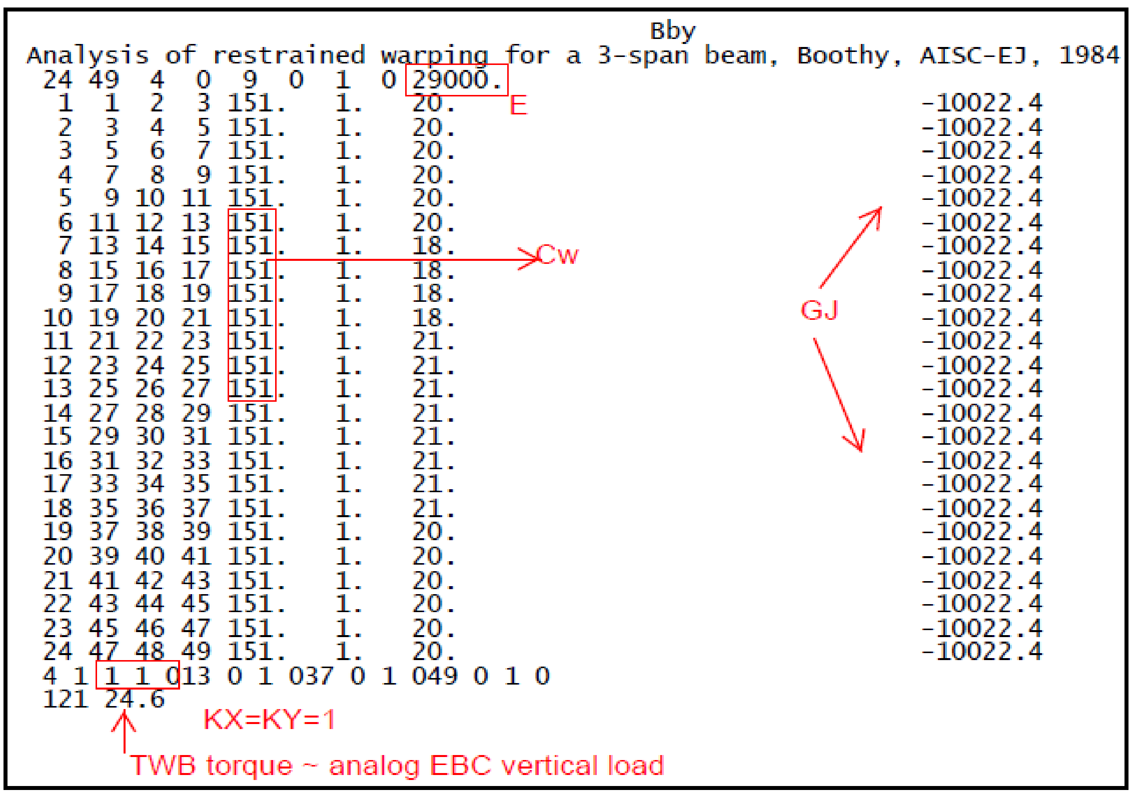


Convention for Normal Stresses Contributions

The analog EBC model used in the input form is shown together with the corresponding input notepad and input forms. The input form used for BMTORSWP was edited to be used in BMCOLDG without resorting to input forms for the bending problem. The purpose was to obtain output data for the bending solution in the exact cross section points where the warping analysis provided data. Afterwards, stresses were superposed and combined. Final results are shown in the charts in this appendix.



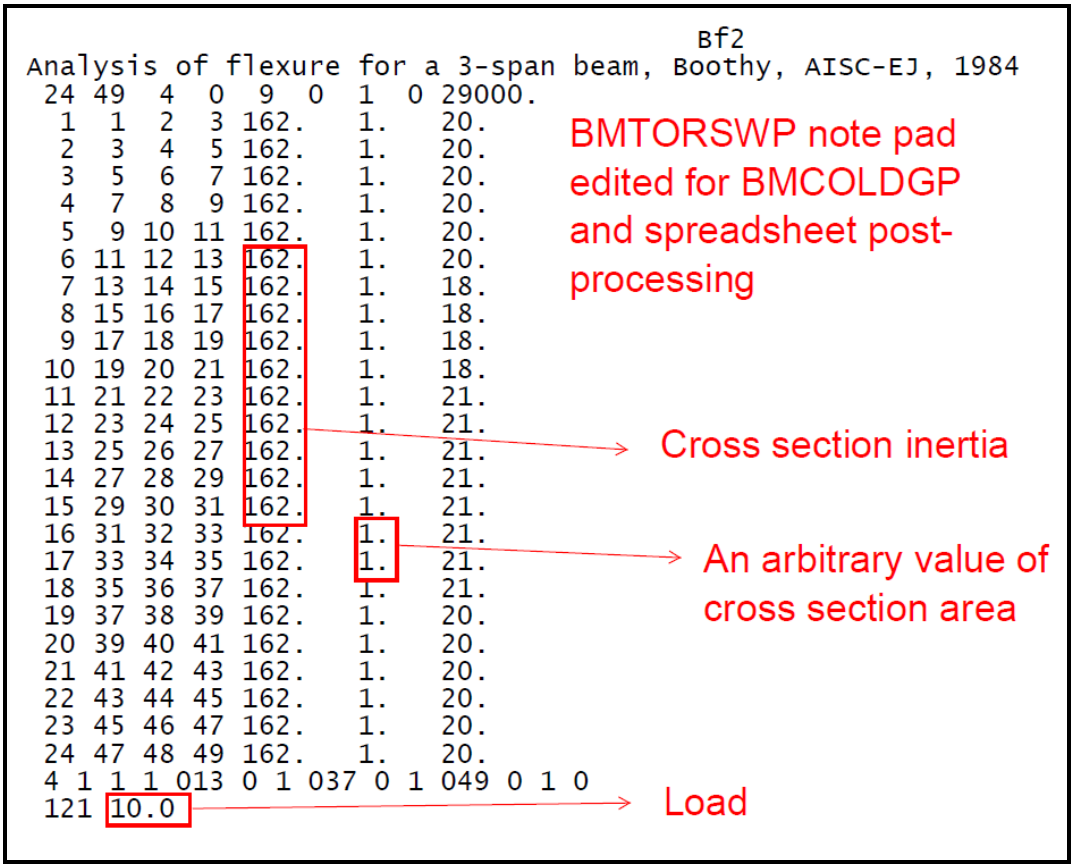

EBC Model for BMTORSWP (Boothby, 1984)



```
                                                Bby
Analysis of restrained warping for a 3-span beam, Boothy, AISC-EJ, 1984
 24 49  4  0  9  0  1  0 29000.
  1  1  2  3 151.   1.   20.                          -10022.4
  2  3  4  5 151.   1.   20.                          -10022.4
  3  5  6  7 151.   1.   20.                          -10022.4
  4  7  8  9 151.   1.   20.                          -10022.4
  5  9 10 11 151.   1.   20.                          -10022.4
  6 11 12 13 151.   1.   20.                          -10022.4
  7 13 14 15 151.   1.   18.                          -10022.4
  8 15 16 17 151.   1.   18.                          -10022.4
  9 17 18 19 151.   1.   18.                          -10022.4
 10 19 20 21 151.   1.   18.                          -10022.4
 11 21 22 23 151.   1.   21.                          -10022.4
 12 23 24 25 151.   1.   21.                          -10022.4
 13 25 26 27 151.   1.   21.                          -10022.4
 14 27 28 29 151.   1.   21.                          -10022.4
 15 29 30 31 151.   1.   21.                          -10022.4
 16 31 32 33 151.   1.   21.                          -10022.4
 17 33 34 35 151.   1.   21.                          -10022.4
 18 35 36 37 151.   1.   21.                          -10022.4
 19 37 38 39 151.   1.   20.                          -10022.4
 20 39 40 41 151.   1.   20.                          -10022.4
 21 41 42 43 151.   1.   20.                          -10022.4
 22 43 44 45 151.   1.   20.                          -10022.4
 23 45 46 47 151.   1.   20.                          -10022.4
 24 47 48 49 151.   1.   20.                          -10022.4
 4 1 1 1 013 0 1 037 0 1 049 0 1 0
 121 24.6
```



Input Notepad of the Torsional Problem for BMTORSWP



```
                                                  Bf2
Analysis of flexure for a 3-span beam, Boothy, AISC-EJ, 1984
 24 49  4  0  9  0  1  0 29000.
  1  1  2  3 162.   1.   20.
  2  3  4  5 162.   1.   20.
  3  5  6  7 162.   1.   20.
  4  7  8  9 162.   1.   20.
  5  9 10 11 162.   1.   20.
  6 11 12 13 162.   1.   20.
  7 13 14 15 162.   1.   18.
  8 15 16 17 162.   1.   18.
  9 17 18 19 162.   1.   18.
 10 19 20 21 162.   1.   18.
 11 21 22 23 162.   1.   21.
 12 23 24 25 162.   1.   21.
 13 25 26 27 162.   1.   21.
 14 27 28 29 162.   1.   21.
 15 29 30 31 162.   1.   21.
 16 31 32 33 162.   1.   21.
 17 33 34 35 162.   1.   21.
 18 35 36 37 162.   1.   21.
 19 37 38 39 162.   1.   20.
 20 39 40 41 162.   1.   20.
 21 41 42 43 162.   1.   20.
 22 43 44 45 162.   1.   20.
 23 45 46 47 162.   1.   20.
 24 47 48 49 162.   1.   20.
 4 1 1 1 013 0 1 037 0 1 049 0 1 0
 121 10.0
```



Input Notepad of the Bending Problem for BMCOLDGP

Perfect coincidence was found in the bimoment diagrams. However the chart of torques by Boothby contains 2 mistakes. At left side of first interior support and right side of last interior support warping torques of 9.2 and 3.5) respectively were not detected by Boothby with corresponding errors of 700 and 300 per cent. See Chart Total and Warping Torque from BMTORSWP and by Boothby (1984.)

It was found that stresses calculations shown by Boothby correspond to the right side of the first interior support. That is why they are similar to calculations made from BMTORSWP output. Nevertheless, another mistake by Boothby was noticed regarding the sectorial coordinate at the flange-web corner. He used Wn2=6.75 corresponding to a C15x50 shape instead of 5.02. See Chart Total and Warping Torque from BMTORSWP and Boothby.

# Program BMTORSWP Input Form

1 2 3 4 5 6 7 8 9 10 11 12 13 14 15 16 17 18 19 20 21 22 23 24 25 26 27 28 29 30 31 32 33 34 35 36 37 38 39 40 41

ALPHAMERIC DESCRIPTION OF THE JOB — FILE NAME:

Analysis of restrained warping for a 3-span beam, Boothy, AISC-EJ, 1984

| NEL | NOD | NSUP | NSPD | JBW | NFX | NFY | NFZ | ELASTICITY (E11.4) |
|---|---|---|---|---|---|---|---|---|
| 24 | 49 | 4 | 0 | 9 | 0 | 1 | 0 | 29000. |

| NE | NI | NC | NJ | Cw or Inert. | AREA | LENGTH | WTANG | SOIL NI | SOI |
|---|---|---|---|---|---|---|---|---|---|
| 1 | 1 | 2 | 3 | 151. | 1. | 20. | | | |
| 2 | 3 | 4 | 5 | 151. | 1. | SAME | | | |
| 3 | 5 | 6 | 7 | 151. | 1. | SAME | | | |
| 4 | 7 | 8 | 9 | 151. | 1. | SAME | | | |
| 5 | 9 | 10 | 11 | 151. | 1. | SAME | | | |
| 6 | 11 | 12 | 13 | 151. | 1. | SAME | | | |
| 7 | 13 | 14 | 15 | 151. | 1. | 18. | | | |
| 8 | 15 | 16 | 17 | 151. | 1. | 18. | | | |
| 9 | 17 | 18 | 19 | 151. | 1. | 18. | | | |
| 10 | 19 | 20 | 21 | 151. | 1. | 18. | | | |
| 11 | 21 | 22 | 23 | 151. | 1. | 21. | | | |
| 12 | 23 | 24 | 25 | 151. | 1. | 21. | | | |
| 13 | 25 | 26 | 27 | 151. | 1. | 21. | | | |

Any value to avoid singularity

| NF | N | KX | KY | KZ | N | KX | KY | KZ | N | KX | KY | KZ | N | KX | KY | KZ | N | KX | KY | K |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| 4 | 1 | 1 | 1 | 0 | 13 | 0 | 1 | 0 | 37 | 0 | 1 | 0 | 49 | 0 | 1 | 0 | | | | |

| NF | N | KD | SPRING | N | KD | SPRING | N | KD |
|---|---|---|---|---|---|---|---|---|
| | | | | | | | | |

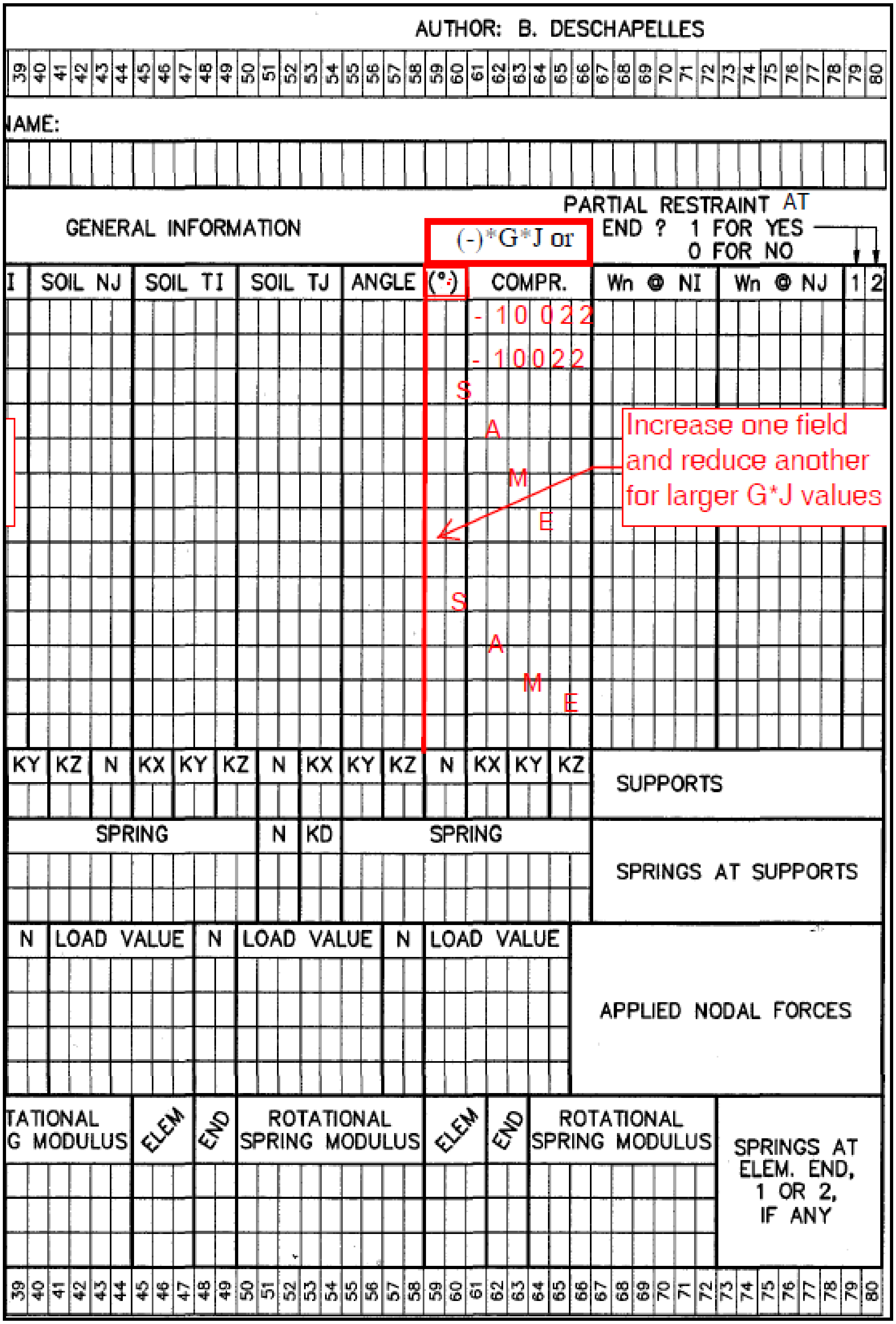
| NF | N | LOAD VALUE | N | LOAD VALUE | N | LOAD VALUE | N | LOAD VALUE | N | L |
|---|---|---|---|---|---|---|---|---|---|---|
| 1 | 21 | 24.6 | | | | | | | | |

keep going in the note pad until reaching element 24

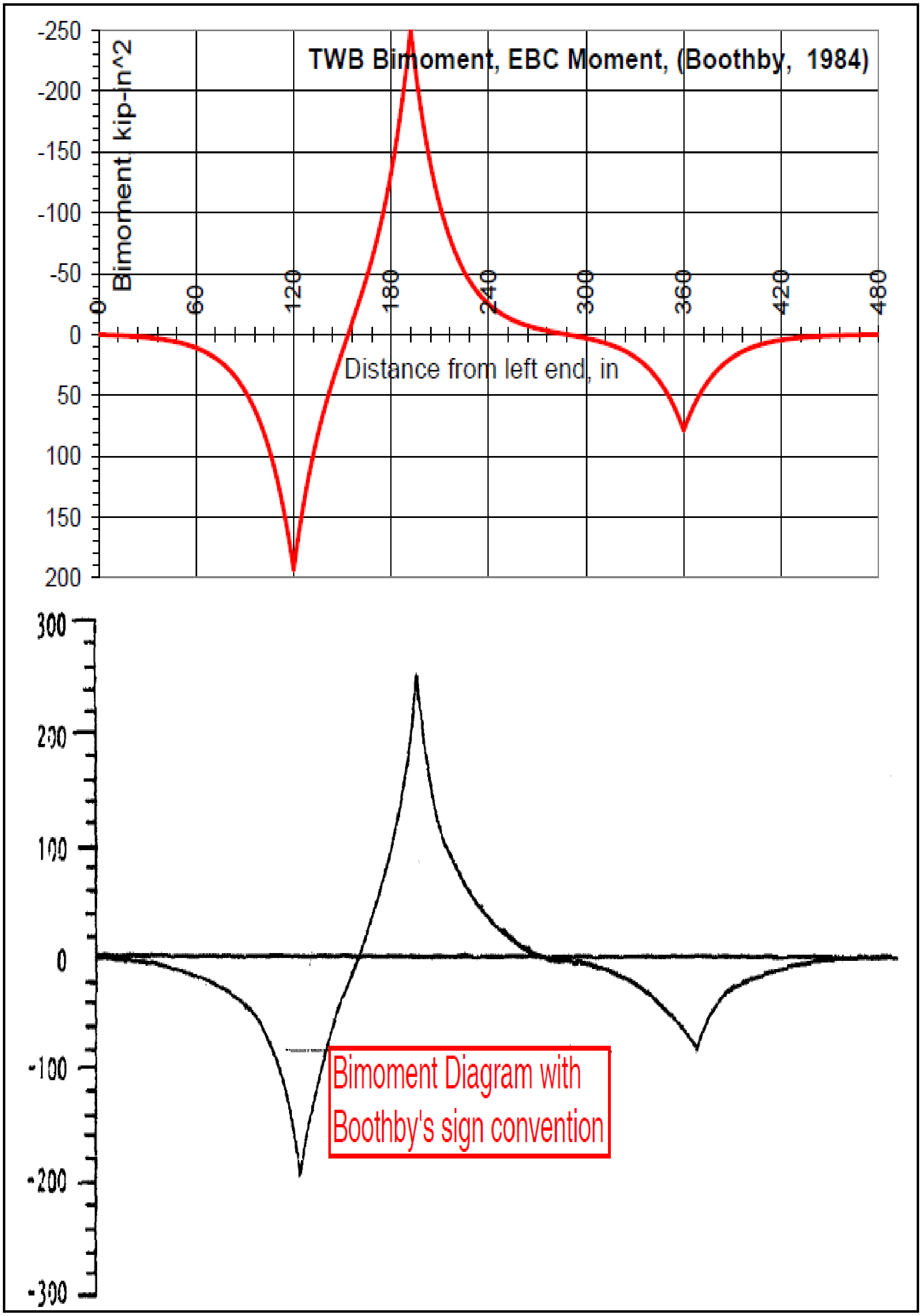
| NF | ELEM | END | ROTATIONAL SPRING MODULUS | ELEM | END | ROTATIONAL SPRING MODULUS | ELEM | END | ROTATIONAL SPRING MOD |
|---|---|---|---|---|---|---|---|---|---|
| | | | | | | | | | |

1 2 3 4 5 6 7 8 9 10 11 12 13 14 15 16 17 18 19 20 21 22 23 24 25 26 27 28 29 30 31 32 33 34 35 36 37 38 39 40 41

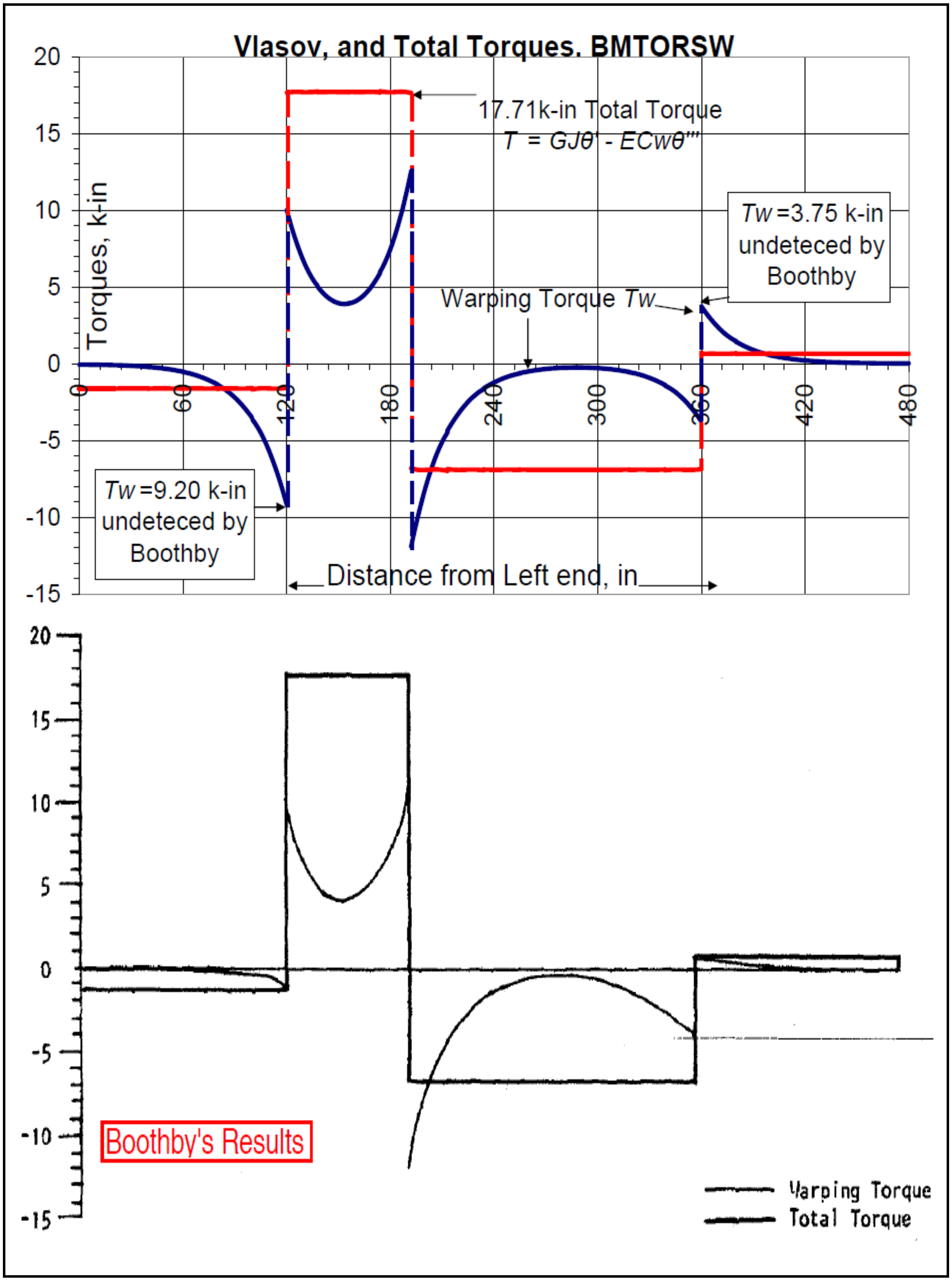
Input Form for BMTORSWP, Left Side




Input Form for BMTORSWP, Right Side




Match between TWB Bimoment from BMTORSWP and Boothby




Match between Total and Warping Torque from BMTORSWP and Boothby

```
                                    BBYOUT

 ------------------------------------------------------------------------
 YOU ARE USING COMPUTER PROGRAM BMTORSW,DEVELOPED BY DR. BERNARDO DESCHAPELLES

         INPUT DATA FILE NAME IS = bby.txt

                 OUTPUT FILE NAME IS = bbyout.txt

           STORAGE FILE FOR POST-PROCESSING WITH EXCEL = bbygrf.grf


 ------------------------------------------------------------------------
 Analysis of restrained warping for a 3-span beam, Boothy, AISC-EJ, 1984

 modulus of elasticity of the material=   29000. k/ft2

 ELEM nodes inertia length distrib. load  AXIAL  SOIL NORMAL MODULUS,Ksf  angle
      i   j  ft.4     ft    at i    at j  LOAD    1st END       2nd END    rad
  1   1   3******** 20.00  0.000  0.000********       0.0          0.0   0.000 00
  2   3   5******** 20.00  0.000  0.000********       0.0          0.0   0.000 00
  3   5   7******** 20.00  0.000  0.000********       0.0          0.0   0.000 00
  4   7   9******** 20.00  0.000  0.000********       0.0          0.0   0.000 00
  5   9  11******** 20.00  0.000  0.000********       0.0          0.0   0.000 00
  6  11  13******** 20.00  0.000  0.000********       0.0          0.0   0.000 00
  7  13  15******** 18.00  0.000  0.000********       0.0          0.0   0.000 00
  8  15  17******** 18.00  0.000  0.000********       0.0          0.0   0.000 00
  9  17  19******** 18.00  0.000  0.000********       0.0          0.0   0.000 00
 10  19  21******** 18.00  0.000  0.000********       0.0          0.0   0.000 00
 11  21  23******** 21.00  0.000  0.000********       0.0          0.0   0.000 00
 12  23  25******** 21.00  0.000  0.000********       0.0          0.0   0.000 00
 13  25  27******** 21.00  0.000  0.000********       0.0          0.0   0.000 00
 14  27  29******** 21.00  0.000  0.000********       0.0          0.0   0.000 00
 15  29  31******** 21.00  0.000  0.000********       0.0          0.0   0.000 00
 16  31  33******** 21.00  0.000  0.000********       0.0          0.0   0.000 00
 17  33  35******** 21.00  0.000  0.000********       0.0          0.0   0.000 00
 18  35  37******** 21.00  0.000  0.000********       0.0          0.0   0.000 00
 19  37  39******** 20.00  0.000  0.000********       0.0          0.0   0.000 00
 20  39  41******** 20.00  0.000  0.000********       0.0          0.0   0.000 00
 21  41  43******** 20.00  0.000  0.000********       0.0          0.0   0.000 00
 22  43  45******** 20.00  0.000  0.000********       0.0          0.0   0.000 00
 23  45  47******** 20.00  0.000  0.000********       0.0          0.0   0.000 00
 24  47  49******** 20.00  0.000  0.000********       0.0          0.0   0.000 00

     INPUT DATA RELATED TO THE  4 SUPPORTS
  4 1 1 1 013 0 1 037 0 1 049 0 1 0

     INPUT OF NODAL FORCES RELATED TO GLOBAL AXIS  2
          121  24.60

       FINAL  SOLUTION   FOUND   AFTER    1   ITERATIONS

      Output of nodal displacements in reference to  global axes

 node  displ.      displ.      displ.     node  displ.      displ.      displ.
     along  x    along  y    around  z         along  x    along  y    around  z
     or nonn1    or nonn2    or nonn 3         or nonn1    or nonn2    or nonn 3

   1 0.0000E+00  0.0000E+00 -0.1551E-03
                                           2 0.0000E+00 -0.1546E-02 -0.2570E-03

   3 0.0000E+00 -0.3082E-02 -0.1521E-03
                                           4 0.0000E+00 -0.4574E-02 -0.2459E-03

                                    Page 1
```

Output Page 1

```
                                   BBYOUT
 5 0.0000E+00 -0.6028E-02 -0.1404E-03
 6 0.0000E+00 -0.7351E-02 -0.2125E-03
 7 0.0000E+00 -0.8568E-02 -0.1084E-03
 8 0.0000E+00 -0.9437E-02 -0.1240E-03
 9 0.0000E+00 -0.1003E-01 -0.2456E-04
10 0.0000E+00 -0.9713E-02  0.1069E-03
11 0.0000E+00 -0.8679E-02  0.1940E-03
12 0.0000E+00 -0.5274E-02  0.7084E-03
13 0.0000E+00  0.0000E+00  0.7633E-03
14 0.0000E+00  0.8776E-02  0.1600E-02
15 0.0000E+00  0.1904E-01  0.1266E-02
16 0.0000E+00  0.3099E-01  0.2026E-02
17 0.0000E+00  0.4327E-01  0.1374E-02
18 0.0000E+00  0.5529E-01  0.1962E-02
19 0.0000E+00  0.6671E-01  0.1174E-02
20 0.0000E+00  0.7575E-01  0.1357E-02
21 0.0000E+00  0.8281E-01  0.5080E-03
22 0.0000E+00  0.8482E-01  0.9450E-04
23 0.0000E+00  0.8421E-01 -0.2502E-03
24 0.0000E+00  0.8036E-01 -0.7284E-03
25 0.0000E+00  0.7556E-01 -0.5273E-03
26 0.0000E+00  0.6958E-01 -0.1028E-02
27 0.0000E+00  0.6326E-01 -0.6274E-03
28 0.0000E+00  0.5652E-01 -0.1134E-02
29 0.0000E+00  0.4967E-01 -0.6604E-03
30 0.0000E+00  0.4270E-01 -0.1162E-02
31 0.0000E+00  0.3573E-01 -0.6623E-03
32 0.0000E+00  0.2884E-01 -0.1142E-02
33 0.0000E+00  0.2205E-01 -0.6352E-03
34 0.0000E+00  0.1560E-01 -0.1052E-02
35 0.0000E+00  0.9451E-02 -0.5495E-03
36 0.0000E+00  0.4315E-02 -0.7945E-03
37 0.0000E+00  0.0000E+00 -0.3111E-03
38 0.0000E+00 -0.2150E-02 -0.2888E-03
39 0.0000E+00 -0.3538E-02 -0.7910E-04
40 0.0000E+00 -0.3960E-02 -0.4359E-04
41 0.0000E+00 -0.4089E-02  0.1001E-04
42 0.0000E+00 -0.3847E-02  0.5056E-04
43 0.0000E+00 -0.3493E-02  0.4421E-04
44 0.0000E+00 -0.2996E-02  0.8662E-04
45 0.0000E+00 -0.2457E-02  0.5725E-04
46 0.0000E+00 -0.1865E-02  0.1002E-03
                                   Page 2
```

Output Page 2

```
                                                    BBYOUT
  47 0.0000E+00 -0.1256E-02  0.6201E-04
                                                        48 0.0000E+00 -0.6302E-03  0.1048E-03

  49 0.0000E+00  0.0000E+00  0.6321E-04

 ----------------------------------------------------------------------
      OUTPUT OF SOIL REACTIONS,STRESSES AND TRANSVERSE DISPLACEMENTS
                                                               Fork
 ----------------------------------------------------------------------
      ELEMENT    1  DISPLACEMENTS IN INCIDENCES    1   2   3
         NODE    1          0.00000E+00         0.00000E+00         -0.15505E-03
         NODE    2          0.00000E+00        -0.15459E-02         -0.25701E-03
         NODE    3          0.00000E+00        -0.30821E-02         -0.15212E-03
      FORCES ACTING ALONG THE 9 DOF
         NODE    1          0.00000E+00         0.16135E+01          0.20670E-12
         NODE    2          0.00000E+00         0.15719E-12          0.28300E-12
         NODE    3          0.00000E+00        -0.16135E+01          0.13802E+01

 ELEMENT   1, FROM NODE   1, TO NODE   3 - LENGTH = 20.00 ft

  left half of span,at tenth points of length
              span       span       span       span       span       span
           0.0        0.1        0.2        0.3        0.4        0.5
 soil,k/ft    0.000      0.000      0.000      0.000      0.000      0.000
 shear,k       1.61       1.61       1.61       1.61       1.61       1.61
 bmom,kft      0.00       0.12       0.24       0.36       0.49       0.62
 tdisp,ft   0.00000   -0.00031   -0.00062   -0.00093   -0.00124   -0.00155
 axial,k       0.00 AT 1st END and       0.00 AT 2nd END

 right half of span,at tenth points of length
              span       span       span       span       span       span
           0.5        0.6        0.7        0.8        0.9        1.0
 soil,k/ft    0.000      0.000      0.000      0.000      0.000      0.000
 shear,k       1.61       1.61       1.61       1.61       1.61       1.61
 bmom,kft      0.62       0.75       0.90       1.05       1.21       1.38
 tdisp,ft  -0.00155   -0.00186   -0.00216   -0.00247   -0.00278   -0.00308
 axial,k       0.00 AT 1st END and       0.00 AT 2nd END

 ----------------------------------------------------------------------
      ELEMENT    2  DISPLACEMENTS IN INCIDENCES    3   4   5
         NODE    3          0.00000E+00        -0.30821E-02         -0.15212E-03
         NODE    4          0.00000E+00        -0.45744E-02         -0.24585E-03
         NODE    5          0.00000E+00        -0.60282E-02         -0.14043E-03
      FORCES ACTING ALONG THE 9 DOF
         NODE    3          0.00000E+00         0.16135E+01         -0.13802E+01
         NODE    4          0.00000E+00         0.43340E-12          0.38510E-11
         NODE    5          0.00000E+00        -0.16135E+01          0.41233E+01

 ELEMENT   2, FROM NODE   3, TO NODE   5 - LENGTH = 20.00 ft

  left half of span,at tenth points of length
              span       span       span       span       span       span
           0.0        0.1        0.2        0.3        0.4        0.5
 soil,k/ft    0.000      0.000      0.000      0.000      0.000      0.000
 shear,k       1.61       1.61       1.61       1.61       1.61       1.61
 bmom,kft      1.38       1.56       1.76       1.98       2.21       2.46
 tdisp,ft  -0.00308   -0.00339   -0.00369   -0.00399   -0.00429   -0.00458
 axial,k       0.00 AT 1st END and       0.00 AT 2nd END

 right half of span,at tenth points of length
              span       span       span       span       span       span
           0.5        0.6        0.7        0.8        0.9        1.0
 soil,k/ft    0.000      0.000      0.000      0.000      0.000      0.000
                                                    Page 3
```

Output Page 3

```
                                      BBYOUT
shear,k        1.61      1.61      1.61      1.61      1.61      1.61
bmom,kft       2.46      2.74      3.04      3.37      3.73      4.12
tdisp,ft   -0.00458  -0.00488  -0.00517  -0.00546  -0.00575  -0.00603
axial,k        0.00 AT 1st END and      0.00 AT 2nd END

----------------------------------------------------------------------
     ELEMENT   3  DISPLACEMENTS IN INCIDENCES    5   6   7
        NODE   5          0.00000E+00          -0.60282E-02         -0.14043E-03
        NODE   6          0.00000E+00          -0.73507E-02         -0.21250E-03
        NODE   7          0.00000E+00          -0.85681E-02         -0.10844E-03
     FORCES ACTING ALONG THE 9 DOF
        NODE   5          0.00000E+00           0.16135E+01         -0.41233E+01
        NODE   6          0.00000E+00           0.17847E-13          0.63055E-11
        NODE   7          0.00000E+00          -0.16135E+01          0.10938E+02

ELEMENT   3, FROM NODE   5, TO NODE   7 - LENGTH = 20.00 ft

 left half of span,at tenth points of length
               span      span      span      span      span      span
           0.0       0.1       0.2       0.3       0.4       0.5
soil,k/ft     0.000     0.000     0.000     0.000     0.000     0.000
shear,k        1.61      1.61      1.61      1.61      1.61      1.61
bmom,kft       4.12      4.55      5.03      5.55      6.12      6.74
tdisp,ft   -0.00603  -0.00631  -0.00658  -0.00685  -0.00712  -0.00738
axial,k        0.00 AT 1st END and      0.00 AT 2nd END

right half of span,at tenth points of length
               span      span      span      span      span      span
           0.5       0.6       0.7       0.8       0.9       1.0
soil,k/ft     0.000     0.000     0.000     0.000     0.000     0.000
shear,k        1.61      1.61      1.61      1.61      1.61      1.61
bmom,kft       6.74      7.43      8.19      9.02      9.93     10.94
tdisp,ft   -0.00738  -0.00763  -0.00788  -0.00812  -0.00835  -0.00857
axial,k        0.00 AT 1st END and      0.00 AT 2nd END

----------------------------------------------------------------------
     ELEMENT   4  DISPLACEMENTS IN INCIDENCES    7   8   9
        NODE   7          0.00000E+00          -0.85681E-02         -0.10844E-03
        NODE   8          0.00000E+00          -0.94369E-02         -0.12403E-03
        NODE   9          0.00000E+00          -0.10030E-01         -0.24560E-04
     FORCES ACTING ALONG THE 9 DOF
        NODE   7          0.00000E+00           0.16135E+01         -0.10938E+02
        NODE   8          0.00000E+00          -0.55385E-12         -0.58053E-11
        NODE   9          0.00000E+00          -0.16135E+01          0.28554E+02

ELEMENT   4, FROM NODE   7, TO NODE   9 - LENGTH = 20.00 ft

 left half of span,at tenth points of length
               span      span      span      span      span      span
           0.0       0.1       0.2       0.3       0.4       0.5
soil,k/ft     0.000     0.000     0.000     0.000     0.000     0.000
shear,k        1.61      1.61      1.61      1.61      1.61      1.61
bmom,kft      10.94     12.04     13.26     14.60     16.07     17.68
tdisp,ft   -0.00857  -0.00878  -0.00898  -0.00917  -0.00934  -0.00950
axial,k        0.00 AT 1st END and      0.00 AT 2nd END

right half of span,at tenth points of length
               span      span      span      span      span      span
           0.5       0.6       0.7       0.8       0.9       1.0
soil,k/ft     0.000     0.000     0.000     0.000     0.000     0.000
shear,k        1.61      1.61      1.61      1.61      1.61      1.61
bmom,kft      17.68     19.46     21.42     23.58     25.95     28.55
tdisp,ft   -0.00950  -0.00965  -0.00978  -0.00988  -0.00997  -0.01003
                                      Page 4
```

Output Page 4

```
                                           BBYOUT
axial,k        0.00 AT 1st END and      0.00 AT 2nd END

----------------------------------------------------------------------
     ELEMENT    5  DISPLACEMENTS IN INCIDENCES    9  10  11
        NODE    9          0.00000E+00         -0.10030E-01        -0.24560E-04
        NODE   10          0.00000E+00         -0.97135E-02         0.10692E-03
        NODE   11          0.00000E+00         -0.86789E-02         0.19405E-03
     FORCES ACTING ALONG THE 9 DOF
        NODE    9          0.00000E+00          0.16135E+01        -0.28554E+02
        NODE   10          0.00000E+00          0.58212E-12        -0.21937E-11
        NODE   11          0.00000E+00         -0.16135E+01         0.74369E+02

ELEMENT  5, FROM NODE  9, TO NODE 11 - LENGTH = 20.00 ft

 left half of span,at tenth points of length
              span      span      span      span      span      span
          0.0       0.1       0.2       0.3       0.4       0.5
soil,k/ft    0.000     0.000     0.000     0.000     0.000     0.000
shear,k       1.61      1.61      1.61      1.61      1.61      1.61
bmom,kft     28.55     31.42     34.58     38.06     41.88     46.09
tdisp,ft  -0.01003  -0.01007  -0.01007  -0.01005  -0.00999  -0.00989
axial,k        0.00 AT 1st END and      0.00 AT 2nd END

right half of span,at tenth points of length
              span      span      span      span      span      span
          0.5       0.6       0.7       0.8       0.9       1.0
soil,k/ft    0.000     0.000     0.000     0.000     0.000     0.000
shear,k       1.61      1.61      1.61      1.61      1.61      1.61
bmom,kft     46.09     50.72     55.81     61.41     67.58     74.37
tdisp,ft  -0.00989  -0.00975  -0.00956  -0.00933  -0.00903  -0.00868
axial,k        0.00 AT 1st END and      0.00 AT 2nd END

----------------------------------------------------------------------
     ELEMENT    6  DISPLACEMENTS IN INCIDENCES   11  12  13
        NODE   11          0.00000E+00         -0.86789E-02         0.19405E-03
        NODE   12          0.00000E+00         -0.52740E-02         0.70843E-03
        NODE   13          0.00000E+00          0.00000E+00         0.76326E-03
     FORCES ACTING ALONG THE 9 DOF
        NODE   11          0.00000E+00          0.16135E+01        -0.74369E+02
        NODE   12          0.00000E+00         -0.11741E-13         0.28773E-11
        NODE   13          0.00000E+00         -0.16135E+01         0.19362E+03

ELEMENT  6, FROM NODE 11, TO NODE 13 - LENGTH = 20.00 ft

 left half of span,at tenth points of length
              span      span      span      span      span      span
          0.0       0.1       0.2       0.3       0.4       0.5
soil,k/ft    0.000     0.000     0.000     0.000     0.000     0.000
shear,k       1.61      1.61      1.61      1.61      1.61      1.61
bmom,kft     74.37     81.84     90.05     99.10    109.05    120.00
tdisp,ft  -0.00868  -0.00826  -0.00776  -0.00718  -0.00651  -0.00574
axial,k        0.00 AT 1st END and      0.00 AT 2nd END

right half of span,at tenth points of length
              span      span      span      span      span      span
          0.5       0.6       0.7       0.8       0.9       1.0
soil,k/ft    0.000     0.000     0.000     0.000     0.000     0.000
shear,k       1.61      1.61      1.61      1.61      1.61      1.61
bmom,kft    120.00    132.05    145.31    159.90    175.95    193.62
tdisp,ft  -0.00574  -0.00486  -0.00385  -0.00272  -0.00144   0.00000
axial,k        0.00 AT 1st END and      0.00 AT 2nd END
                                          (193.62+1.38)120=1.625 ~ 01.61
----------------------------------------------------------------------
                                           Page 5
```

Output Page 5

```
                                  BBYOUT
     ELEMENT    7  DISPLACEMENTS IN INCIDENCES  13  14  15
        NODE   13          0.00000E+00          0.00000E+00          0.76326E-03
        NODE   14          0.00000E+00          0.87756E-02          0.15996E-02
        NODE   15          0.00000E+00          0.19041E-01          0.12660E-02
     FORCES ACTING ALONG THE 9 DOF
        NODE   13          0.00000E+00         -0.17698E+02         -0.19362E+03
        NODE   14          0.00000E+00          0.35749E-12         -0.28040E-11
        NODE   15          0.00000E+00          0.17698E+02          0.65896E+02

ELEMENT   7, FROM NODE 13, TO NODE 15 - LENGTH = 18.00 ft

 left half of span,at tenth points of length
             span      span      span      span      span      span
          0.0       0.1       0.2       0.3       0.4       0.5
soil,k/ft    0.000     0.000     0.000     0.000     0.000     0.000
shear,k    -17.70    -17.70    -17.70    -17.70    -17.70    -17.70
bmom,kft   193.62    176.23    160.15    145.25    131.44    118.60
tdisp,ft  0.00000   0.00144   0.00302   0.00471   0.00651   0.00841
axial,k       0.00 AT 1st END and      0.00 AT 2nd END

right half of span,at tenth points of length
             span      span      span      span      span      span
          0.5       0.6       0.7       0.8       0.9       1.0
soil,k/ft    0.000     0.000     0.000     0.000     0.000     0.000
shear,k    -17.70    -17.70    -17.70    -17.70    -17.70    -17.70
bmom,kft   118.60    106.63     95.46     85.00     75.17     65.90
tdisp,ft  0.00841   0.01039   0.01246   0.01459   0.01679   0.01904
axial,k       0.00 AT 1st END and      0.00 AT 2nd END

----------------------------------------------------------------------
     ELEMENT    8  DISPLACEMENTS IN INCIDENCES  15  16  17
        NODE   15          0.00000E+00          0.19041E-01          0.12660E-02
        NODE   16          0.00000E+00          0.30993E-01          0.20264E-02
        NODE   17          0.00000E+00          0.43266E-01          0.13745E-02
     FORCES ACTING ALONG THE 9 DOF
        NODE   15          0.00000E+00         -0.17698E+02         -0.65896E+02
        NODE   16          0.00000E+00         -0.22204E-12          0.35884E-10
        NODE   17          0.00000E+00          0.17698E+02         -0.98709E+01

ELEMENT   8, FROM NODE 15, TO NODE 17 - LENGTH = 18.00 ft

 left half of span,at tenth points of length
             span      span      span      span      span      span
          0.0       0.1       0.2       0.3       0.4       0.5
soil,k/ft    0.000     0.000     0.000     0.000     0.000     0.000
shear,k    -17.70    -17.70    -17.70    -17.70    -17.70    -17.70
bmom,kft    65.90     57.11     48.75     40.75     33.05     25.60
tdisp,ft  0.01904   0.02134   0.02369   0.02607   0.02848   0.03091
axial,k       0.00 AT 1st END and      0.00 AT 2nd END

right half of span,at tenth points of length
             span      span      span      span      span      span
          0.5       0.6       0.7       0.8       0.9       1.0
soil,k/ft    0.000     0.000     0.000     0.000     0.000     0.000
shear,k    -17.70    -17.70    -17.70    -17.70    -17.70    -17.70
bmom,kft    25.60     18.34     11.22      4.17     -2.84     -9.87
tdisp,ft  0.03091   0.03337   0.03583   0.03831   0.04079   0.04327
axial,k       0.00 AT 1st END and      0.00 AT 2nd END

----------------------------------------------------------------------
     ELEMENT    9  DISPLACEMENTS IN INCIDENCES  17  18  19
        NODE   17          0.00000E+00          0.43266E-01          0.13745E-02
        NODE   18          0.00000E+00          0.55287E-01          0.19625E-02
                                  Page 6
```

Output Page 6

```
                                        BBYOUT
        NODE  19          0.00000E+00          0.66715E-01          0.11744E-02
     FORCES ACTING ALONG THE 9 DOF
        NODE  17          0.00000E+00         -0.17698E+02          0.98709E+01
        NODE  18          0.00000E+00         -0.19454E-11          0.14981E-10
        NODE  19          0.00000E+00          0.17698E+02         -0.93421E+02

ELEMENT  9, FROM NODE 17, TO NODE 19 - LENGTH = 18.00 ft

 left half of span,at tenth points of length
             span      span      span      span      span      span
          0.0       0.1       0.2       0.3       0.4       0.5
soil,k/ft   0.000     0.000     0.000     0.000     0.000     0.000
shear,k    -17.70    -17.70    -17.70    -17.70    -17.70    -17.70
bmom,kft    -9.87    -16.98    -24.21    -31.62    -39.27    -47.20
tdisp,ft  0.04327   0.04574   0.04819   0.05063   0.05305   0.05543
axial,k       0.00 AT 1st END and       0.00 AT 2nd END

right half of span,at tenth points of length
             span      span      span      span      span      span
          0.5       0.6       0.7       0.8       0.9       1.0
soil,k/ft   0.000     0.000     0.000     0.000     0.000     0.000
shear,k    -17.70    -17.70    -17.70    -17.70    -17.70    -17.70
bmom,kft   -47.20    -55.49    -64.19    -73.36    -83.08    -93.42
tdisp,ft  0.05543   0.05779   0.06010   0.06236   0.06457   0.06671
axial,k       0.00 AT 1st END and       0.00 AT 2nd END

----------------------------------------------------------------------
     ELEMENT  10  DISPLACEMENTS IN INCIDENCES  19  20  21
        NODE  19          0.00000E+00          0.66715E-01          0.11744E-02
        NODE  20          0.00000E+00          0.75752E-01          0.13574E-02
        NODE  21          0.00000E+00          0.82814E-01          0.50795E-03
     FORCES ACTING ALONG THE 9 DOF
        NODE  19          0.00000E+00         -0.17698E+02          0.93421E+02
        NODE  20          0.00000E+00         -0.95102E-12         -0.30931E-11
        NODE  21          0.00000E+00          0.17698E+02         -0.25063E+03

ELEMENT 10, FROM NODE 19, TO NODE 21 - LENGTH = 18.00 ft

 left half of span,at tenth points of length
             span      span      span      span      span      span
          0.0       0.1       0.2       0.3       0.4       0.5
soil,k/ft   0.000     0.000     0.000     0.000     0.000     0.000
shear,k    -17.70    -17.70    -17.70    -17.70    -17.70    -17.70
bmom,kft   -93.42   -104.45   -116.25   -128.92   -142.55   -157.23
tdisp,ft  0.06671   0.06879   0.07079   0.07271   0.07453   0.07624
axial,k       0.00 AT 1st END and       0.00 AT 2nd END

right half of span,at tenth points of length
             span      span      span      span      span      span
          0.5       0.6       0.7       0.8       0.9       1.0
soil,k/ft   0.000     0.000     0.000     0.000     0.000     0.000
shear,k    -17.70    -17.70    -17.70    -17.70    -17.70    -17.70
bmom,kft  -157.23   -173.08   -190.21   -208.75   -228.84   -250.63
tdisp,ft  0.07624   0.07784   0.07931   0.08064   0.08181   0.08281
axial,k       0.00 AT 1st END and       0.00 AT 2nd END

----------------------------------------------------------------------
     ELEMENT  11  DISPLACEMENTS IN INCIDENCES  21  22  23
        NODE  21          0.00000E+00          0.82814E-01          0.50795E-03
        NODE  22          0.00000E+00          0.84816E-01          0.94498E-04
        NODE  23          0.00000E+00          0.84208E-01         -0.25021E-03
     FORCES ACTING ALONG THE 9 DOF
        NODE  21          0.00000E+00          0.69021E+01          0.25063E+03
                                        Page 7
```

Sum. of moments (17.7x72-193.62-250.63)/10022 = 0.08283 ~ 0.08281 OK

193.62 k-in from elem. 7, node 13

Output Page 7

```
                                 BBYOUT
       NODE   22          0.00000E+00         -0.28362E-11         -0.18572E-10
       NODE   23          0.00000E+00         -0.69021E+01         -0.91715E+02

ELEMENT 11, FROM NODE 21, TO NODE 23 - LENGTH = 21.00 ft

 left half of span,at tenth points of length
              span      span      span      span      span      span
          0.0       0.1       0.2       0.3       0.4       0.5
soil,k/ft    0.000     0.000     0.000     0.000     0.000     0.000
shear,k       6.90      6.90      6.90      6.90      6.90      6.90
bmom,kft   -250.63   -226.67   -205.00   -185.40   -167.67   -151.64
tdisp,ft   0.08281   0.08376   0.08447   0.08498   0.08531   0.08546
axial,k       0.00 AT 1st END and      0.00 AT 2nd END

right half of span,at tenth points of length
              span      span      span      span      span      span
          0.5       0.6       0.7       0.8       0.9       1.0
soil,k/ft    0.000     0.000     0.000     0.000     0.000     0.000
shear,k       6.90      6.90      6.90      6.90      6.90      6.90
bmom,kft   -151.64   -137.14   -124.02   -112.15   -101.42    -91.71
tdisp,ft   0.08546   0.08546   0.08532   0.08506   0.08469   0.08421
axial,k       0.00 AT 1st END and      0.00 AT 2nd END

----------------------------------------------------------------------
     ELEMENT   12  DISPLACEMENTS IN INCIDENCES  23  24  25
        NODE   23          0.00000E+00          0.84208E-01         -0.25021E-03
        NODE   24          0.00000E+00          0.80362E-01         -0.72842E-03
        NODE   25          0.00000E+00          0.75563E-01         -0.52733E-03
     FORCES ACTING ALONG THE 9 DOF
        NODE   23          0.00000E+00          0.69021E+01          0.91715E+02
        NODE   24          0.00000E+00          0.22534E-11         -0.25101E-10
        NODE   25          0.00000E+00         -0.69021E+01         -0.33418E+02

ELEMENT 12, FROM NODE 23, TO NODE 25 - LENGTH = 21.00 ft

 left half of span,at tenth points of length
              span      span      span      span      span      span
          0.0       0.1       0.2       0.3       0.4       0.5
soil,k/ft    0.000     0.000     0.000     0.000     0.000     0.000
shear,k       6.90      6.90      6.90      6.90      6.90      6.90
bmom,kft    -91.71    -82.93    -74.99    -67.81    -61.31    -55.43
tdisp,ft   0.08421   0.08364   0.08298   0.08225   0.08146   0.08060
axial,k       0.00 AT 1st END and      0.00 AT 2nd END

right half of span,at tenth points of length
              span      span      span      span      span      span
          0.5       0.6       0.7       0.8       0.9       1.0
soil,k/ft    0.000     0.000     0.000     0.000     0.000     0.000
shear,k       6.90      6.90      6.90      6.90      6.90      6.90
bmom,kft    -55.43    -50.10    -45.29    -40.93    -36.99    -33.42
tdisp,ft   0.08060   0.07968   0.07872   0.07771   0.07665   0.07556
axial,k       0.00 AT 1st END and      0.00 AT 2nd END

----------------------------------------------------------------------
     ELEMENT   13  DISPLACEMENTS IN INCIDENCES  25  26  27
        NODE   25          0.00000E+00          0.75563E-01         -0.52733E-03
        NODE   26          0.00000E+00          0.69583E-01         -0.10283E-02
        NODE   27          0.00000E+00          0.63259E-01         -0.62744E-03
     FORCES ACTING ALONG THE 9 DOF
        NODE   25          0.00000E+00          0.69021E+01          0.33418E+02
        NODE   26          0.00000E+00         -0.30136E-11         -0.12480E-10
        NODE   27          0.00000E+00         -0.69021E+01         -0.11786E+02

                                 Page 8
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Output Page 8

```
                                  BBYOUT
ELEMENT 13, FROM NODE 25, TO NODE 27 - LENGTH = 21.00 ft

 left half of span,at tenth points of length
              span       span       span       span       span       span
          0.0        0.1        0.2        0.3        0.4        0.5
soil,k/ft    0.000      0.000      0.000      0.000      0.000      0.000
shear,k       6.90       6.90       6.90       6.90       6.90       6.90
bmom,kft    -33.42     -30.19     -27.26     -24.61     -22.20     -20.02
tdisp,ft   0.07556    0.07444    0.07328    0.07210    0.07090    0.06967
axial,k       0.00 AT 1st END and       0.00 AT 2nd END

right half of span,at tenth points of length
              span       span       span       span       span       span
          0.5        0.6        0.7        0.8        0.9        1.0
soil,k/ft    0.000      0.000      0.000      0.000      0.000      0.000
shear,k       6.90       6.90       6.90       6.90       6.90       6.90
bmom,kft    -20.02     -18.05     -16.25     -14.62     -13.14     -11.79
tdisp,ft   0.06967    0.06842    0.06715    0.06587    0.06457    0.06326
axial,k       0.00 AT 1st END and       0.00 AT 2nd END

----------------------------------------------------------------------
     ELEMENT  14  DISPLACEMENTS IN INCIDENCES  27  28  29
        NODE  27         0.00000E+00          0.63259E-01        -0.62744E-03
        NODE  28         0.00000E+00          0.56519E-01        -0.11340E-02
        NODE  29         0.00000E+00          0.49665E-01        -0.66038E-03
     FORCES ACTING ALONG THE 9 DOF
        NODE  27         0.00000E+00          0.69021E+01         0.11786E+02
        NODE  28         0.00000E+00          0.64125E-11        -0.84248E-10
        NODE  29         0.00000E+00         -0.69021E+01        -0.30859E+01

ELEMENT 14, FROM NODE 27, TO NODE 29 - LENGTH = 21.00 ft

 left half of span,at tenth points of length
              span       span       span       span       span       span
          0.0        0.1        0.2        0.3        0.4        0.5
soil,k/ft    0.000      0.000      0.000      0.000      0.000      0.000
shear,k       6.90       6.90       6.90       6.90       6.90       6.90
bmom,kft    -11.79     -10.56      -9.43      -8.40      -7.46      -6.59
tdisp,ft   0.06326    0.06194    0.06060    0.05926    0.05791    0.05655
axial,k       0.00 AT 1st END and       0.00 AT 2nd END

right half of span,at tenth points of length
              span       span       span       span       span       span
          0.5        0.6        0.7        0.8        0.9        1.0
soil,k/ft    0.000      0.000      0.000      0.000      0.000      0.000
shear,k       6.90       6.90       6.90       6.90       6.90       6.90
bmom,kft     -6.59      -5.78      -5.04      -4.35      -3.70      -3.09
tdisp,ft   0.05655    0.05518    0.05381    0.05243    0.05105    0.04967
axial,k       0.00 AT 1st END and       0.00 AT 2nd END

----------------------------------------------------------------------
     ELEMENT  15  DISPLACEMENTS IN INCIDENCES  29  30  31
        NODE  29         0.00000E+00          0.49665E-01        -0.66038E-03
        NODE  30         0.00000E+00          0.42702E-01        -0.11617E-02
        NODE  31         0.00000E+00          0.35733E-01        -0.66228E-03
     FORCES ACTING ALONG THE 9 DOF
        NODE  29         0.00000E+00          0.69021E+01         0.30859E+01
        NODE  30         0.00000E+00          0.46169E-12         0.54601E-11
        NODE  31         0.00000E+00         -0.69021E+01         0.22290E+01

ELEMENT 15, FROM NODE 29, TO NODE 31 - LENGTH = 21.00 ft

 left half of span,at tenth points of length
                                  Page 9
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Output Page 9

```
                                          BBYOUT
                 span       span       span       span       span       span
             0.0        0.1        0.2        0.3        0.4        0.5
soil,k/ft      0.000      0.000      0.000      0.000      0.000      0.000
shear,k         6.90       6.90       6.90       6.90       6.90       6.90
bmom,kft       -3.09      -2.51      -1.95      -1.41      -0.89      -0.38
tdisp,ft     0.04967    0.04828    0.04689    0.04549    0.04410    0.04270
axial,k         0.00 AT 1st END and      0.00 AT 2nd END

right half of span,at tenth points of length
                 span       span       span       span       span       span
             0.5        0.6        0.7        0.8        0.9        1.0
soil,k/ft      0.000      0.000      0.000      0.000      0.000      0.000
shear,k         6.90       6.90       6.90       6.90       6.90       6.90
bmom,kft       -0.38       0.13       0.64       1.16       1.68       2.23
tdisp,ft     0.04270    0.04131    0.03991    0.03852    0.03713    0.03573
axial,k         0.00 AT 1st END and      0.00 AT 2nd END

----------------------------------------------------------------------
     ELEMENT   16  DISPLACEMENTS IN INCIDENCES   31  32  33
        NODE   31         0.00000E+00         0.35733E-01         -0.66228E-03
        NODE   32         0.00000E+00         0.28843E-01         -0.11417E-02
        NODE   33         0.00000E+00         0.22046E-01         -0.63522E-03
     FORCES ACTING ALONG THE 9 DOF
        NODE   31         0.00000E+00         0.69021E+01         -0.22290E+01
        NODE   32         0.00000E+00        -0.14607E-11         -0.14266E-10
        NODE   33         0.00000E+00        -0.69021E+01          0.99895E+01

ELEMENT 16, FROM NODE 31, TO NODE 33 - LENGTH = 21.00 ft

 left half of span,at tenth points of length
                 span       span       span       span       span       span
             0.0        0.1        0.2        0.3        0.4        0.5
soil,k/ft      0.000      0.000      0.000      0.000      0.000      0.000
shear,k         6.90       6.90       6.90       6.90       6.90       6.90
bmom,kft        2.23       2.80       3.39       4.02       4.69       5.41
tdisp,ft     0.03573    0.03434    0.03296    0.03157    0.03019    0.02882
axial,k         0.00 AT 1st END and      0.00 AT 2nd END

right half of span,at tenth points of length
                 span       span       span       span       span       span
             0.5        0.6        0.7        0.8        0.9        1.0
soil,k/ft      0.000      0.000      0.000      0.000      0.000      0.000
shear,k         6.90       6.90       6.90       6.90       6.90       6.90
bmom,kft        5.41       6.18       7.02       7.93       8.91       9.99
tdisp,ft     0.02882    0.02745    0.02609    0.02473    0.02338    0.02205
axial,k         0.00 AT 1st END and      0.00 AT 2nd END

----------------------------------------------------------------------
     ELEMENT   17  DISPLACEMENTS IN INCIDENCES   33  34  35
        NODE   33         0.00000E+00         0.22046E-01         -0.63522E-03
        NODE   34         0.00000E+00         0.15601E-01         -0.10521E-02
        NODE   35         0.00000E+00         0.94514E-02         -0.54951E-03
     FORCES ACTING ALONG THE 9 DOF
        NODE   33         0.00000E+00         0.69021E+01         -0.99895E+01
        NODE   34         0.00000E+00        -0.39885E-13          0.82379E-11
        NODE   35         0.00000E+00        -0.69021E+01          0.28710E+02

ELEMENT 17, FROM NODE 33, TO NODE 35 - LENGTH = 21.00 ft

 left half of span,at tenth points of length
                 span       span       span       span       span       span
             0.0        0.1        0.2        0.3        0.4        0.5
soil,k/ft      0.000      0.000      0.000      0.000      0.000      0.000
                                         Page 10
```

Output Page 10

```
                                             BBYOUT
shear,k        6.90      6.90      6.90      6.90      6.90      6.90
bmom,kft       9.99     11.17     12.46     13.87     15.43     17.14
tdisp,ft    0.02205   0.02072   0.01940   0.01809   0.01680   0.01553
axial,k        0.00 AT 1st END and        0.00 AT 2nd END

right half of span,at tenth points of length
               span      span      span      span      span      span
          0.5       0.6       0.7       0.8       0.9       1.0
soil,k/ft     0.000     0.000     0.000     0.000     0.000     0.000
shear,k        6.90      6.90      6.90      6.90      6.90      6.90
bmom,kft      17.14     19.03     21.10     23.39     25.92     28.71
tdisp,ft    0.01553   0.01427   0.01303   0.01181   0.01062   0.00945
axial,k        0.00 AT 1st END and        0.00 AT 2nd END

----------------------------------------------------------------------
     ELEMENT  18  DISPLACEMENTS IN INCIDENCES  35  36  37
        NODE  35          0.00000E+00          0.94514E-02          -0.54951E-03
        NODE  36          0.00000E+00          0.43154E-02          -0.79447E-03
        NODE  37          0.00000E+00          0.00000E+00          -0.31114E-03
     FORCES ACTING ALONG THE 9 DOF
        NODE  35          0.00000E+00          0.69021E+01          -0.28710E+02
        NODE  36          0.00000E+00          0.61062E-15          -0.29798E-11
        NODE  37          0.00000E+00         -0.69021E+01           0.78929E+02

ELEMENT 18, FROM NODE 35, TO NODE 37 - LENGTH = 21.00 ft

 left half of span,at tenth points of length
               span      span      span      span      span      span
          0.0       0.1       0.2       0.3       0.4       0.5
soil,k/ft     0.000     0.000     0.000     0.000     0.000     0.000
shear,k        6.90      6.90      6.90      6.90      6.90      6.90
bmom,kft      28.71     31.79     35.19     38.94     43.09     47.68
tdisp,ft    0.00945   0.00831   0.00721   0.00613   0.00510   0.00411
axial,k        0.00 AT 1st END and        0.00 AT 2nd END

right half of span,at tenth points of length
               span      span      span      span      span      span
          0.5       0.6       0.7       0.8       0.9       1.0
soil,k/ft     0.000     0.000     0.000     0.000     0.000     0.000
shear,k        6.90      6.90      6.90      6.90      6.90      6.90
bmom,kft      47.68     52.74     58.34     64.53     71.37     78.93
tdisp,ft    0.00411   0.00317   0.00228   0.00146   0.00069   0.00000
axial,k        0.00 AT 1st END and        0.00 AT 2nd END

----------------------------------------------------------------------
     ELEMENT  19  DISPLACEMENTS IN INCIDENCES  37  38  39
        NODE  37          0.00000E+00          0.00000E+00          -0.31114E-03
        NODE  38          0.00000E+00         -0.21499E-02          -0.28879E-03
        NODE  39          0.00000E+00         -0.35379E-02          -0.79102E-04
     FORCES ACTING ALONG THE 9 DOF
        NODE  37          0.00000E+00         -0.65774E+00          -0.78929E+02
        NODE  38          0.00000E+00          0.82531E-13           0.65675E-12
        NODE  39          0.00000E+00          0.65774E+00           0.30316E+02

ELEMENT 19, FROM NODE 37, TO NODE 39 - LENGTH = 20.00 ft

 left half of span,at tenth points of length
               span      span      span      span      span      span
          0.0       0.1       0.2       0.3       0.4       0.5
soil,k/ft     0.000     0.000     0.000     0.000     0.000     0.000
shear,k       -0.66     -0.66     -0.66     -0.66     -0.66     -0.66
bmom,kft      78.93     71.73     65.18     59.23     53.83     48.92
tdisp,ft    0.00000  -0.00059  -0.00111  -0.00157  -0.00198  -0.00234
                                            Page 11
```

48.92/120=0.41 ~ 0.66

Output Page 11

```
                                                    BBYOUT
axial,k        0.00 AT 1st END and      0.00 AT 2nd END

right half of span,at tenth points of length
              span      span      span      span      span      span
          0.5       0.6       0.7       0.8       0.9       1.0
soil,k/ft    0.000     0.000     0.000     0.000     0.000     0.000
shear,k      -0.66     -0.66     -0.66     -0.66     -0.66     -0.66
bmom,kft     48.92     44.45     40.40     36.71     33.36     30.32
tdisp,ft  -0.00234  -0.00265  -0.00293  -0.00316  -0.00337  -0.00354
axial,k        0.00 AT 1st END and      0.00 AT 2nd END

----------------------------------------------------------------------
     ELEMENT  20  DISPLACEMENTS IN INCIDENCES  39  40  41
        NODE  39         0.00000E+00        -0.35379E-02        -0.79102E-04
        NODE  40         0.00000E+00        -0.39596E-02        -0.43587E-04
        NODE  41         0.00000E+00        -0.40888E-02         0.10012E-04
     FORCES ACTING ALONG THE 9 DOF
        NODE  39         0.00000E+00        -0.65774E+00        -0.30316E+02
        NODE  40         0.00000E+00        -0.37181E-12        -0.12364E-11
        NODE  41         0.00000E+00         0.65774E+00         0.11640E+02

ELEMENT 20, FROM NODE 39, TO NODE 41 - LENGTH = 20.00 ft

 left half of span,at tenth points of length
              span      span      span      span      span      span
          0.0       0.1       0.2       0.3       0.4       0.5
soil,k/ft    0.000     0.000     0.000     0.000     0.000     0.000
shear,k      -0.66     -0.66     -0.66     -0.66     -0.66     -0.66
bmom,kft     30.32     27.55     25.03     22.75     20.67     18.79
tdisp,ft  -0.00354  -0.00368  -0.00380  -0.00390  -0.00397  -0.00403
axial,k        0.00 AT 1st END and      0.00 AT 2nd END

right half of span,at tenth points of length
              span      span      span      span      span      span
          0.5       0.6       0.7       0.8       0.9       1.0
soil,k/ft    0.000     0.000     0.000     0.000     0.000     0.000
shear,k      -0.66     -0.66     -0.66     -0.66     -0.66     -0.66
bmom,kft     18.79     17.07     15.51     14.10     12.81     11.64
tdisp,ft  -0.00403  -0.00407  -0.00410  -0.00411  -0.00410  -0.00409
axial,k        0.00 AT 1st END and      0.00 AT 2nd END

----------------------------------------------------------------------
     ELEMENT  21  DISPLACEMENTS IN INCIDENCES  41  42  43
        NODE  41         0.00000E+00        -0.40888E-02         0.10012E-04
        NODE  42         0.00000E+00        -0.38469E-02         0.50560E-04
        NODE  43         0.00000E+00        -0.34927E-02         0.44206E-04
     FORCES ACTING ALONG THE 9 DOF
        NODE  41         0.00000E+00        -0.65774E+00        -0.11640E+02
        NODE  42         0.00000E+00        -0.18067E-12         0.41511E-11
        NODE  43         0.00000E+00         0.65774E+00         0.44588E+01

ELEMENT 21, FROM NODE 41, TO NODE 43 - LENGTH = 20.00 ft

 left half of span,at tenth points of length
              span      span      span      span      span      span
          0.0       0.1       0.2       0.3       0.4       0.5
soil,k/ft    0.000     0.000     0.000     0.000     0.000     0.000
shear,k      -0.66     -0.66     -0.66     -0.66     -0.66     -0.66
bmom,kft     11.64     10.58      9.61      8.73      7.93      7.21
tdisp,ft  -0.00409  -0.00406  -0.00403  -0.00399  -0.00393  -0.00387
axial,k        0.00 AT 1st END and      0.00 AT 2nd END

right half of span,at tenth points of length
                                                   Page 12
```

Output Page 12

```
                                          BBYOUT
                span      span      span      span      span      span
            0.5       0.6       0.7       0.8       0.9       1.0
soil,k/ft     0.000     0.000     0.000     0.000     0.000     0.000
shear,k       -0.66     -0.66     -0.66     -0.66     -0.66     -0.66
bmom,kft       7.21      6.55      5.95      5.40      4.91      4.46
tdisp,ft   -0.00387  -0.00381  -0.00374  -0.00366  -0.00358  -0.00349
axial,k         0.00 AT 1st END and       0.00 AT 2nd END

----------------------------------------------------------------------
     ELEMENT  22  DISPLACEMENTS IN INCIDENCES  43  44  45
        NODE  43         0.00000E+00         -0.34927E-02         0.44206E-04
        NODE  44         0.00000E+00         -0.29965E-02         0.86625E-04
        NODE  45         0.00000E+00         -0.24574E-02         0.57247E-04
     FORCES ACTING ALONG THE 9 DOF
        NODE  43         0.00000E+00         -0.65774E+00        -0.44588E+01
        NODE  44         0.00000E+00         -0.31086E-14         0.35572E-11
        NODE  45         0.00000E+00          0.65774E+00         0.16808E+01

ELEMENT 22, FROM NODE 43, TO NODE 45 - LENGTH = 20.00 ft

 left half of span,at tenth points of length
                span      span      span      span      span      span
            0.0       0.1       0.2       0.3       0.4       0.5
soil,k/ft     0.000     0.000     0.000     0.000     0.000     0.000
shear,k       -0.66     -0.66     -0.66     -0.66     -0.66     -0.66
bmom,kft       4.46      4.05      3.68      3.34      3.03      2.75
tdisp,ft   -0.00349  -0.00340  -0.00331  -0.00321  -0.00311  -0.00301
axial,k         0.00 AT 1st END and       0.00 AT 2nd END

right half of span,at tenth points of length
                span      span      span      span      span      span
            0.5       0.6       0.7       0.8       0.9       1.0
soil,k/ft     0.000     0.000     0.000     0.000     0.000     0.000
shear,k       -0.66     -0.66     -0.66     -0.66     -0.66     -0.66
bmom,kft       2.75      2.49      2.26      2.05      1.86      1.68
tdisp,ft   -0.00301  -0.00290  -0.00279  -0.00268  -0.00257  -0.00246
axial,k         0.00 AT 1st END and       0.00 AT 2nd END

----------------------------------------------------------------------
     ELEMENT  23  DISPLACEMENTS IN INCIDENCES  45  46  47
        NODE  45         0.00000E+00         -0.24574E-02         0.57247E-04
        NODE  46         0.00000E+00         -0.18647E-02         0.10022E-03
        NODE  47         0.00000E+00         -0.12564E-02         0.62012E-04
     FORCES ACTING ALONG THE 9 DOF
        NODE  45         0.00000E+00         -0.65774E+00        -0.16808E+01
        NODE  46         0.00000E+00          0.49689E-13        -0.59464E-12
        NODE  47         0.00000E+00          0.65774E+00         0.56262E+00

ELEMENT 23, FROM NODE 45, TO NODE 47 - LENGTH = 20.00 ft

 left half of span,at tenth points of length
                span      span      span      span      span      span
            0.0       0.1       0.2       0.3       0.4       0.5
soil,k/ft     0.000     0.000     0.000     0.000     0.000     0.000
shear,k       -0.66     -0.66     -0.66     -0.66     -0.66     -0.66
bmom,kft       1.68      1.52      1.37      1.24      1.12      1.00
tdisp,ft   -0.00246  -0.00234  -0.00223  -0.00211  -0.00199  -0.00187
axial,k         0.00 AT 1st END and       0.00 AT 2nd END

right half of span,at tenth points of length
                span      span      span      span      span      span
            0.5       0.6       0.7       0.8       0.9       1.0
soil,k/ft     0.000     0.000     0.000     0.000     0.000     0.000
                                         Page 13
```

Output Page 13

```
                                        BBYOUT
shear,k        -0.66     -0.66     -0.66     -0.66     -0.66     -0.66
bmom,kft        1.00      0.90      0.81      0.72      0.64      0.56
tdisp,ft    -0.00187  -0.00175  -0.00163  -0.00150  -0.00138  -0.00126
axial,k         0.00 AT 1st END and       0.00 AT 2nd END

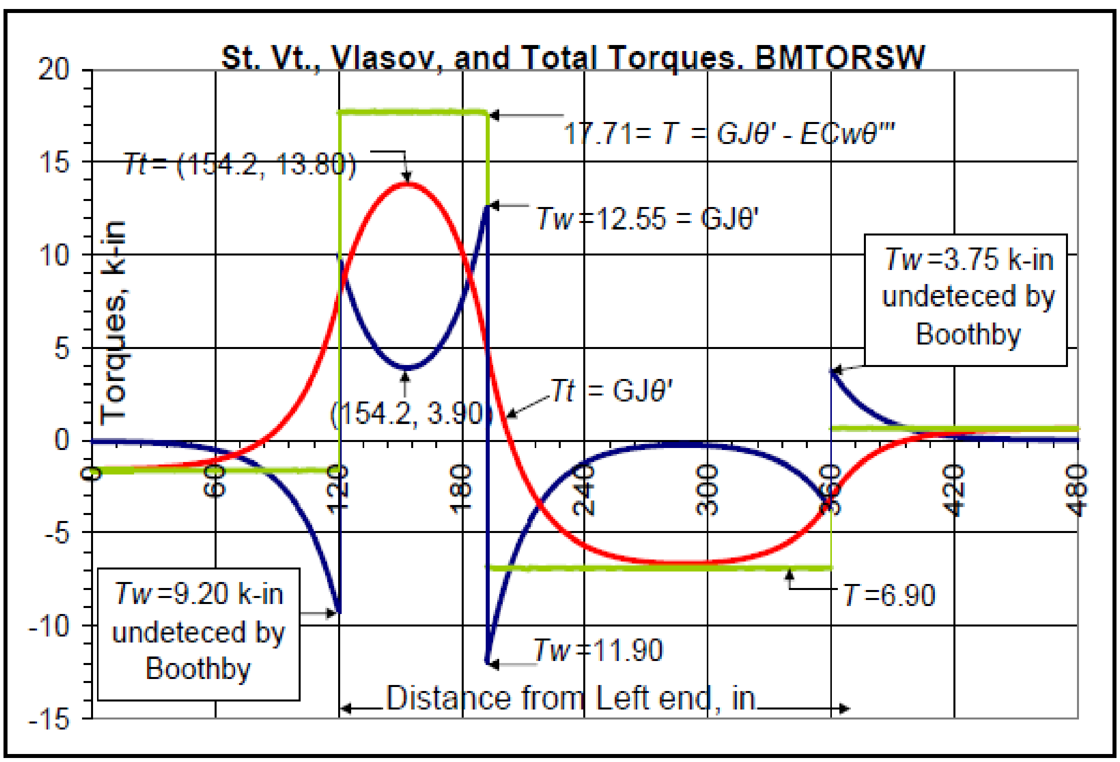
----------------------------------------------------------------------
     ELEMENT  24  DISPLACEMENTS IN INCIDENCES  47  48  49
        NODE  47         0.00000E+00         -0.12564E-02         0.62012E-04
        NODE  48         0.00000E+00         -0.63016E-03         0.10477E-03
        NODE  49         0.00000E+00          0.00000E+00         0.63207E-04
     FORCES ACTING ALONG THE 9 DOF
        NODE  47         0.00000E+00         -0.65774E+00        -0.56262E+00
        NODE  48         0.00000E+00          0.27485E-13         0.29610E-12
        NODE  49         0.00000E+00          0.65774E+00         0.26197E-12

ELEMENT 24, FROM NODE 47, TO NODE 49 - LENGTH = 20.00 ft

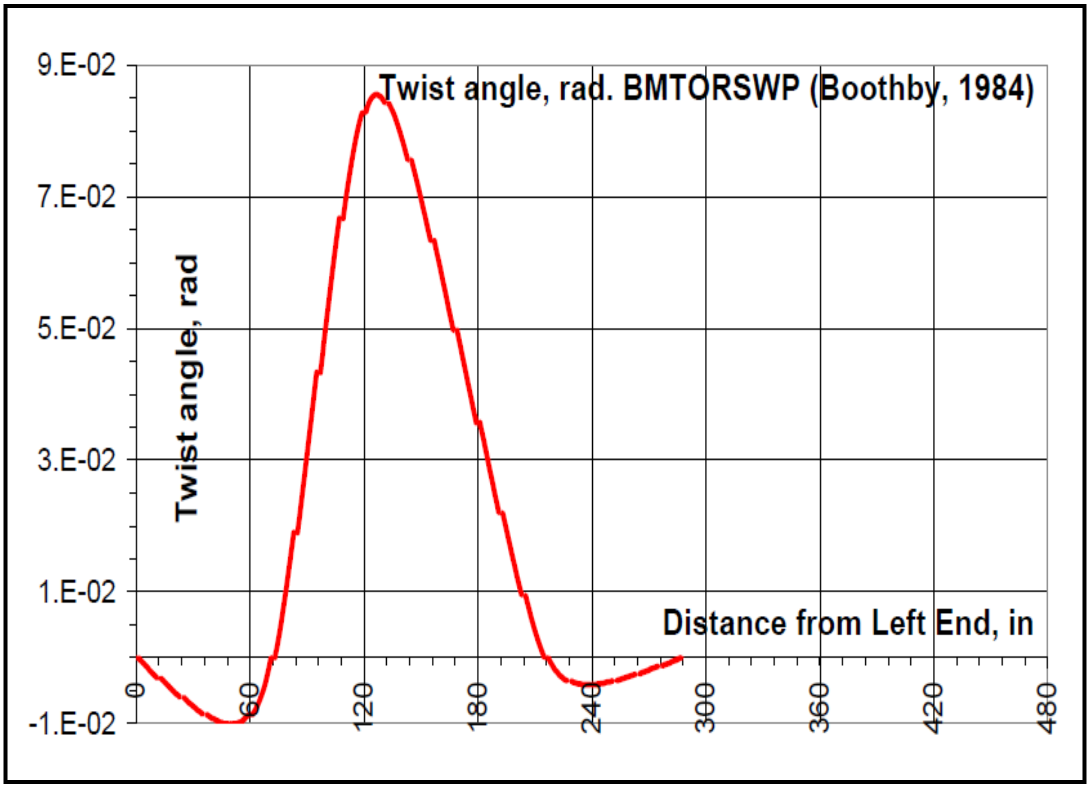
 left half of span,at tenth points of length
             span      span      span      span      span      span
          0.0       0.1       0.2       0.3       0.4       0.5
soil,k/ft      0.000     0.000     0.000     0.000     0.000     0.000
shear,k        -0.66     -0.66     -0.66     -0.66     -0.66     -0.66
bmom,kft        0.56      0.49      0.43      0.37      0.31      0.25
tdisp,ft    -0.00126  -0.00113  -0.00101  -0.00088  -0.00076  -0.00063
axial,k         0.00 AT 1st END and       0.00 AT 2nd END

right half of span,at tenth points of length
             span      span      span      span      span      span
          0.5       0.6       0.7       0.8       0.9       1.0
soil,k/ft      0.000     0.000     0.000     0.000     0.000     0.000
shear,k        -0.66     -0.66     -0.66     -0.66     -0.66     -0.66
bmom,kft        0.25      0.20      0.15      0.10      0.05      0.00
tdisp,ft    -0.00063  -0.00051  -0.00038  -0.00025  -0.00013   0.00000
axial,k         0.00 AT 1st END and       0.00 AT 2nd END
```

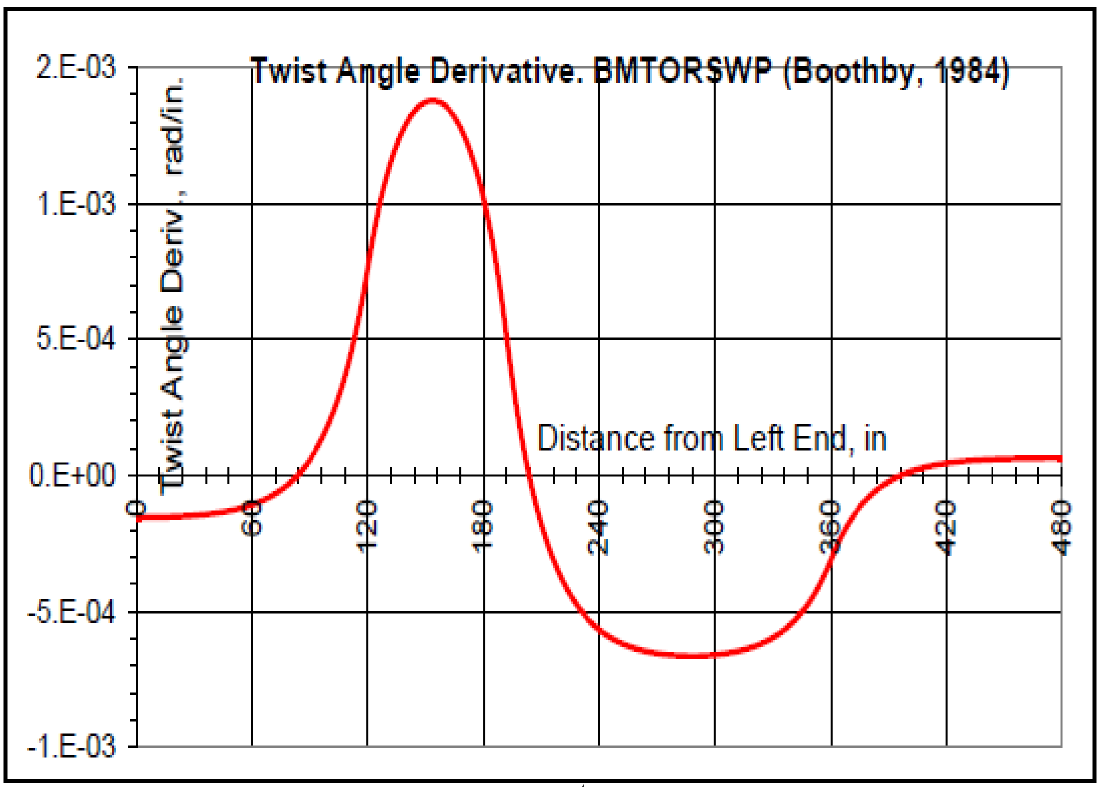
Output Final Page



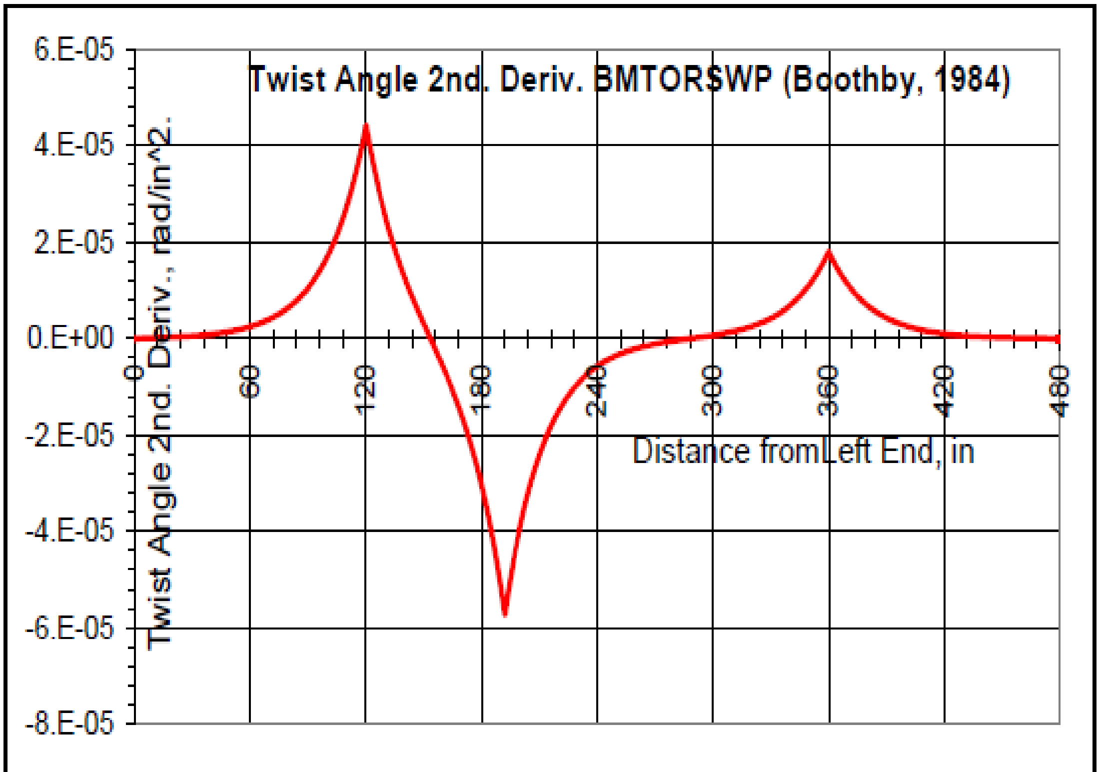

Uniform, Non Uniform and Total Torques from BMTORSWP



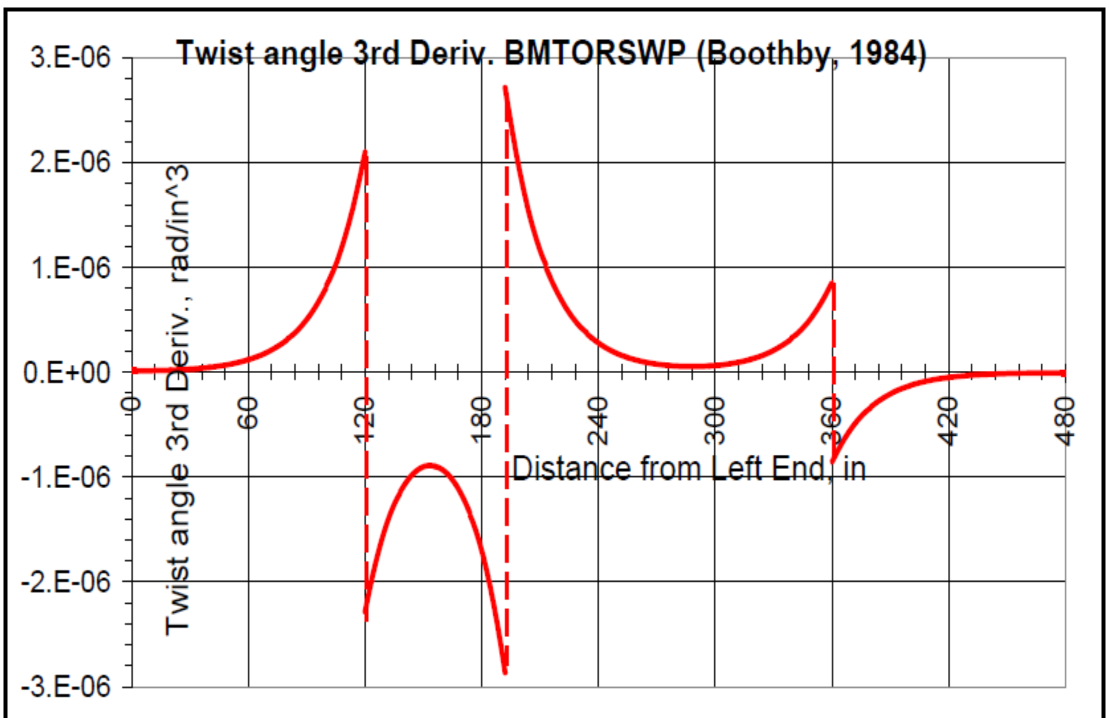

Torque Angle from BMTORSWP



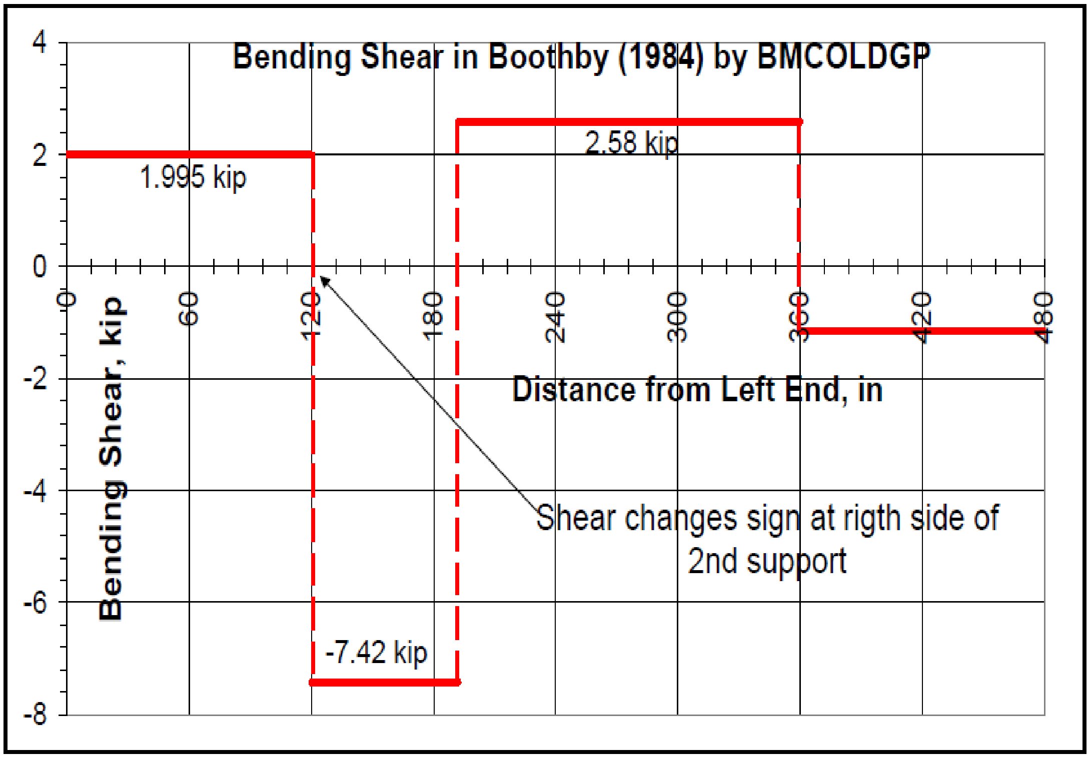

Torque Angle 1st Derivative



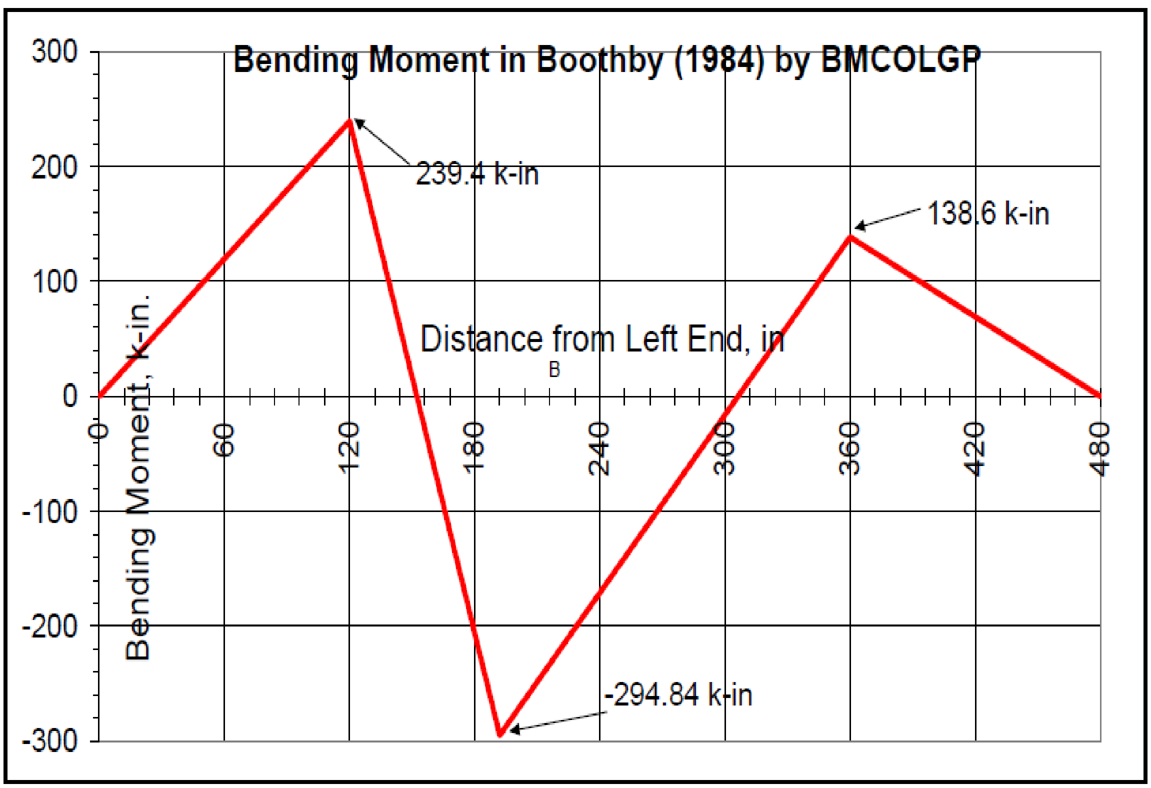

Torque Angle 2nd Derivative



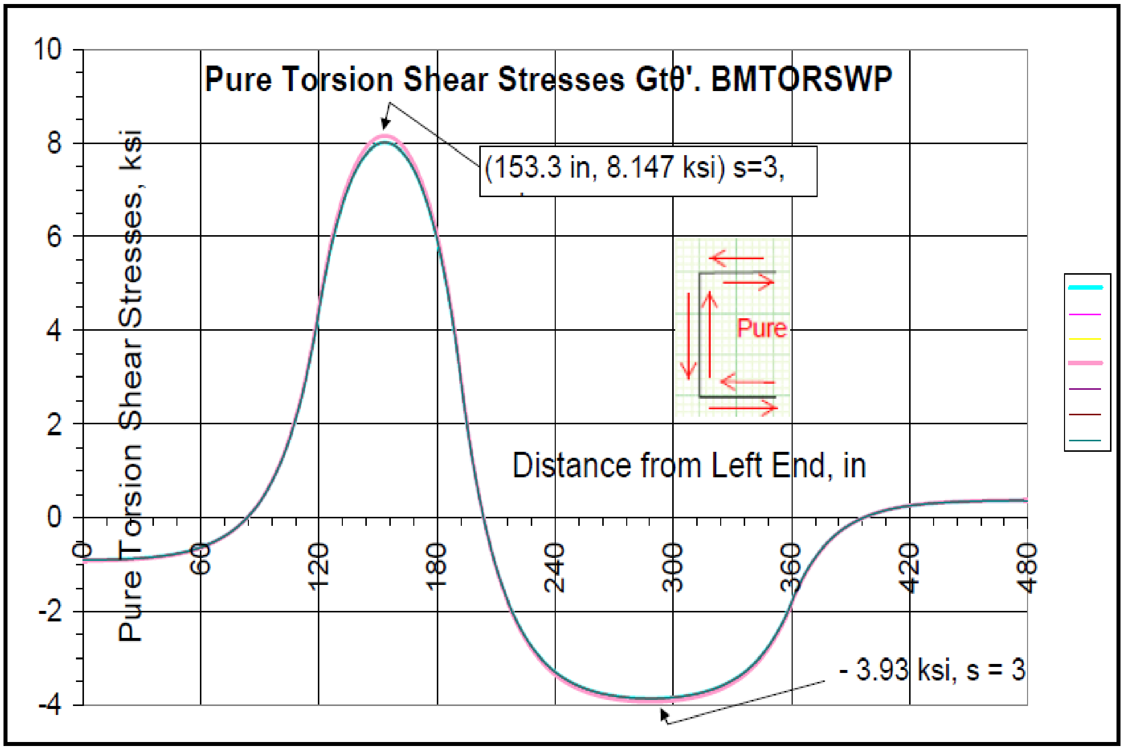

Torque Angle 3rd Derivative



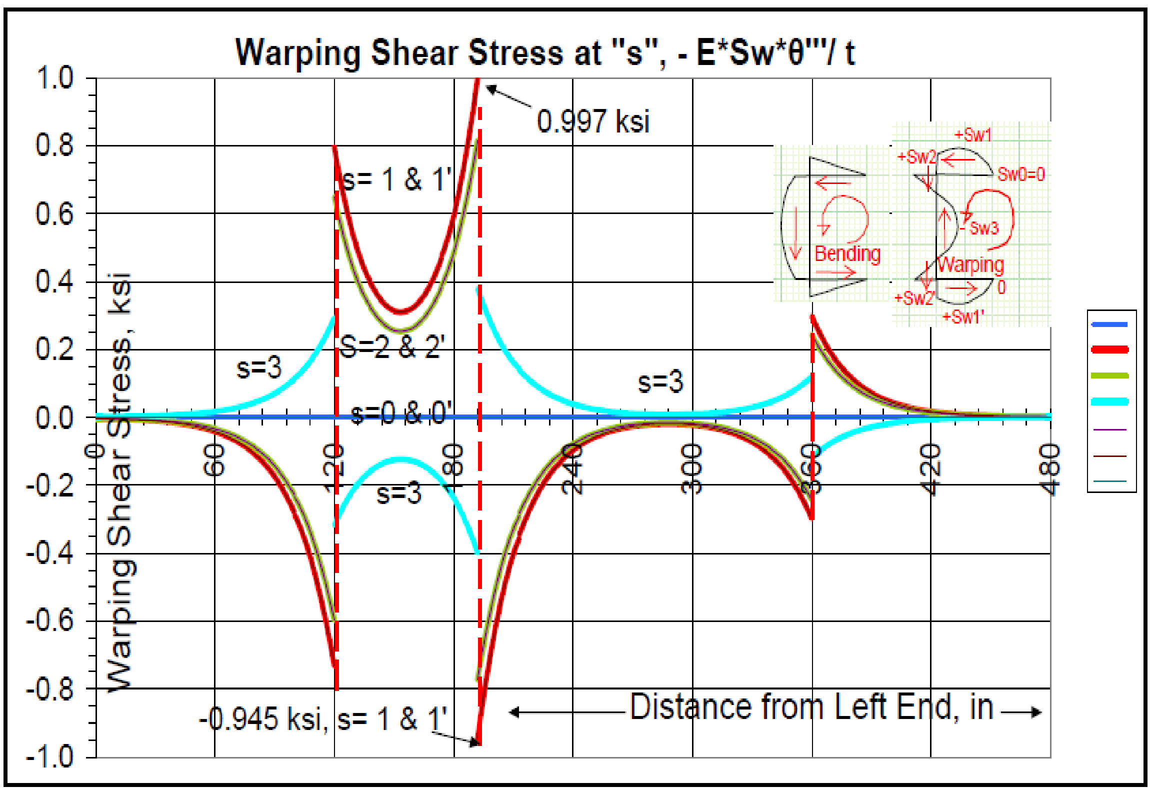

Bending Shear from BMCOLDGP



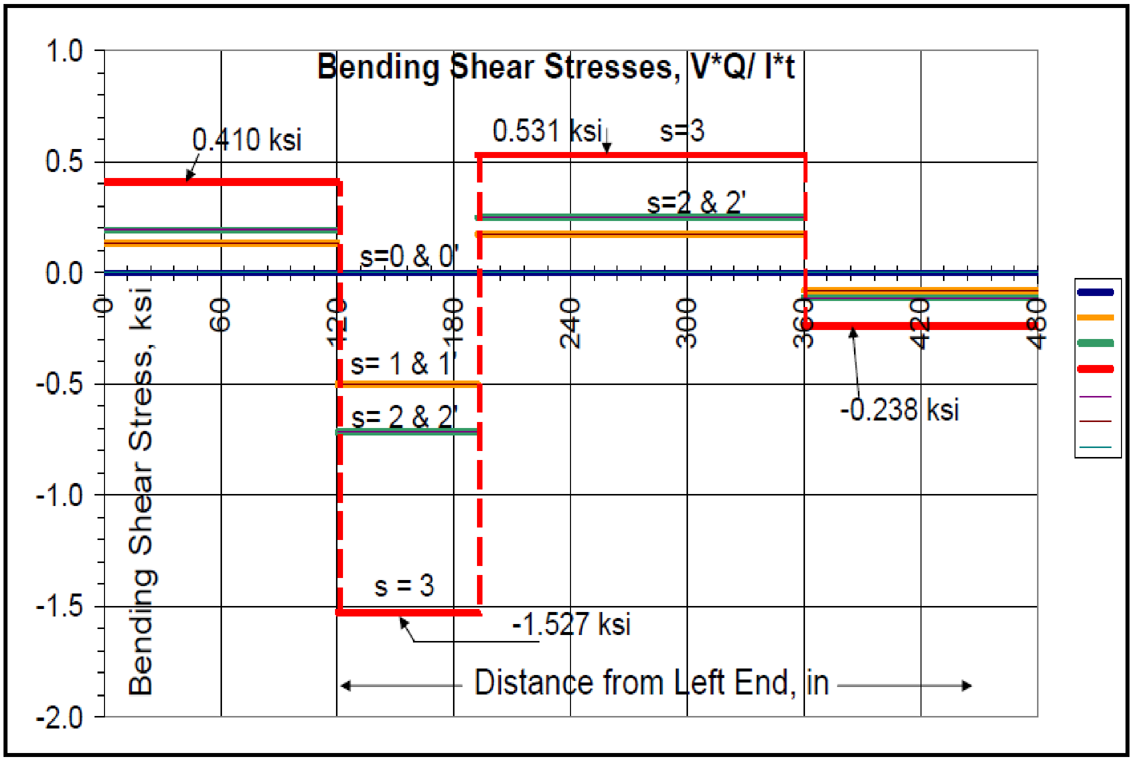

Bending Moment from BMCOLGP



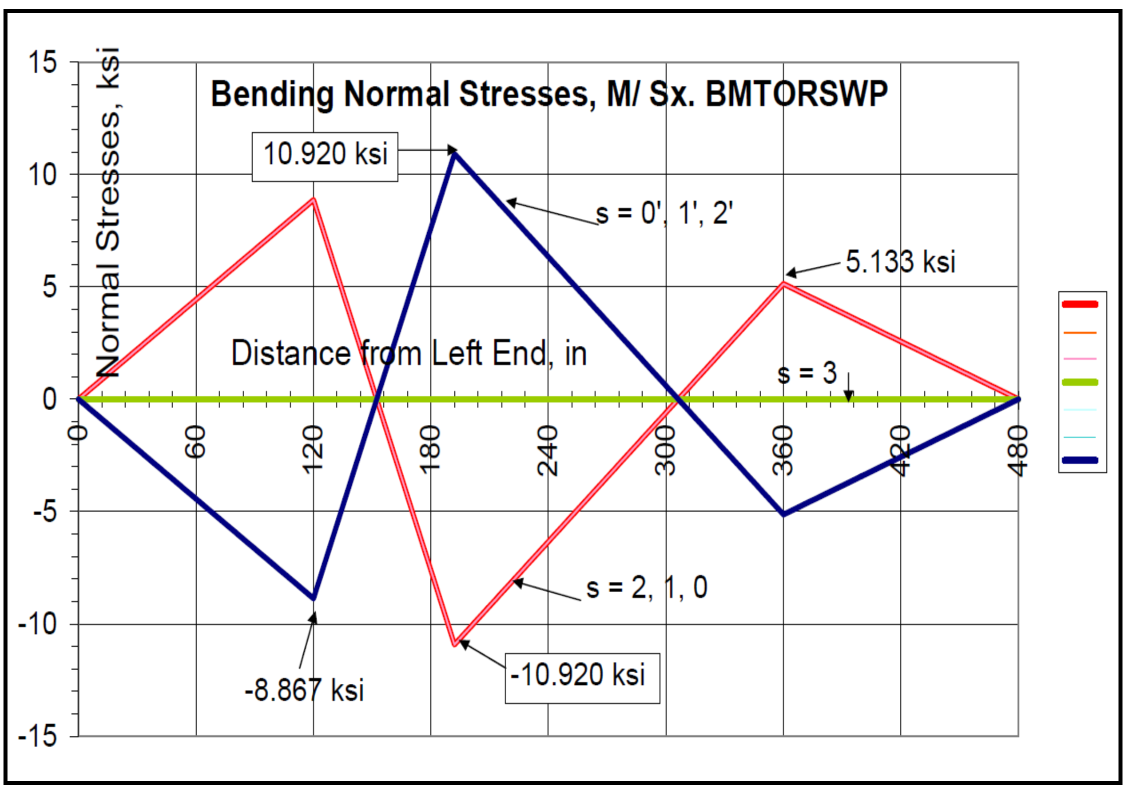

Pure Torsion Shear Stresses at s= 0, 1, 2, 3, 2', 1' and 0'



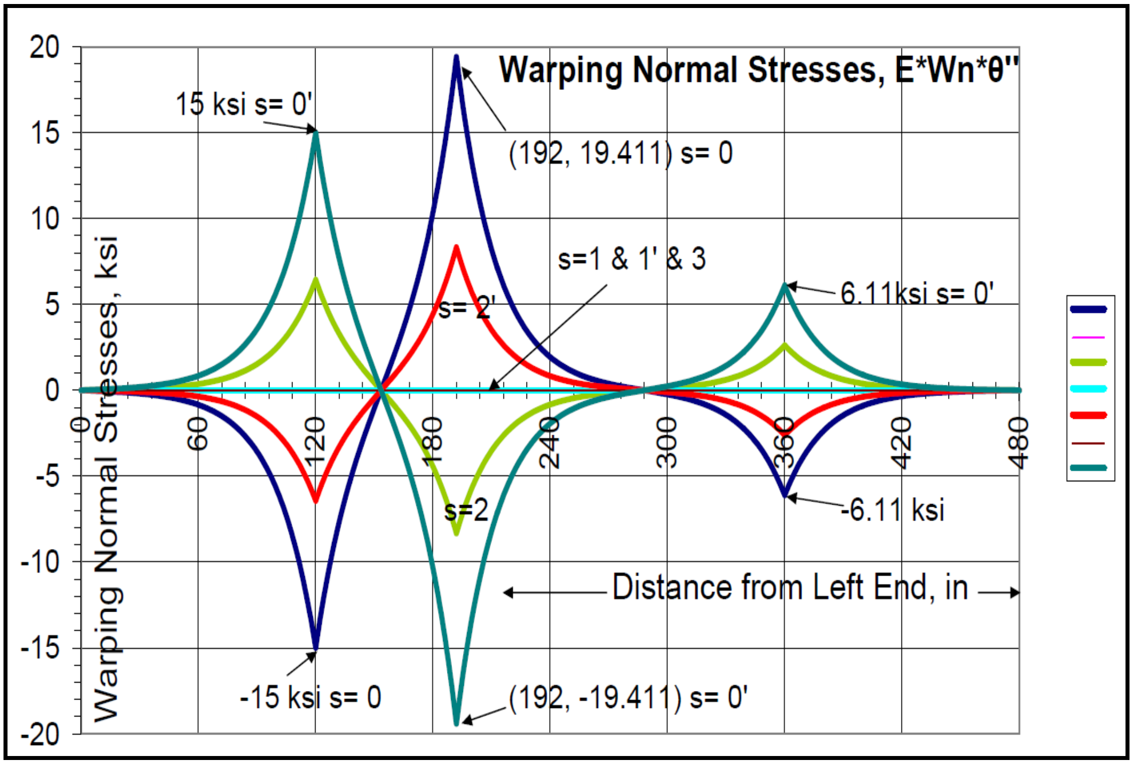

Warping Shear Stresses along the Spans and Profile Points, Legend in Order (0, 1, 2, 3, 2', 1', 0')



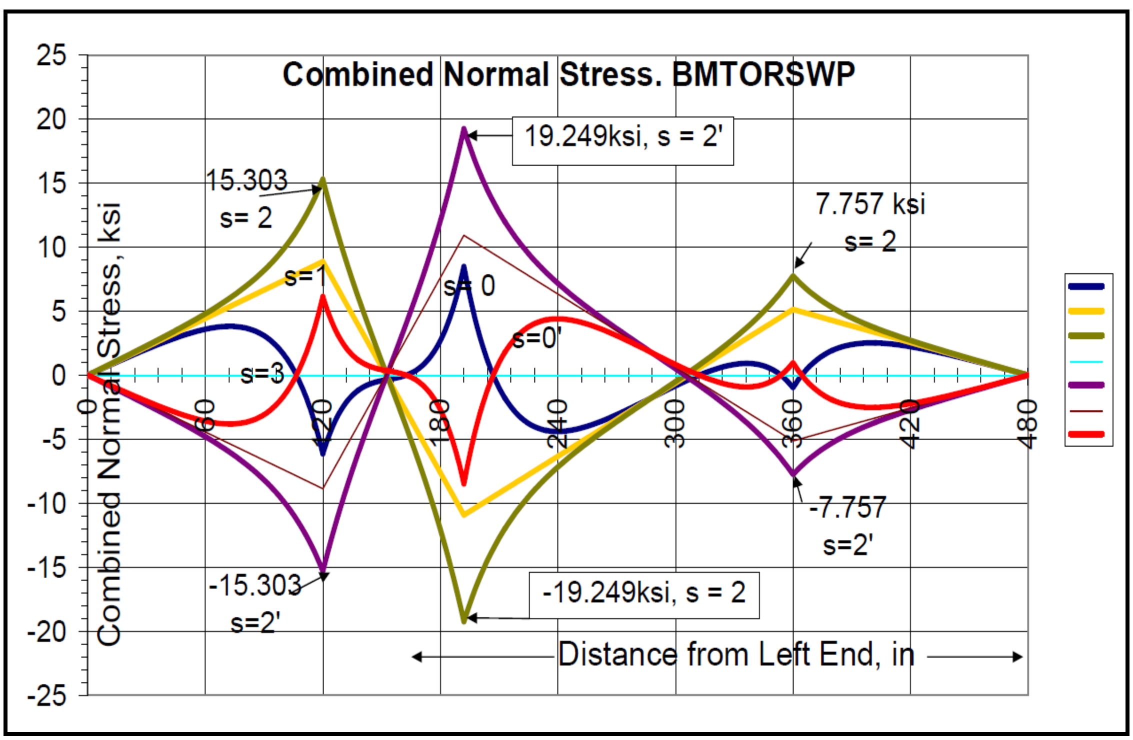

Bending Shear Stresses along the Spans and Profile Points, Legend in Order



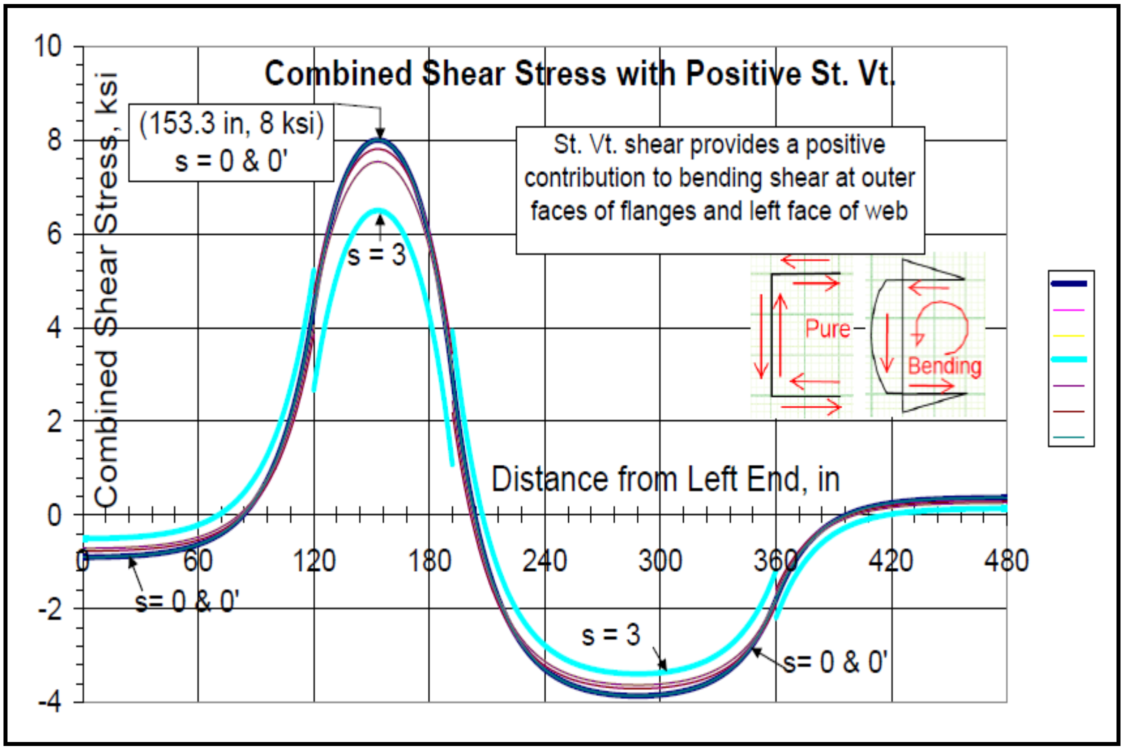

Bending Normal Stresses along the Spans and Profile Points, Legend in Order (0, 1, 2, 3, 2', 1', 0')



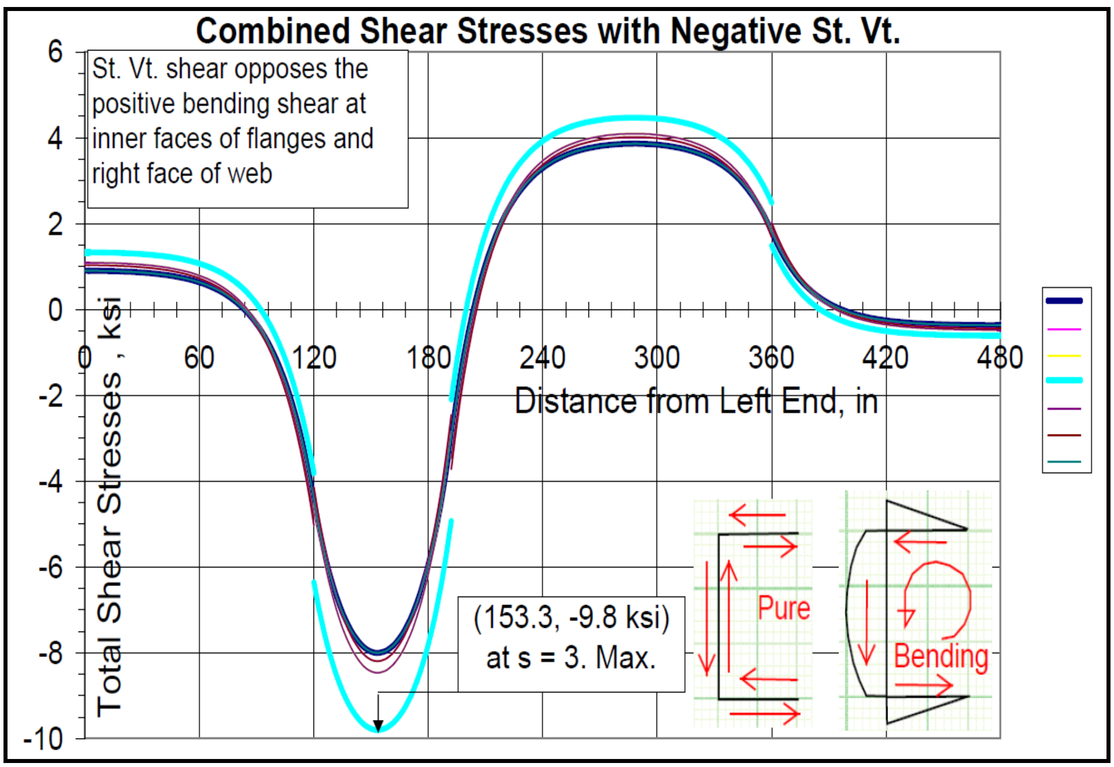

Warping Normal Stresses along the Spans and Profile Points, Legend in Order (0, 1, 2, 3, 2', 1', 0')




Combined Normal Stress along the Spans and Profile Points, Legend in Order (0, 1, 2, 3, 2', 1', 0')




First Case of Combined Shear Stress along the Spans and Profile Points, Legend in Order (0, 1, 2, 3, 2', 1', 0')




Second Case of Combined Shear Stress along the Spans and Profile Points, Legend in Order (0, 1, 2, 3, 2', 1', 0')

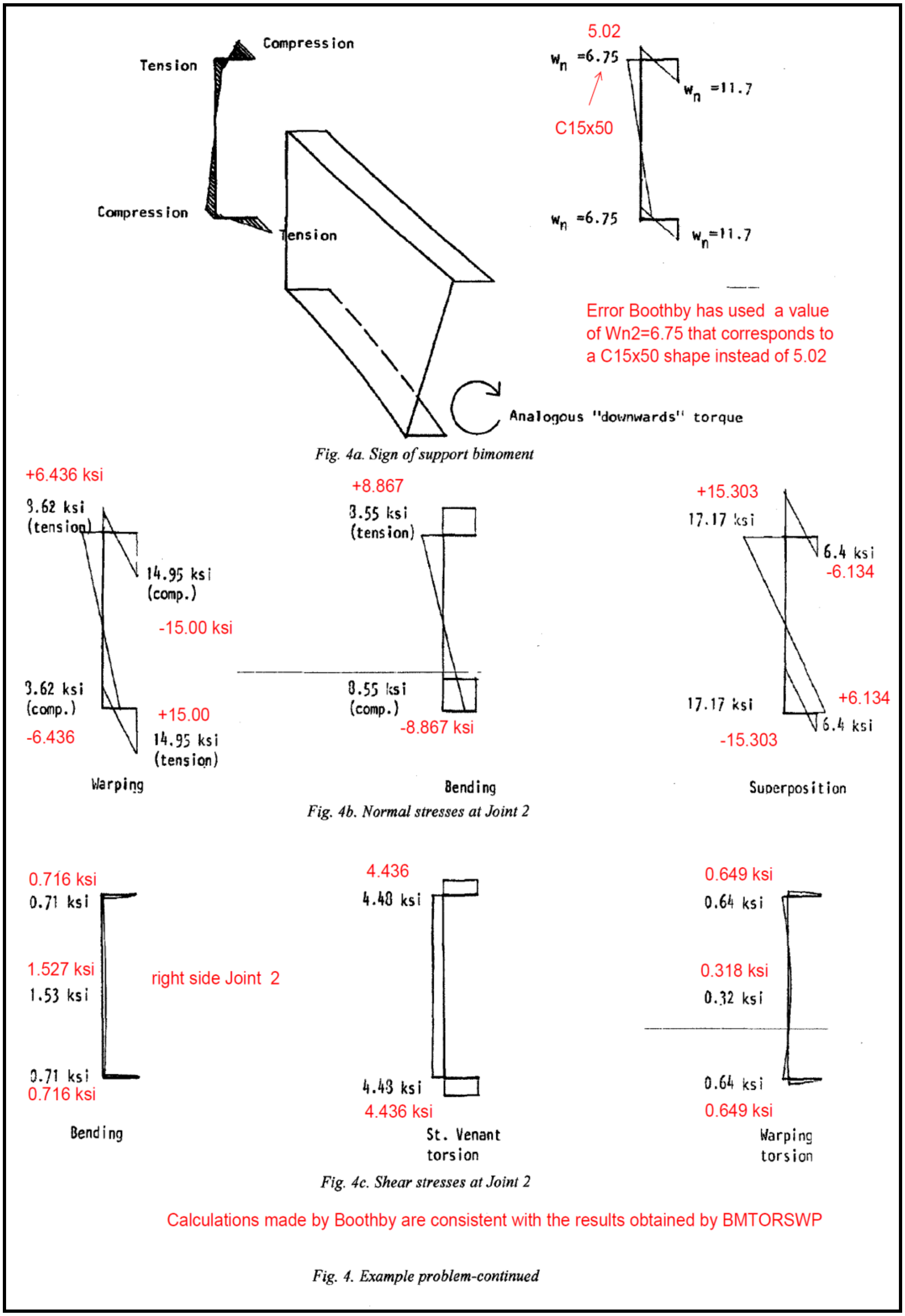


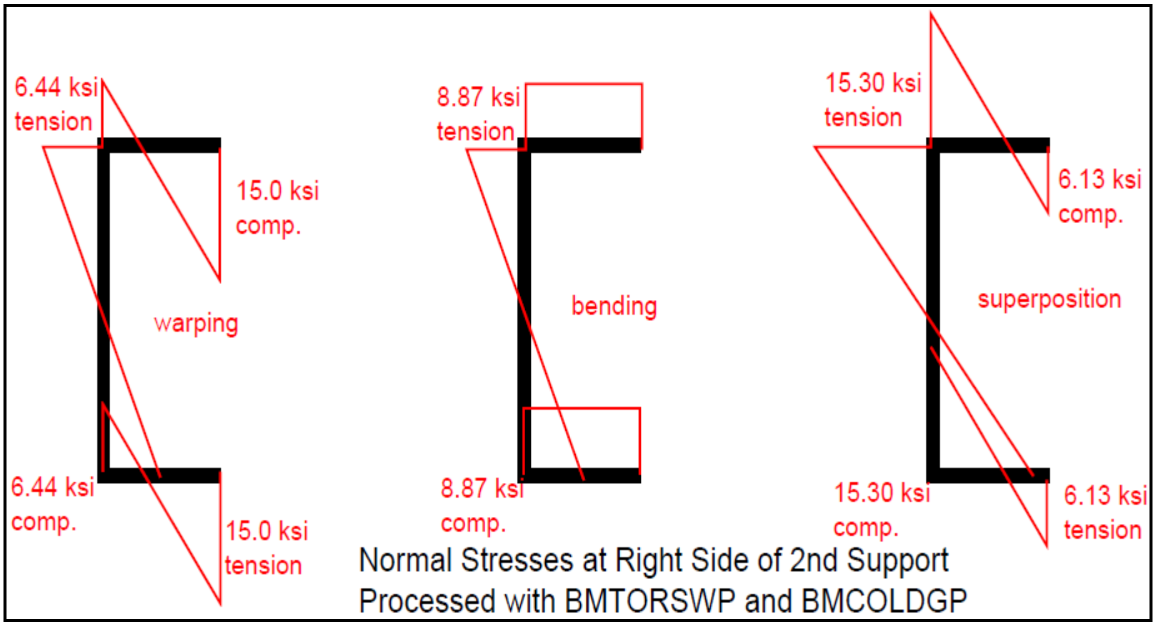

*Fig. 4a. Sign of support bimoment*

*Fig. 4b. Normal stresses at Joint 2*

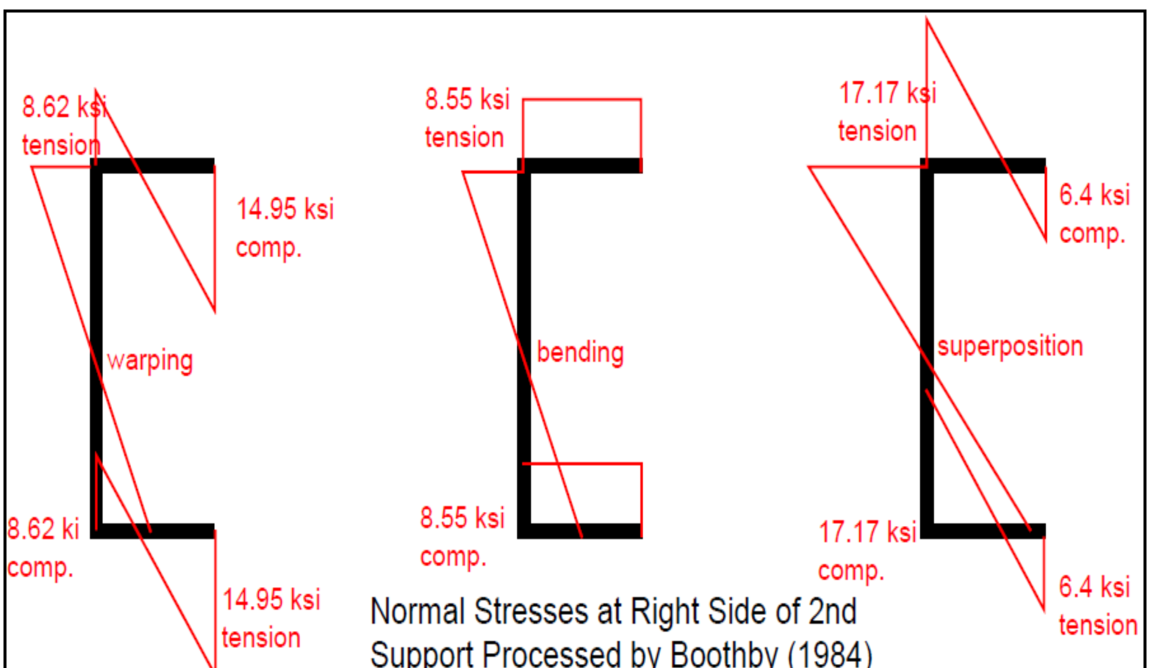
*Fig. 4c. Shear stresses at Joint 2*

*Fig. 4. Example problem-continued*

BMTORSWP and Boothby's Calculations for Right Side of First Interior Support




Another View of the Normal Stresses at Right Side of 2nd Support Processed with BMTORSWP and BMCOLDGP




Normal Stresses at Right Side of 2nd Support Processed by Boothby (1984)

The error committed by Boothby for the sectorial coordinate at the flange-web corner is illustrated in the previous page. Regarding shear stresses, no discrepancies were found as shown in the same page.

| Torques and Torque Angles and Derivatives and Bimoment | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|
| z | *Tw* | *Tt* | *T* | El | θ | θ' | θ'' | θ''' | B(z) |
| | -ECwθ''' + GJθ' = T | | | 1 | | | | | Bim |
| 0.0 | -0.060 | -1.554 | -1.614 | | 0.00E+00 | -1.551E-04 | -6.305E-11 | 1.365E-08 | 0.00 |
| 2.0 | -0.060 | -1.553 | -1.613 | | -3.00E-04 | -1.550E-04 | 2.723E-08 | 1.367E-08 | 0.12 |
| 4.0 | -0.061 | -1.552 | -1.613 | | -6.00E-04 | -1.549E-04 | 5.471E-08 | 1.384E-08 | 0.24 |
| 6.0 | -0.062 | -1.551 | -1.613 | | -9.00E-04 | -1.548E-04 | 8.267E-08 | 1.414E-08 | 0.36 |
| 8.0 | -0.064 | -1.549 | -1.613 | | -1.20E-03 | -1.546E-04 | 1.114E-07 | 1.459E-08 | 0.49 |
| 10.0 | -0.066 | -1.547 | -1.614 | | -1.50E-03 | -1.544E-04 | 1.411E-07 | 1.517E-08 | 0.62 |
| 12.0 | -0.070 | -1.543 | -1.613 | | -1.90E-03 | -1.540E-04 | 1.721E-07 | 1.590E-08 | 0.75 |
| 14.0 | -0.073 | -1.540 | -1.614 | | -2.20E-03 | -1.537E-04 | 2.048E-07 | 1.677E-08 | 0.90 |
| 16.0 | -0.078 | -1.535 | -1.613 | | -2.50E-03 | -1.532E-04 | 2.393E-07 | 1.777E-08 | 1.05 |
| 18.0 | -0.083 | -1.530 | -1.613 | | -2.80E-03 | -1.527E-04 | 2.760E-07 | 1.892E-08 | 1.21 |
| 20.0 | -0.088 | -1.524 | -1.613 | | -3.10E-03 | -1.521E-04 | 3.151E-07 | 2.021E-08 | 1.38 |
| | | | | **2** | | | | | |
| 20.0 | -0.090 | -1.524 | -1.614 | | -3.10E-03 | -1.521E-04 | 3.149E-07 | 2.056E-08 | 1.38 |
| 22.0 | -0.096 | -1.518 | -1.614 | | -3.40E-03 | -1.515E-04 | 3.573E-07 | 2.192E-08 | 1.56 |
| 24.0 | -0.103 | -1.510 | -1.614 | | -3.70E-03 | -1.507E-04 | 4.028E-07 | 2.356E-08 | 1.76 |
| 26.0 | -0.112 | -1.501 | -1.613 | | -4.00E-03 | -1.498E-04 | 4.518E-07 | 2.548E-08 | 1.98 |
| 28.0 | -0.121 | -1.492 | -1.614 | | -4.30E-03 | -1.489E-04 | 5.049E-07 | 2.768E-08 | 2.21 |
| 30.0 | -0.132 | -1.481 | -1.613 | | -4.60E-03 | -1.478E-04 | 5.626E-07 | 3.015E-08 | 2.46 |
| 32.0 | -0.144 | -1.469 | -1.613 | | -4.90E-03 | -1.466E-04 | 6.257E-07 | 3.291E-08 | 2.74 |
| 34.0 | -0.157 | -1.456 | -1.614 | | -5.20E-03 | -1.453E-04 | 6.945E-07 | 3.595E-08 | 3.04 |
| 36.0 | -0.172 | -1.441 | -1.613 | | -5.50E-03 | -1.438E-04 | 7.697E-07 | 3.927E-08 | 3.37 |
| 38.0 | -0.188 | -1.425 | -1.613 | | -5.70E-03 | -1.422E-04 | 8.517E-07 | 4.286E-08 | 3.73 |
| 40.0 | -0.205 | -1.407 | -1.612 | | -6.00E-03 | -1.404E-04 | 9.413E-07 | 4.674E-08 | 4.12 |
| | | | | **3** | | | | | |
| 40.0 | -0.209 | -1.407 | -1.616 | | -6.00E-03 | -1.404E-04 | 9.409E-07 | 4.777E-08 | 4.12 |
| 42.0 | -0.227 | -1.388 | -1.615 | | -6.30E-03 | -1.385E-04 | 1.040E-06 | 5.181E-08 | 4.55 |
| 44.0 | -0.248 | -1.366 | -1.614 | | -6.60E-03 | -1.363E-04 | 1.149E-06 | 5.654E-08 | 5.03 |
| 46.0 | -0.271 | -1.342 | -1.613 | | -6.90E-03 | -1.339E-04 | 1.267E-06 | 6.197E-08 | 5.55 |
| 48.0 | -0.298 | -1.315 | -1.613 | | -7.10E-03 | -1.312E-04 | 1.397E-06 | 6.810E-08 | 6.12 |
| 50.0 | -0.328 | -1.286 | -1.614 | | -7.40E-03 | -1.283E-04 | 1.540E-06 | 7.491E-08 | 6.74 |
| 52.0 | -0.361 | -1.253 | -1.614 | | -7.60E-03 | -1.250E-04 | 1.697E-06 | 8.243E-08 | 7.43 |
| 54.0 | -0.397 | -1.218 | -1.615 | | -7.90E-03 | -1.215E-04 | 1.870E-06 | 9.063E-08 | 8.19 |
| 56.0 | -0.436 | -1.178 | -1.614 | | -8.10E-03 | -1.175E-04 | 2.060E-06 | 9.954E-08 | 9.02 |
| 58.0 | -0.478 | -1.135 | -1.612 | | -8.30E-03 | -1.132E-04 | 2.269E-06 | 1.091E-07 | 9.93 |
| 60.0 | -0.523 | -1.086 | -1.609 | | -8.60E-03 | -1.084E-04 | 2.497E-06 | 1.194E-07 | 10.94 |
| | | | | **4** | | | | | |
| 60.0 | -0.535 | -1.086 | -1.622 | | -8.60E-03 | -1.084E-04 | 2.496E-06 | 1.222E-07 | 10.94 |
| 62.0 | -0.582 | -1.034 | -1.616 | | -8.80E-03 | -1.032E-04 | 2.751E-06 | 1.329E-07 | 12.04 |
| 64.0 | -0.637 | -0.976 | -1.613 | | -9.00E-03 | -9.742E-05 | 3.029E-06 | 1.454E-07 | 13.26 |
| 66.0 | -0.699 | -0.913 | -1.612 | | -9.20E-03 | -9.107E-05 | 3.333E-06 | 1.597E-07 | 14.60 |
| 68.0 | -0.770 | -0.843 | -1.612 | | -9.30E-03 | -8.407E-05 | 3.668E-06 | 1.758E-07 | 16.07 |

(continued) Torques and Torque Angles and Derivatives and Bimoment

| z | *Tw* | *Tt* | *T* | Elm | θ | θ' | θ'' | θ''' | B(z) |
|---|---|---|---|---|---|---|---|---|---|
| 70.0 | -0.848 | -0.765 | -1.614 | | -9.50E-03 | -7.637E-05 | 4.038E-06 | 1.937E-07 | 17.68 |
| 72.0 | -0.934 | -0.681 | -1.615 | | -9.60E-03 | -6.790E-05 | 4.444E-06 | 2.133E-07 | 19.46 |
| 74.0 | -1.028 | -0.587 | -1.615 | | -9.80E-03 | -5.857E-05 | 4.892E-06 | 2.348E-07 | 21.42 |
| 76.0 | -1.130 | -0.484 | -1.614 | | -9.90E-03 | -4.830E-05 | 5.385E-06 | 2.581E-07 | 23.58 |
| 78.0 | -1.240 | -0.371 | -1.611 | | -1.00E-02 | -3.700E-05 | 5.926E-06 | 2.832E-07 | 25.95 |
| 80.0 | -1.357 | -0.246 | -1.604 | | -1.00E-02 | -2.456E-05 | 6.519E-06 | 3.100E-07 | 28.55 |
| | | | | **5** | | | | | |
| 80.0 | -1.389 | -0.246 | -1.635 | | -1.00E-02 | -2.456E-05 | 6.516E-06 | 3.172E-07 | 28.55 |
| 82.0 | -1.511 | -0.109 | -1.620 | | -1.01E-02 | -1.088E-05 | 7.177E-06 | 3.451E-07 | 31.42 |
| 84.0 | -1.654 | 0.042 | -1.612 | | -1.01E-02 | 4.189E-06 | 7.899E-06 | 3.777E-07 | 34.58 |
| 86.0 | -1.817 | 0.208 | -1.609 | | -1.00E-02 | 2.077E-05 | 8.691E-06 | 4.150E-07 | 38.06 |
| 88.0 | -2.001 | 0.391 | -1.610 | | -1.00E-02 | 3.901E-05 | 9.563E-06 | 4.570E-07 | 41.88 |
| 90.0 | -2.205 | 0.592 | -1.613 | | -9.90E-03 | 5.908E-05 | 1.052E-05 | 5.036E-07 | 46.09 |
| 92.0 | -2.430 | 0.813 | -1.616 | | -9.80E-03 | 8.116E-05 | 1.158E-05 | 5.549E-07 | 50.72 |
| 94.0 | -2.675 | 1.057 | -1.618 | | -9.60E-03 | 1.055E-04 | 1.275E-05 | 6.109E-07 | 55.81 |
| 96.0 | -2.940 | 1.325 | -1.616 | | -9.30E-03 | 1.322E-04 | 1.403E-05 | 6.715E-07 | 61.41 |
| 98.0 | -3.227 | 1.621 | -1.606 | | -9.00E-03 | 1.617E-04 | 1.543E-05 | 7.369E-07 | 67.58 |
| 100.0 | -3.533 | 1.944 | -1.589 | | -8.70E-03 | 1.940E-04 | 1.698E-05 | 8.068E-07 | 74.37 |
| | | | | **6** | | | | | |
| 100.0 | -3.614 | 1.944 | -1.670 | | -8.70E-03 | 1.940E-04 | 1.697E-05 | 8.254E-07 | 74.37 |
| 102.0 | -3.933 | 2.302 | -1.631 | | -8.30E-03 | 2.297E-04 | 1.869E-05 | 8.982E-07 | 81.84 |
| 104.0 | -4.305 | 2.695 | -1.610 | | -7.80E-03 | 2.689E-04 | 2.057E-05 | 9.831E-07 | 90.05 |
| 106.0 | -4.729 | 3.128 | -1.601 | | -7.20E-03 | 3.121E-04 | 2.263E-05 | 1.080E-06 | 99.10 |
| 108.0 | -5.211 | 3.604 | -1.607 | | -6.50E-03 | 3.596E-04 | 2.490E-05 | 1.190E-06 | 109.05 |
| 110.0 | -5.741 | 4.127 | -1.614 | | -5.70E-03 | 4.118E-04 | 2.740E-05 | 1.311E-06 | 120.00 |
| 112.0 | -6.328 | 4.704 | -1.624 | | -4.90E-03 | 4.693E-04 | 3.015E-05 | 1.445E-06 | 132.05 |
| 114.0 | -6.963 | 5.338 | -1.625 | | -3.90E-03 | 5.326E-04 | 3.318E-05 | 1.590E-06 | 145.31 |
| 116.0 | -7.654 | 6.036 | -1.618 | | -2.70E-03 | 6.023E-04 | 3.652E-05 | 1.748E-06 | 159.90 |
| 118.0 | -8.399 | 6.804 | -1.595 | | -1.40E-03 | 6.789E-04 | 4.018E-05 | 1.918E-06 | 175.95 |
| 120.0 | -9.196 | 7.650 | -1.546 | | 0.00E+00 | 7.633E-04 | 4.420E-05 | 2.100E-06 | 193.62 |
| | | | | **7** | | | | | |
| 120.0 | 10.002 | 7.650 | 17.652 | | 0.00E+00 | 7.633E-04 | 4.421E-05 | -2.284E-06 | 193.62 |
| 121.8 | 9.275 | 8.411 | 17.686 | | 1.40E-03 | 8.392E-04 | 4.025E-05 | -2.118E-06 | 176.23 |
| 123.6 | 8.596 | 9.103 | 17.699 | | 3.00E-03 | 9.083E-04 | 3.658E-05 | -1.963E-06 | 160.15 |
| 125.4 | 7.974 | 9.733 | 17.707 | | 4.70E-03 | 9.711E-04 | 3.317E-05 | -1.821E-06 | 145.25 |
| 127.2 | 7.401 | 10.303 | 17.704 | | 6.50E-03 | 1.028E-03 | 3.001E-05 | -1.690E-06 | 131.44 |
| 129.0 | 6.879 | 10.814 | 17.694 | | 8.40E-03 | 1.079E-03 | 2.708E-05 | -1.571E-06 | 118.60 |
| 130.8 | 6.411 | 11.275 | 17.686 | | 1.04E-02 | 1.125E-03 | 2.435E-05 | -1.464E-06 | 106.63 |
| 132.6 | 5.995 | 11.696 | 17.691 | | 1.25E-02 | 1.167E-03 | 2.180E-05 | -1.369E-06 | 95.46 |
| 134.4 | 5.627 | 12.067 | 17.694 | | 1.46E-02 | 1.204E-03 | 1.941E-05 | -1.285E-06 | 85.00 |
| 136.2 | 5.312 | 12.398 | 17.709 | | 1.68E-02 | 1.237E-03 | 1.717E-05 | -1.213E-06 | 75.17 |
| 138.0 | 5.049 | 12.688 | 17.737 | | 1.90E-02 | 1.266E-03 | 1.504E-05 | -1.153E-06 | 65.90 |

(continued) Torques and Torque Angles and Derivatives and Bimoment

| z | *Tw* | *Tt* | *T* | Elm | θ | θ' | θ" | θ''' | B(z) |
|---|---|---|---|---|---|---|---|---|---|
| | | | | **8** | | | | | |
| 138.0 | 4.996 | 12.688 | 17.685 | | 1.90E-02 | 1.266E-03 | 1.505E-05 | -1.141E-06 | 65.90 |
| 139.8 | 4.756 | 12.939 | 17.695 | | 2.13E-02 | 1.291E-03 | 1.304E-05 | -1.086E-06 | 57.11 |
| 141.6 | 4.541 | 13.159 | 17.700 | | 2.37E-02 | 1.313E-03 | 1.113E-05 | -1.037E-06 | 48.75 |
| 143.4 | 4.357 | 13.340 | 17.696 | | 2.61E-02 | 1.331E-03 | 9.306E-06 | -9.949E-07 | 40.75 |
| 145.2 | 4.204 | 13.500 | 17.704 | | 2.85E-02 | 1.347E-03 | 7.548E-06 | -9.600E-07 | 33.05 |
| 147.0 | 4.082 | 13.620 | 17.702 | | 3.09E-02 | 1.359E-03 | 5.846E-06 | -9.321E-07 | 25.60 |
| 148.8 | 3.990 | 13.711 | 17.701 | | 3.34E-02 | 1.368E-03 | 4.188E-06 | -9.112E-07 | 18.34 |
| 150.6 | 3.929 | 13.771 | 17.700 | | 3.58E-02 | 1.374E-03 | 2.561E-06 | -8.973E-07 | 11.22 |
| 150.6 | 3.929 | 13.771 | 17.700 | | 3.58E-02 | 1.374E-03 | 2.561E-06 | -8.973E-07 | 11.22 |
| 152.4 | 3.899 | 13.801 | 17.699 | | 3.83E-02 | 1.377E-03 | 9.537E-07 | -8.903E-07 | 4.17 |
| 154.2 | 3.899 | 13.801 | 17.699 | | 4.08E-02 | 1.377E-03 | -6.479E-07 | -8.903E-07 | -2.84 |
| 156.0 | 3.930 | 13.771 | 17.700 | | 4.33E-02 | 1.374E-03 | -2.256E-06 | -8.974E-07 | -9.87 |
| | | | | **9** | | | | | |
| 156.0 | 3.937 | 13.771 | 17.708 | | 4.33E-02 | 1.374E-03 | -2.251E-06 | -8.991E-07 | -9.87 |
| 157.8 | 3.983 | 13.721 | 17.703 | | 4.57E-02 | 1.369E-03 | -3.877E-06 | -9.095E-07 | -16.98 |
| 159.6 | 4.062 | 13.630 | 17.692 | | 4.82E-02 | 1.360E-03 | -5.530E-06 | -9.275E-07 | -24.21 |
| 161.4 | 4.175 | 13.520 | 17.695 | | 5.06E-02 | 1.349E-03 | -7.221E-06 | -9.533E-07 | -31.62 |
| 163.2 | 4.321 | 13.370 | 17.691 | | 5.30E-02 | 1.334E-03 | -8.966E-06 | -9.867E-07 | -39.27 |
| 165.0 | 4.502 | 13.200 | 17.701 | | 5.54E-02 | 1.317E-03 | -1.078E-05 | -1.028E-06 | -47.20 |
| 166.8 | 4.716 | 12.989 | 17.705 | | 5.78E-02 | 1.296E-03 | -1.267E-05 | -1.077E-06 | -55.49 |
| 168.6 | 4.961 | 12.738 | 17.700 | | 6.01E-02 | 1.271E-03 | -1.466E-05 | -1.133E-06 | -64.19 |
| 170.4 | 5.246 | 12.458 | 17.704 | | 6.24E-02 | 1.243E-03 | -1.676E-05 | -1.198E-06 | -73.36 |
| 172.2 | 5.557 | 12.137 | 17.694 | | 6.46E-02 | 1.211E-03 | -1.897E-05 | -1.269E-06 | -83.08 |
| 174.0 | 5.907 | 11.766 | 17.674 | | 6.67E-02 | 1.174E-03 | -2.133E-05 | -1.349E-06 | -93.42 |
| | | | | **10** | | | | | |
| 174.0 | 5.982 | 11.766 | 17.748 | | 6.67E-02 | 1.174E-03 | -2.132E-05 | -1.366E-06 | -93.42 |
| 175.8 | 6.350 | 11.365 | 17.715 | | 6.88E-02 | 1.134E-03 | -2.386E-05 | -1.450E-06 | -104.45 |
| 177.6 | 6.787 | 10.904 | 17.692 | | 7.08E-02 | 1.088E-03 | -2.655E-05 | -1.550E-06 | -116.25 |
| 179.4 | 7.282 | 10.403 | 17.686 | | 7.27E-02 | 1.038E-03 | -2.944E-05 | -1.663E-06 | -128.92 |
| 181.2 | 7.843 | 9.845 | 17.688 | | 7.45E-02 | 9.823E-04 | -3.255E-05 | -1.791E-06 | -142.55 |
| 183.0 | 8.469 | 9.228 | 17.697 | | 7.62E-02 | 9.207E-04 | -3.590E-05 | -1.934E-06 | -157.23 |
| 184.8 | 9.156 | 8.548 | 17.705 | | 7.78E-02 | 8.529E-04 | -3.952E-05 | -2.091E-06 | -173.08 |
| 186.6 | 9.910 | 7.800 | 17.710 | | 7.93E-02 | 7.783E-04 | -4.344E-05 | -2.263E-06 | -190.21 |
| 188.4 | 10.724 | 6.979 | 17.703 | | 8.06E-02 | 6.963E-04 | -4.768E-05 | -2.449E-06 | -208.75 |
| 190.2 | 11.604 | 6.078 | 17.682 | | 8.18E-02 | 6.064E-04 | -5.226E-05 | -2.650E-06 | -228.84 |
| 192.0 | 12.546 | 5.091 | 17.637 | | 8.28E-02 | 5.080E-04 | -5.721E-05 | -2.865E-06 | -250.64 |
| | | | | **11** | | | | | |
| 192.0 | -11.898 | 5.091 | -6.806 | | 8.28E-02 | 5.080E-04 | -5.721E-05 | 2.717E-06 | -250.64 |
| 194.1 | -10.821 | 3.945 | -6.876 | | 8.38E-02 | 3.936E-04 | -5.177E-05 | 2.471E-06 | -226.67 |
| 196.2 | -9.818 | 2.909 | -6.909 | | 8.45E-02 | 2.902E-04 | -4.682E-05 | 2.242E-06 | -205.00 |
| 198.3 | -8.889 | 1.970 | -6.919 | | 8.50E-02 | 1.966E-04 | -4.234E-05 | 2.030E-06 | -185.40 |

| (continued) Torques and Torque Angles and Derivatives and Bimoment | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|
| z | *Tw* | *Tt* | *T* | Elm | θ | θ' | θ" | θ"' | B(z) |
| 200.4 | -8.035 | 1.123 | -6.913 | | 8.53E-02 | 1.120E-04 | -3.828E-05 | 1.835E-06 | -167.67 |
| 202.5 | -7.256 | 0.356 | -6.900 | | 8.55E-02 | 3.556E-05 | -3.462E-05 | 1.657E-06 | -151.64 |
| 204.6 | -6.551 | -0.337 | -6.888 | | 8.55E-02 | -3.361E-05 | -3.131E-05 | 1.496E-06 | -137.14 |
| 206.7 | -5.920 | -0.964 | -6.884 | | 8.53E-02 | -9.617E-05 | -2.832E-05 | 1.352E-06 | -124.02 |
| 208.8 | -5.364 | -1.531 | -6.896 | | 8.51E-02 | -1.528E-04 | -2.562E-05 | 1.225E-06 | -112.15 |
| 210.9 | -4.883 | -2.045 | -6.927 | | 8.47E-02 | -2.040E-04 | -2.317E-05 | 1.115E-06 | -101.42 |
| 213.0 | -4.475 | -2.508 | -6.983 | | 8.42E-02 | -2.502E-04 | -2.092E-05 | 1.022E-06 | -91.71 |
| | | | | **12** | | | | | |
| 213.0 | -4.359 | -2.508 | -6.866 | | 8.42E-02 | -2.502E-04 | -2.094E-05 | 9.954E-07 | -91.71 |
| 215.1 | -3.965 | -2.928 | -6.893 | | 8.36E-02 | -2.921E-04 | -1.894E-05 | 9.055E-07 | -82.93 |
| 217.2 | -3.599 | -3.306 | -6.905 | | 8.30E-02 | -3.299E-04 | -1.713E-05 | 8.218E-07 | -74.99 |
| 219.3 | -3.259 | -3.649 | -6.908 | | 8.23E-02 | -3.641E-04 | -1.549E-05 | 7.443E-07 | -67.81 |
| 221.4 | -2.947 | -3.959 | -6.906 | | 8.15E-02 | -3.950E-04 | -1.400E-05 | 6.730E-07 | -61.31 |
| 223.5 | -2.662 | -4.239 | -6.902 | | 8.06E-02 | -4.230E-04 | -1.265E-05 | 6.080E-07 | -55.43 |
| 225.6 | -2.405 | -4.493 | -6.898 | | 7.97E-02 | -4.483E-04 | -1.144E-05 | 5.491E-07 | -50.10 |
| 227.7 | -2.174 | -4.722 | -6.896 | | 7.87E-02 | -4.711E-04 | -1.034E-05 | 4.965E-07 | -45.29 |
| 229.8 | -1.971 | -4.929 | -6.900 | | 7.77E-02 | -4.918E-04 | -9.350E-06 | 4.502E-07 | -40.93 |
| 231.9 | -1.795 | -5.116 | -6.912 | | 7.67E-02 | -5.105E-04 | -8.448E-06 | 4.100E-07 | -36.99 |
| 234.0 | -1.647 | -5.285 | -6.932 | | 7.56E-02 | -5.273E-04 | -7.624E-06 | 3.761E-07 | -33.42 |
| | | | | **13** | | | | | |
| 234.0 | -1.604 | -5.285 | -6.889 | | 7.56E-02 | -5.273E-04 | -7.629E-06 | 3.663E-07 | -33.42 |
| 236.1 | -1.461 | -5.438 | -6.899 | | 7.44E-02 | -5.426E-04 | -6.894E-06 | 3.336E-07 | -30.19 |
| 238.2 | -1.327 | -5.575 | -6.903 | | 7.33E-02 | -5.563E-04 | -6.226E-06 | 3.031E-07 | -27.26 |
| 240.3 | -1.204 | -5.701 | -6.905 | | 7.21E-02 | -5.688E-04 | -5.619E-06 | 2.750E-07 | -24.61 |
| 242.4 | -1.091 | -5.813 | -6.904 | | 7.09E-02 | -5.800E-04 | -5.069E-06 | 2.491E-07 | -22.20 |
| 244.5 | -0.988 | -5.914 | -6.902 | | 6.97E-02 | -5.901E-04 | -4.571E-06 | 2.256E-07 | -20.02 |
| 246.6 | -0.895 | -6.005 | -6.900 | | 6.84E-02 | -5.992E-04 | -4.120E-06 | 2.044E-07 | -18.05 |
| 248.7 | -0.812 | -6.088 | -6.900 | | 6.72E-02 | -6.074E-04 | -3.711E-06 | 1.855E-07 | -16.25 |
| 250.8 | -0.740 | -6.162 | -6.901 | | 6.59E-02 | -6.148E-04 | -3.340E-06 | 1.689E-07 | -14.62 |
| 252.9 | -0.677 | -6.229 | -6.906 | | 6.46E-02 | -6.215E-04 | -3.001E-06 | 1.546E-07 | -13.14 |
| 255.0 | -0.624 | -6.288 | -6.912 | | 6.33E-02 | -6.274E-04 | -2.689E-06 | 1.426E-07 | -11.79 |
| | | | | **14** | | | | | |
| 255.0 | -0.610 | -6.288 | -6.898 | | 6.33E-02 | -6.274E-04 | -2.691E-06 | 1.392E-07 | -11.79 |
| 257.1 | -0.559 | -6.342 | -6.901 | | 6.19E-02 | -6.328E-04 | -2.411E-06 | 1.276E-07 | -10.56 |
| 259.2 | -0.512 | -6.390 | -6.903 | | 6.06E-02 | -6.376E-04 | -2.154E-06 | 1.170E-07 | -9.43 |
| 261.3 | -0.470 | -6.433 | -6.903 | | 5.93E-02 | -6.419E-04 | -1.919E-06 | 1.073E-07 | -8.40 |
| 263.4 | -0.432 | -6.471 | -6.903 | | 5.79E-02 | -6.457E-04 | -1.703E-06 | 9.856E-08 | -7.46 |
| 265.5 | -0.397 | -6.505 | -6.902 | | 5.65E-02 | -6.490E-04 | -1.504E-06 | 9.074E-08 | -6.59 |
| 267.6 | -0.367 | -6.535 | -6.902 | | 5.52E-02 | -6.520E-04 | -1.321E-06 | 8.384E-08 | -5.78 |
| 269.7 | -0.341 | -6.561 | -6.902 | | 5.38E-02 | -6.546E-04 | -1.151E-06 | 7.787E-08 | -5.04 |
| 271.8 | -0.319 | -6.583 | -6.902 | | 5.24E-02 | -6.568E-04 | -9.931E-07 | 7.283E-08 | -4.35 |

continued) Torques and Torque Angles and Derivatives and Bimoment

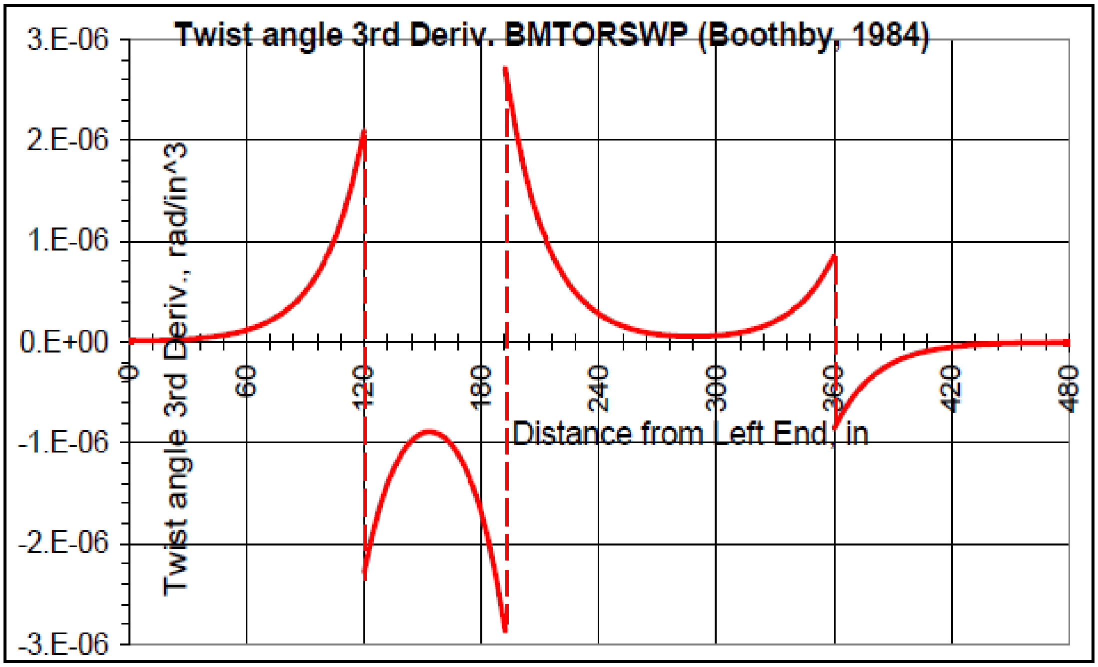
| z | *Tw* | *Tt* | *T* | Elm | θ | θ' | θ'' | θ''' | B(z) |
|---|---|---|---|---|---|---|---|---|---|
| 273.9 | -0.301 | -6.603 | -6.904 | | 5.11E-02 | -6.588E-04 | -8.446E-07 | 6.872E-08 | -3.70 |
| 276.0 | -0.287 | -6.619 | -6.906 | | 4.97E-02 | -6.604E-04 | -7.038E-07 | 6.554E-08 | -3.09 |
| | | | | **15** | | | | | |
| 276.0 | -0.283 | -6.619 | -6.902 | | 4.97E-02 | -6.604E-04 | -7.046E-07 | 6.464E-08 | -3.09 |
| 278.1 | -0.270 | -6.632 | -6.902 | | 4.83E-02 | -6.617E-04 | -5.721E-07 | 6.166E-08 | -2.51 |
| 280.2 | -0.259 | -6.643 | -6.902 | | 4.69E-02 | -6.628E-04 | -4.452E-07 | 5.925E-08 | -1.95 |
| 282.3 | -0.251 | -6.651 | -6.902 | | 4.55E-02 | -6.636E-04 | -3.228E-07 | 5.741E-08 | -1.41 |
| 284.4 | -0.246 | -6.656 | -6.902 | | 4.41E-02 | -6.641E-04 | -2.037E-07 | 5.614E-08 | -0.89 |
| 286.5 | -0.243 | -6.659 | -6.902 | | 4.27E-02 | -6.644E-04 | -8.665E-08 | 5.543E-08 | -0.38 |
| 288.6 | -0.242 | -6.660 | -6.902 | | 4.13E-02 | -6.645E-04 | 2.950E-08 | 5.529E-08 | 0.13 |
| 290.7 | -0.244 | -6.658 | -6.902 | | 3.99E-02 | -6.643E-04 | 1.460E-07 | 5.571E-08 | 0.64 |
| 292.8 | -0.248 | -6.654 | -6.902 | | 3.85E-02 | -6.639E-04 | 2.639E-07 | 5.671E-08 | 1.16 |
| 294.9 | -0.255 | -6.647 | -6.902 | | 3.71E-02 | -6.632E-04 | 3.845E-07 | 5.827E-08 | 1.68 |
| 297.0 | -0.264 | -6.638 | -6.902 | | 3.57E-02 | -6.623E-04 | 5.090E-07 | 6.040E-08 | 2.23 |
| | | | | **16** | | | | | |
| 297.0 | -0.267 | -6.638 | -6.905 | | 3.57E-02 | -6.623E-04 | 5.083E-07 | 6.105E-08 | 2.23 |
| 299.1 | -0.277 | -6.626 | -6.903 | | 3.43E-02 | -6.611E-04 | 6.388E-07 | 6.337E-08 | 2.80 |
| 301.2 | -0.291 | -6.611 | -6.902 | | 3.30E-02 | -6.596E-04 | 7.751E-07 | 6.652E-08 | 3.39 |
| 303.3 | -0.309 | -6.593 | -6.901 | | 3.16E-02 | -6.578E-04 | 9.188E-07 | 7.049E-08 | 4.02 |
| 305.4 | -0.330 | -6.572 | -6.901 | | 3.02E-02 | -6.557E-04 | 1.072E-06 | 7.530E-08 | 4.69 |
| 307.5 | -0.354 | -6.548 | -6.902 | | 2.88E-02 | -6.533E-04 | 1.236E-06 | 8.093E-08 | 5.41 |
| 309.6 | -0.383 | -6.520 | -6.902 | | 2.75E-02 | -6.505E-04 | 1.412E-06 | 8.739E-08 | 6.18 |
| 311.7 | -0.415 | -6.489 | -6.903 | | 2.61E-02 | -6.474E-04 | 1.603E-06 | 9.469E-08 | 7.02 |
| 313.8 | -0.450 | -6.452 | -6.903 | | 2.47E-02 | -6.438E-04 | 1.810E-06 | 1.028E-07 | 7.93 |
| 315.9 | -0.490 | -6.411 | -6.901 | | 2.34E-02 | -6.397E-04 | 2.036E-06 | 1.118E-07 | 8.91 |
| 318.0 | -0.532 | -6.366 | -6.898 | | 2.20E-02 | -6.352E-04 | 2.280E-06 | 1.215E-07 | 9.99 |
| | | | | **17** | | | | | |
| 318.0 | -0.545 | -6.366 | -6.911 | | 2.20E-02 | -6.352E-04 | 2.279E-06 | 1.244E-07 | 9.99 |
| 320.1 | -0.589 | -6.315 | -6.905 | | 2.07E-02 | -6.301E-04 | 2.551E-06 | 1.346E-07 | 11.17 |
| 322.2 | -0.643 | -6.259 | -6.902 | | 1.94E-02 | -6.245E-04 | 2.846E-06 | 1.468E-07 | 12.46 |
| 324.3 | -0.705 | -6.196 | -6.900 | | 1.81E-02 | -6.182E-04 | 3.168E-06 | 1.609E-07 | 13.87 |
| 326.4 | -0.776 | -6.126 | -6.901 | | 1.68E-02 | -6.112E-04 | 3.523E-06 | 1.771E-07 | 15.43 |
| 328.5 | -0.855 | -6.048 | -6.902 | | 1.55E-02 | -6.034E-04 | 3.913E-06 | 1.952E-07 | 17.14 |
| 330.6 | -0.943 | -5.960 | -6.904 | | 1.43E-02 | -5.947E-04 | 4.344E-06 | 2.154E-07 | 19.03 |
| 332.7 | -1.040 | -5.864 | -6.904 | | 1.30E-02 | -5.851E-04 | 4.820E-06 | 2.375E-07 | 21.10 |
| 334.8 | -1.146 | -5.757 | -6.903 | | 1.18E-02 | -5.744E-04 | 5.343E-06 | 2.617E-07 | 23.39 |
| 336.9 | -1.260 | -5.639 | -6.899 | | 1.06E-02 | -5.626E-04 | 5.920E-06 | 2.878E-07 | 25.92 |
| 339.0 | -1.384 | -5.507 | -6.891 | | 9.50E-03 | -5.495E-04 | 6.554E-06 | 3.160E-07 | 28.71 |
| | | | | **18** | | | | | |
| 339.0 | -1.421 | -5.507 | -6.928 | | 9.50E-03 | -5.495E-04 | 6.550E-06 | 3.244E-07 | 28.71 |
| 341.1 | -1.548 | -5.362 | -6.910 | | 8.30E-03 | -5.350E-04 | 7.261E-06 | 3.535E-07 | 31.79 |
| 343.2 | -1.699 | -5.202 | -6.901 | | 7.20E-03 | -5.190E-04 | 8.038E-06 | 3.880E-07 | 35.19 |

(continued) Torques and Torque Angles and Derivatives and Bimoment

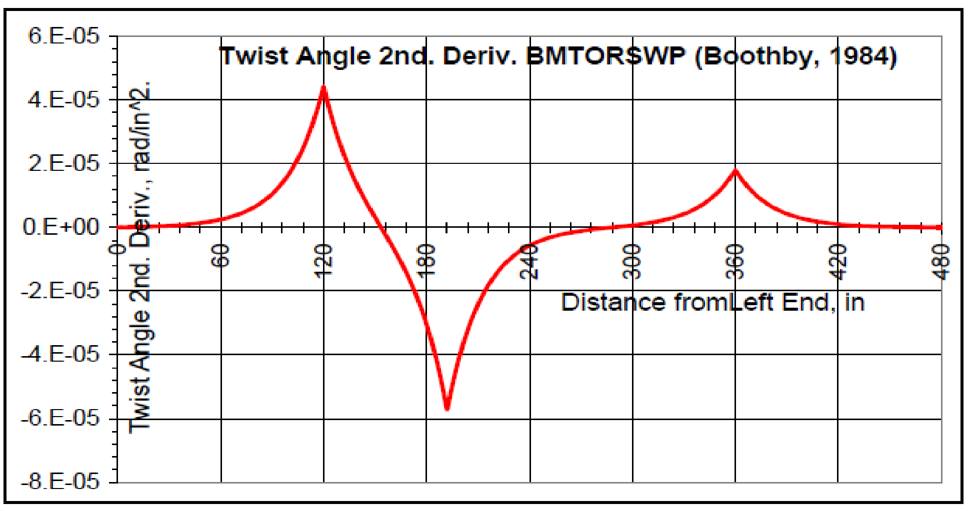
| z | *Tw* | *Tt* | *T* | Elm | θ | θ' | θ" | θ"' | B(z) |
|---|---|---|---|---|---|---|---|---|---|
| 345.3 | -1.874 | -5.023 | -6.897 | | 6.10E-03 | -5.012E-04 | 8.894E-06 | 4.279E-07 | 38.94 |
| 347.4 | -2.072 | -4.827 | -6.898 | | 5.10E-03 | -4.816E-04 | 9.839E-06 | 4.731E-07 | 43.09 |
| 349.5 | -2.293 | -4.608 | -6.902 | | 4.10E-03 | -4.598E-04 | 1.089E-05 | 5.237E-07 | 47.68 |
| 351.6 | -2.539 | -4.368 | -6.906 | | 3.20E-03 | -4.358E-04 | 1.204E-05 | 5.797E-07 | 52.74 |
| 353.7 | -2.807 | -4.100 | -6.907 | | 2.30E-03 | -4.091E-04 | 1.332E-05 | 6.410E-07 | 58.34 |
| 355.8 | -3.099 | -3.806 | -6.905 | | 1.50E-03 | -3.797E-04 | 1.474E-05 | 7.077E-07 | 64.53 |
| 357.9 | -3.414 | -3.479 | -6.893 | | 7.00E-04 | -3.471E-04 | 1.630E-05 | 7.797E-07 | 71.37 |
| 360.0 | -3.753 | -3.118 | -6.871 | | 0.00E+00 | -3.111E-04 | 1.802E-05 | 8.571E-07 | 78.93 |
| | | | | **19** | | | | | |
| 360.0 | 3.749 | -3.118 | 0.631 | | 0.00E+00 | -3.111E-04 | 1.802E-05 | -8.562E-07 | 78.93 |
| 362.0 | 3.424 | -2.774 | 0.650 | | -6.00E-04 | -2.768E-04 | 1.638E-05 | -7.819E-07 | 71.73 |
| 364.0 | 3.120 | -2.460 | 0.660 | | -1.10E-03 | -2.455E-04 | 1.489E-05 | -7.126E-07 | 65.18 |
| 366.0 | 2.838 | -2.176 | 0.663 | | -1.60E-03 | -2.171E-04 | 1.353E-05 | -6.482E-07 | 59.23 |
| 368.0 | 2.578 | -1.917 | 0.661 | | -2.00E-03 | -1.913E-04 | 1.229E-05 | -5.888E-07 | 53.83 |
| 370.0 | 2.340 | -1.683 | 0.657 | | -2.30E-03 | -1.679E-04 | 1.117E-05 | -5.344E-07 | 48.92 |
| 372.0 | 2.123 | -1.469 | 0.654 | | -2.70E-03 | -1.466E-04 | 1.015E-05 | -4.849E-07 | 44.45 |
| 374.0 | 1.929 | -1.275 | 0.654 | | -2.90E-03 | -1.272E-04 | 9.226E-06 | -4.404E-07 | 40.40 |
| 376.0 | 1.755 | -1.098 | 0.657 | | -3.20E-03 | -1.096E-04 | 8.385E-06 | -4.008E-07 | 36.71 |
| 378.0 | 1.603 | -0.938 | 0.665 | | -3.40E-03 | -9.363E-05 | 7.619E-06 | -3.661E-07 | 33.36 |
| 380.0 | 1.474 | -0.793 | 0.681 | | -3.50E-03 | -7.910E-05 | 6.918E-06 | -3.365E-07 | 30.32 |
| | | | | 20 | | | | | |
| 380.0 | 1.440 | -0.793 | 0.647 | | -3.50E-03 | -7.910E-05 | 6.921E-06 | -3.289E-07 | 30.32 |
| 382.0 | 1.315 | -0.660 | 0.655 | | -3.70E-03 | -6.590E-05 | 6.292E-06 | -3.004E-07 | 27.55 |
| 384.0 | 1.199 | -0.540 | 0.658 | | -3.80E-03 | -5.390E-05 | 5.718E-06 | -2.737E-07 | 25.03 |
| 386.0 | 1.090 | -0.431 | 0.660 | | -3.90E-03 | -4.299E-05 | 5.195E-06 | -2.490E-07 | 22.75 |
| 388.0 | 0.991 | -0.332 | 0.659 | | -4.00E-03 | -3.308E-05 | 4.721E-06 | -2.262E-07 | 20.67 |
| 390.0 | 0.899 | -0.241 | 0.658 | | -4.00E-03 | -2.408E-05 | 4.289E-06 | -2.053E-07 | 18.79 |
| 392.0 | 0.816 | -0.159 | 0.656 | | -4.10E-03 | -1.590E-05 | 3.898E-06 | -1.863E-07 | 17.07 |
| 394.0 | 0.741 | -0.085 | 0.656 | | -4.10E-03 | -8.466E-06 | 3.543E-06 | -1.692E-07 | 15.51 |
| 396.0 | 0.674 | -0.017 | 0.657 | | -4.10E-03 | -1.708E-06 | 3.220E-06 | -1.540E-07 | 14.10 |
| 398.0 | 0.616 | 0.044 | 0.661 | | -4.10E-03 | 4.434E-06 | 2.926E-06 | -1.407E-07 | 12.81 |
| 400.0 | 0.566 | 0.100 | 0.667 | | -4.10E-03 | 1.001E-05 | 2.656E-06 | -1.293E-07 | 11.64 |
| | | | | **21** | | | | | |
| 400.0 | 0.554 | 0.100 | 0.654 | | -4.10E-03 | 1.001E-05 | 2.657E-06 | -1.264E-07 | 11.64 |
| 402.0 | 0.505 | 0.151 | 0.656 | | -4.10E-03 | 1.508E-05 | 2.416E-06 | -1.154E-07 | 10.58 |
| 404.0 | 0.461 | 0.197 | 0.658 | | -4.00E-03 | 1.969E-05 | 2.195E-06 | -1.052E-07 | 9.61 |
| 406.0 | 0.419 | 0.239 | 0.658 | | -4.00E-03 | 2.387E-05 | 1.994E-06 | -9.572E-08 | 8.73 |
| 408.0 | 0.381 | 0.277 | 0.658 | | -3.90E-03 | 2.768E-05 | 1.812E-06 | -8.697E-08 | 7.93 |
| 410.0 | 0.346 | 0.312 | 0.658 | | -3.90E-03 | 3.113E-05 | 1.646E-06 | -7.894E-08 | 7.21 |
| 412.0 | 0.314 | 0.343 | 0.657 | | -3.80E-03 | 3.427E-05 | 1.495E-06 | -7.165E-08 | 6.55 |
| 414.0 | 0.285 | 0.372 | 0.657 | | -3.70E-03 | 3.712E-05 | 1.359E-06 | -6.509E-08 | 5.95 |
| 416.0 | 0.259 | 0.398 | 0.657 | | -3.70E-03 | 3.971E-05 | 1.235E-06 | -5.926E-08 | 5.40 |

(continued) Torques and Torque Angles and Derivatives and Bimoment

| z | *Tw* | *Tt* | *T* | Elm | θ | θ' | θ'' | θ''' | B(z) |
|---|---|---|---|---|---|---|---|---|---|
| 418.0 | 0.237 | 0.422 | 0.659 | | -3.60E-03 | 4.207E-05 | 1.121E-06 | -5.416E-08 | 4.91 |
| 420.0 | 0.218 | 0.443 | 0.661 | | -3.50E-03 | 4.421E-05 | 1.017E-06 | -4.980E-08 | 4.46 |
| | | | | **22** | | | | | |
| 420.0 | 0.213 | 0.443 | 0.656 | | -3.50E-03 | 4.421E-05 | 1.018E-06 | -4.868E-08 | 4.46 |
| 422.0 | 0.195 | 0.463 | 0.657 | | -3.40E-03 | 4.615E-05 | 9.248E-07 | -4.449E-08 | 4.05 |
| 424.0 | 0.178 | 0.480 | 0.658 | | -3.30E-03 | 4.791E-05 | 8.398E-07 | -4.058E-08 | 3.68 |
| 426.0 | 0.162 | 0.496 | 0.658 | | -3.20E-03 | 4.951E-05 | 7.623E-07 | -3.695E-08 | 3.34 |
| 428.0 | 0.147 | 0.511 | 0.658 | | -3.10E-03 | 5.096E-05 | 6.918E-07 | -3.360E-08 | 3.03 |
| 430.0 | 0.134 | 0.524 | 0.658 | | -3.00E-03 | 5.228E-05 | 6.277E-07 | -3.054E-08 | 2.75 |
| 432.0 | 0.122 | 0.536 | 0.658 | | -2.90E-03 | 5.348E-05 | 5.694E-07 | -2.776E-08 | 2.49 |
| 434.0 | 0.111 | 0.547 | 0.657 | | -2.80E-03 | 5.456E-05 | 5.165E-07 | -2.526E-08 | 2.26 |
| 436.0 | 0.101 | 0.557 | 0.658 | | -2.70E-03 | 5.555E-05 | 4.682E-07 | -2.305E-08 | 2.05 |
| 438.0 | 0.092 | 0.566 | 0.658 | | -2.60E-03 | 5.644E-05 | 4.241E-07 | -2.112E-08 | 1.86 |
| 440.0 | 0.085 | 0.574 | 0.659 | | -2.50E-03 | 5.725E-05 | 3.835E-07 | -1.947E-08 | 1.68 |
| | | | | **23** | | | | | |
| 440.0 | 0.083 | 0.574 | 0.657 | | -2.50E-03 | 5.725E-05 | 3.837E-07 | -1.905E-08 | 1.68 |
| 442.0 | 0.077 | 0.581 | 0.658 | | -2.30E-03 | 5.798E-05 | 3.472E-07 | -1.747E-08 | 1.52 |
| 444.0 | 0.070 | 0.588 | 0.658 | | -2.20E-03 | 5.864E-05 | 3.137E-07 | -1.601E-08 | 1.37 |
| 446.0 | 0.064 | 0.594 | 0.658 | | -2.10E-03 | 5.923E-05 | 2.831E-07 | -1.465E-08 | 1.24 |
| 448.0 | 0.059 | 0.599 | 0.658 | | -2.00E-03 | 5.977E-05 | 2.550E-07 | -1.342E-08 | 1.12 |
| 450.0 | 0.054 | 0.604 | 0.658 | | -1.90E-03 | 6.026E-05 | 2.294E-07 | -1.229E-08 | 1.00 |
| 452.0 | 0.049 | 0.608 | 0.658 | | -1.70E-03 | 6.069E-05 | 2.058E-07 | -1.128E-08 | 0.90 |
| 454.0 | 0.045 | 0.612 | 0.658 | | -1.60E-03 | 6.108E-05 | 1.842E-07 | -1.039E-08 | 0.81 |
| 456.0 | 0.042 | 0.616 | 0.658 | | -1.50E-03 | 6.143E-05 | 1.642E-07 | -9.603E-09 | 0.72 |
| 458.0 | 0.039 | 0.619 | 0.658 | | -1.40E-03 | 6.174E-05 | 1.457E-07 | -8.935E-09 | 0.64 |
| 460.0 | 0.037 | 0.621 | 0.658 | | -1.30E-03 | 6.201E-05 | 1.284E-07 | -8.380E-09 | 0.56 |
| | | | | **24** | | | | | |
| 460.0 | 0.036 | 0.621 | 0.658 | | -1.30E-03 | 6.201E-05 | 1.284E-07 | -8.240E-09 | 0.56 |
| 462.0 | 0.034 | 0.624 | 0.658 | | -1.10E-03 | 6.225E-05 | 1.125E-07 | -7.714E-09 | 0.49 |
| 464.0 | 0.032 | 0.626 | 0.658 | | -1.00E-03 | 6.246E-05 | 9.755E-08 | -7.246E-09 | 0.43 |
| 466.0 | 0.030 | 0.628 | 0.658 | | -9.00E-04 | 6.264E-05 | 8.348E-08 | -6.835E-09 | 0.37 |
| 468.0 | 0.028 | 0.629 | 0.658 | | -8.00E-04 | 6.280E-05 | 7.018E-08 | -6.481E-09 | 0.31 |
| 470.0 | 0.027 | 0.631 | 0.658 | | -6.00E-04 | 6.292E-05 | 5.752E-08 | -6.185E-09 | 0.25 |
| 472.0 | 0.026 | 0.632 | 0.658 | | -5.00E-04 | 6.303E-05 | 4.540E-08 | -5.946E-09 | 0.20 |
| 474.0 | 0.025 | 0.633 | 0.658 | | -4.00E-04 | 6.311E-05 | 3.370E-08 | -5.764E-09 | 0.15 |
| 476.0 | 0.025 | 0.633 | 0.658 | | -3.00E-04 | 6.316E-05 | 2.230E-08 | -5.640E-09 | 0.10 |
| 478.0 | 0.024 | 0.633 | 0.658 | | -1.00E-04 | 6.320E-05 | 1.110E-08 | -5.573E-09 | 0.05 |
| 480.0 | 0.024 | 0.634 | 0.658 | | 0.00E+00 | 6.321E-05 | -2.570E-11 | -5.563E-09 | 0.00 |



Third Derivative of Torque Angle



Second Derivative of Torque Angle

St. Vt. and Warping Shear Stresses

| Pure Torsion Shear Stresses G*t*θ' | | | | | | | Warping shear stress at "s", - E*Sw*θ'''/ t | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| “t” at XS s= 1, 2, 3, 2’, 1’, 0’ | | | | | | | axis | “*s*” and *Sw* | | | | | | |
| 0 | 1 | 2 | 3 | 2' | 1' | 0' | z | 0 | 1 | 2 | 3 | 2' | 1' | 0' |
| 0.501 | 0.501 | 0.501 | 0.510 | 0.501 | 0.501 | 0.501 | axial | 0 | 6.01 | 4.91 | -2.45 | 4.91 | 6.01 | 0 |
| | | | | | | | | | | | | | | |
| -0.90 | -0.90 | -0.90 | -0.92 | -0.90 | -0.90 | -0.90 | 0.0 | 0 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0 |
| -0.90 | -0.90 | -0.90 | -0.92 | -0.90 | -0.90 | -0.90 | 2.0 | 0 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0 |
| -0.90 | -0.90 | -0.90 | -0.92 | -0.90 | -0.90 | -0.90 | 4.0 | 0 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0 |
| -0.90 | -0.90 | -0.90 | -0.92 | -0.90 | -0.90 | -0.90 | 6.0 | 0 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0 |
| -0.90 | -0.90 | -0.90 | -0.91 | -0.90 | -0.90 | -0.90 | 8.0 | 0 | -0.01 | 0.00 | 0.00 | 0.00 | -0.01 | 0 |
| -0.90 | -0.90 | -0.90 | -0.91 | -0.90 | -0.90 | -0.90 | 10.0 | 0 | -0.01 | 0.00 | 0.00 | 0.00 | -0.01 | 0 |
| -0.89 | -0.89 | -0.89 | -0.91 | -0.89 | -0.89 | -0.89 | 12.0 | 0 | -0.01 | 0.00 | 0.00 | 0.00 | -0.01 | 0 |
| -0.89 | -0.89 | -0.89 | -0.91 | -0.89 | -0.89 | -0.89 | 14.0 | 0 | -0.01 | 0.00 | 0.00 | 0.00 | -0.01 | 0 |
| -0.89 | -0.89 | -0.89 | -0.91 | -0.89 | -0.89 | -0.89 | 16.0 | 0 | -0.01 | -0.01 | 0.00 | -0.01 | -0.01 | 0 |
| -0.89 | -0.89 | -0.89 | -0.90 | -0.89 | -0.89 | -0.89 | 18.0 | 0 | -0.01 | -0.01 | 0.00 | -0.01 | -0.01 | 0 |
| -0.88 | -0.88 | -0.88 | -0.90 | -0.88 | -0.88 | -0.88 | 20.0 | 0 | -0.01 | -0.01 | 0.00 | -0.01 | -0.01 | 0 |
| 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | | 0 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0 |
| -0.88 | -0.88 | -0.88 | -0.90 | -0.88 | -0.88 | -0.88 | 20.0 | 0 | -0.01 | -0.01 | 0.00 | -0.01 | -0.01 | 0 |
| -0.88 | -0.88 | -0.88 | -0.90 | -0.88 | -0.88 | -0.88 | 22.0 | 0 | -0.01 | -0.01 | 0.00 | -0.01 | -0.01 | 0 |
| -0.88 | -0.88 | -0.88 | -0.89 | -0.88 | -0.88 | -0.88 | 24.0 | 0 | -0.01 | -0.01 | 0.00 | -0.01 | -0.01 | 0 |
| -0.87 | -0.87 | -0.87 | -0.89 | -0.87 | -0.87 | -0.87 | 26.0 | 0 | -0.01 | -0.01 | 0.00 | -0.01 | -0.01 | 0 |
| -0.87 | -0.87 | -0.87 | -0.88 | -0.87 | -0.87 | -0.87 | 28.0 | 0 | -0.01 | -0.01 | 0.00 | -0.01 | -0.01 | 0 |
| -0.86 | -0.86 | -0.86 | -0.87 | -0.86 | -0.86 | -0.86 | 30.0 | 0 | -0.01 | -0.01 | 0.00 | -0.01 | -0.01 | 0 |
| -0.85 | -0.85 | -0.85 | -0.87 | -0.85 | -0.85 | -0.85 | 32.0 | 0 | -0.01 | -0.01 | 0.00 | -0.01 | -0.01 | 0 |
| -0.84 | -0.84 | -0.84 | -0.86 | -0.84 | -0.84 | -0.84 | 34.0 | 0 | -0.01 | -0.01 | 0.01 | -0.01 | -0.01 | 0 |
| -0.84 | -0.84 | -0.84 | -0.85 | -0.84 | -0.84 | -0.84 | 36.0 | 0 | -0.01 | -0.01 | 0.01 | -0.01 | -0.01 | 0 |
| -0.83 | -0.83 | -0.83 | -0.84 | -0.83 | -0.83 | -0.83 | 38.0 | 0 | -0.01 | -0.01 | 0.01 | -0.01 | -0.01 | 0 |
| -0.82 | -0.82 | -0.82 | -0.83 | -0.82 | -0.82 | -0.82 | 40.0 | 0 | -0.02 | -0.01 | 0.01 | -0.01 | -0.02 | 0 |
| 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | | 0 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0 |
| -0.82 | -0.82 | -0.82 | -0.83 | -0.82 | -0.82 | -0.82 | 40.0 | 0 | -0.02 | -0.01 | 0.01 | -0.01 | -0.02 | 0 |
| -0.80 | -0.80 | -0.80 | -0.82 | -0.80 | -0.80 | -0.80 | 42.0 | 0 | -0.02 | -0.01 | 0.01 | -0.01 | -0.02 | 0 |
| -0.79 | -0.79 | -0.79 | -0.81 | -0.79 | -0.79 | -0.79 | 44.0 | 0 | -0.02 | -0.02 | 0.01 | -0.02 | -0.02 | 0 |
| -0.78 | -0.78 | -0.78 | -0.79 | -0.78 | -0.78 | -0.78 | 46.0 | 0 | -0.02 | -0.02 | 0.01 | -0.02 | -0.02 | 0 |
| -0.76 | -0.76 | -0.76 | -0.78 | -0.76 | -0.76 | -0.76 | 48.0 | 0 | -0.02 | -0.02 | 0.01 | -0.02 | -0.02 | 0 |
| -0.75 | -0.75 | -0.75 | -0.76 | -0.75 | -0.75 | -0.75 | 50.0 | 0 | -0.03 | -0.02 | 0.01 | -0.02 | -0.03 | 0 |
| -0.73 | -0.73 | -0.73 | -0.74 | -0.73 | -0.73 | -0.73 | 52.0 | 0 | -0.03 | -0.02 | 0.01 | -0.02 | -0.03 | 0 |
| -0.71 | -0.71 | -0.71 | -0.72 | -0.71 | -0.71 | -0.71 | 54.0 | 0 | -0.03 | -0.03 | 0.01 | -0.03 | -0.03 | 0 |
| -0.68 | -0.68 | -0.68 | -0.70 | -0.68 | -0.68 | -0.68 | 56.0 | 0 | -0.03 | -0.03 | 0.01 | -0.03 | -0.03 | 0 |
| -0.66 | -0.66 | -0.66 | -0.67 | -0.66 | -0.66 | -0.66 | 58.0 | 0 | -0.04 | -0.03 | 0.02 | -0.03 | -0.04 | 0 |
| -0.63 | -0.63 | -0.63 | -0.64 | -0.63 | -0.63 | -0.63 | 60.0 | 0 | -0.04 | -0.03 | 0.02 | -0.03 | -0.04 | 0 |
| 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | | 0 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0 |
| -0.63 | -0.63 | -0.63 | -0.64 | -0.63 | -0.63 | -0.63 | 60.0 | 0 | -0.04 | -0.03 | 0.02 | -0.03 | -0.04 | 0 |
| -0.60 | -0.60 | -0.60 | -0.61 | -0.60 | -0.60 | -0.60 | 62.0 | 0 | -0.05 | -0.04 | 0.02 | -0.04 | -0.05 | 0 |
| -0.57 | -0.57 | -0.57 | -0.58 | -0.57 | -0.57 | -0.57 | 64.0 | 0 | -0.05 | -0.04 | 0.02 | -0.04 | -0.05 | 0 |

| 0 | 1 | 2 | 3 | 2' | 1' | 0' | z | 0 | 1 | 2 | 3 | 2' | 1' | 0' |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| -0.53 | -0.53 | -0.53 | -0.54 | -0.53 | -0.53 | -0.53 | 66.0 | 0 | -0.06 | -0.05 | 0.02 | -0.05 | -0.06 | 0 |
| -0.49 | -0.49 | -0.49 | -0.50 | -0.49 | -0.49 | -0.49 | 68.0 | 0 | -0.06 | -0.05 | 0.02 | -0.05 | -0.06 | 0 |
| -0.44 | -0.44 | -0.44 | -0.45 | -0.44 | -0.44 | -0.44 | 70.0 | 0 | -0.07 | -0.06 | 0.03 | -0.06 | -0.07 | 0 |
| -0.39 | -0.39 | -0.39 | -0.40 | -0.39 | -0.39 | -0.39 | 72.0 | 0 | -0.07 | -0.06 | 0.03 | -0.06 | -0.07 | 0 |
| -0.34 | -0.34 | -0.34 | -0.35 | -0.34 | -0.34 | -0.34 | 74.0 | 0 | -0.08 | -0.07 | 0.03 | -0.07 | -0.08 | 0 |
| -0.28 | -0.28 | -0.28 | -0.29 | -0.28 | -0.28 | -0.28 | 76.0 | 0 | -0.09 | -0.07 | 0.04 | -0.07 | -0.09 | 0 |
| -0.22 | -0.22 | -0.22 | -0.22 | -0.22 | -0.22 | -0.22 | 78.0 | 0 | -0.10 | -0.08 | 0.04 | -0.08 | -0.10 | 0 |
| -0.14 | -0.14 | -0.14 | -0.15 | -0.14 | -0.14 | -0.14 | 80.0 | 0 | -0.11 | -0.09 | 0.04 | -0.09 | -0.11 | 0 |
| 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | | 0 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0 |
| -0.14 | -0.14 | -0.14 | -0.15 | -0.14 | -0.14 | -0.14 | 80.0 | 0 | -0.11 | -0.09 | 0.04 | -0.09 | -0.11 | 0 |
| -0.06 | -0.06 | -0.06 | -0.06 | -0.06 | -0.06 | -0.06 | 82.0 | 0 | -0.12 | -0.10 | 0.05 | -0.10 | -0.12 | 0 |
| 0.02 | 0.02 | 0.02 | 0.02 | 0.02 | 0.02 | 0.02 | 84.0 | 0 | -0.13 | -0.11 | 0.05 | -0.11 | -0.13 | 0 |
| 0.12 | 0.12 | 0.12 | 0.12 | 0.12 | 0.12 | 0.12 | 86.0 | 0 | -0.14 | -0.12 | 0.06 | -0.12 | -0.14 | 0 |
| 0.23 | 0.23 | 0.23 | 0.23 | 0.23 | 0.23 | 0.23 | 88.0 | 0 | -0.16 | -0.13 | 0.06 | -0.13 | -0.16 | 0 |
| 0.34 | 0.34 | 0.34 | 0.35 | 0.34 | 0.34 | 0.34 | 90.0 | 0 | -0.18 | -0.14 | 0.07 | -0.14 | -0.18 | 0 |
| 0.47 | 0.47 | 0.47 | 0.48 | 0.47 | 0.47 | 0.47 | 92.0 | 0 | -0.19 | -0.16 | 0.08 | -0.16 | -0.19 | 0 |
| 0.61 | 0.61 | 0.61 | 0.62 | 0.61 | 0.61 | 0.61 | 94.0 | 0 | -0.21 | -0.17 | 0.09 | -0.17 | -0.21 | 0 |
| 0.77 | 0.77 | 0.77 | 0.78 | 0.77 | 0.77 | 0.77 | 96.0 | 0 | -0.23 | -0.19 | 0.09 | -0.19 | -0.23 | 0 |
| 0.94 | 0.94 | 0.94 | 0.96 | 0.94 | 0.94 | 0.94 | 98.0 | 0 | -0.26 | -0.21 | 0.10 | -0.21 | -0.26 | 0 |
| 1.13 | 1.13 | 1.13 | 1.15 | 1.13 | 1.13 | 1.13 | 100.0 | 0 | -0.28 | -0.23 | 0.11 | -0.23 | -0.28 | 0 |
| 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | | 0 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0 |
| 1.13 | 1.13 | 1.13 | 1.15 | 1.13 | 1.13 | 1.13 | 100.0 | 0 | -0.29 | -0.23 | 0.11 | -0.23 | -0.29 | 0 |
| 1.33 | 1.33 | 1.33 | 1.36 | 1.33 | 1.33 | 1.33 | 102.0 | 0 | -0.31 | -0.26 | 0.13 | -0.26 | -0.31 | 0 |
| 1.56 | 1.56 | 1.56 | 1.59 | 1.56 | 1.56 | 1.56 | 104.0 | 0 | -0.34 | -0.28 | 0.14 | -0.28 | -0.34 | 0 |
| 1.81 | 1.81 | 1.81 | 1.85 | 1.81 | 1.81 | 1.81 | 106.0 | 0 | -0.38 | -0.31 | 0.15 | -0.31 | -0.38 | 0 |
| 2.09 | 2.09 | 2.09 | 2.13 | 2.09 | 2.09 | 2.09 | 108.0 | 0 | -0.41 | -0.34 | 0.17 | -0.34 | -0.41 | 0 |
| 2.39 | 2.39 | 2.39 | 2.44 | 2.39 | 2.39 | 2.39 | 110.0 | 0 | -0.46 | -0.37 | 0.18 | -0.37 | -0.46 | 0 |
| 2.73 | 2.73 | 2.73 | 2.78 | 2.73 | 2.73 | 2.73 | 112.0 | 0 | -0.50 | -0.41 | 0.20 | -0.41 | -0.50 | 0 |
| 3.10 | 3.10 | 3.10 | 3.15 | 3.10 | 3.10 | 3.10 | 114.0 | 0 | -0.55 | -0.45 | 0.22 | -0.45 | -0.55 | 0 |
| 3.50 | 3.50 | 3.50 | 3.56 | 3.50 | 3.50 | 3.50 | 116.0 | 0 | -0.61 | -0.50 | 0.24 | -0.50 | -0.61 | 0 |
| 3.95 | 3.95 | 3.95 | 4.02 | 3.95 | 3.95 | 3.95 | 118.0 | 0 | -0.67 | -0.55 | 0.27 | -0.55 | -0.67 | 0 |
| 4.44 | 4.44 | 4.44 | 4.52 | 4.44 | 4.44 | 4.44 | 120.0 | 0 | -0.73 | -0.60 | 0.29 | -0.60 | -0.73 | 0 |
| | | | | 0.00 | 0.00 | 0.00 | | 0 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0 |
| 4.44 | 4.44 | 4.44 | 4.52 | 4.44 | 4.44 | 4.44 | 120.0 | 0 | 0.79 | 0.65 | -0.32 | 0.65 | 0.79 | 0 |
| 4.88 | 4.88 | 4.88 | 4.96 | 4.88 | 4.88 | 4.88 | 121.8 | 0 | 0.74 | 0.60 | -0.30 | 0.60 | 0.74 | 0 |
| 5.28 | 5.28 | 5.28 | 5.37 | 5.28 | 5.28 | 5.28 | 123.6 | 0 | 0.68 | 0.56 | -0.27 | 0.56 | 0.68 | 0 |
| 5.64 | 5.64 | 5.64 | 5.75 | 5.64 | 5.64 | 5.64 | 125.4 | 0 | 0.63 | 0.52 | -0.25 | 0.52 | 0.63 | 0 |
| 5.97 | 5.97 | 5.97 | 6.08 | 5.97 | 5.97 | 5.97 | 127.2 | 0 | 0.59 | 0.48 | -0.24 | 0.48 | 0.59 | 0 |
| 6.27 | 6.27 | 6.27 | 6.38 | 6.27 | 6.27 | 6.27 | 129.0 | 0 | 0.55 | 0.45 | -0.22 | 0.45 | 0.55 | 0 |
| 6.54 | 6.54 | 6.54 | 6.66 | 6.54 | 6.54 | 6.54 | 130.8 | 0 | 0.51 | 0.42 | -0.20 | 0.42 | 0.51 | 0 |
| 6.78 | 6.78 | 6.78 | 6.90 | 6.78 | 6.78 | 6.78 | 132.6 | 0 | 0.48 | 0.39 | -0.19 | 0.39 | 0.48 | 0 |
| 7.00 | 7.00 | 7.00 | 7.12 | 7.00 | 7.00 | 7.00 | 134.4 | 0 | 0.45 | 0.37 | -0.18 | 0.37 | 0.45 | 0 |
| 7.19 | 7.19 | 7.19 | 7.32 | 7.19 | 7.19 | 7.19 | 136.2 | 0 | 0.42 | 0.34 | -0.17 | 0.34 | 0.42 | 0 |
| 7.36 | 7.36 | 7.36 | 7.49 | 7.36 | 7.36 | 7.36 | 138.0 | 0 | 0.40 | 0.33 | -0.16 | 0.33 | 0.40 | 0 |
| | | | | 0.00 | 0.00 | 0.00 | | 0 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0 |

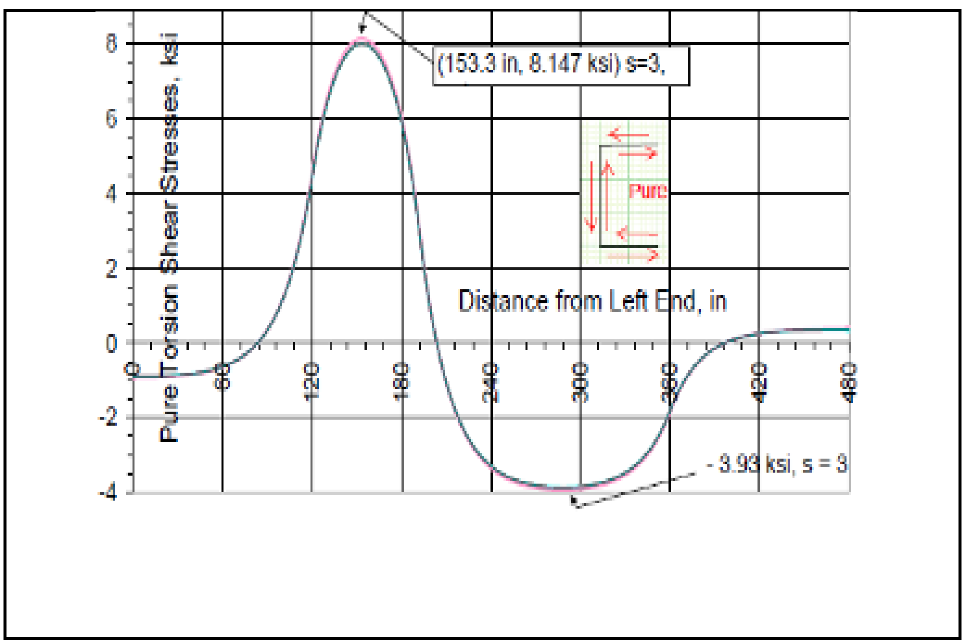
| 0 | 1 | 2 | 3 | 2' | 1' | 0' | z | 0 | 1 | 2 | 3 | 2' | 1' | 0' |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| 7.36 | 7.36 | 7.36 | 7.49 | 7.36 | 7.36 | 7.36 | 138.0 | 0 | 0.40 | 0.32 | -0.16 | 0.32 | 0.40 | 0 |
| 7.50 | 7.50 | 7.50 | 7.64 | 7.50 | 7.50 | 7.50 | 139.8 | 0 | 0.38 | 0.31 | -0.15 | 0.31 | 0.38 | 0 |
| 7.63 | 7.63 | 7.63 | 7.77 | 7.63 | 7.63 | 7.63 | 141.6 | 0 | 0.36 | 0.29 | -0.14 | 0.29 | 0.36 | 0 |
| 7.74 | 7.74 | 7.74 | 7.87 | 7.74 | 7.74 | 7.74 | 143.4 | 0 | 0.35 | 0.28 | -0.14 | 0.28 | 0.35 | 0 |
| 7.83 | 7.83 | 7.83 | 7.97 | 7.83 | 7.83 | 7.83 | 145.2 | 0 | 0.33 | 0.27 | -0.13 | 0.27 | 0.33 | 0 |
| 7.90 | 7.90 | 7.90 | 8.04 | 7.90 | 7.90 | 7.90 | 147.0 | 0 | 0.32 | 0.26 | -0.13 | 0.26 | 0.32 | 0 |
| 7.95 | 7.95 | 7.95 | 8.09 | 7.95 | 7.95 | 7.95 | 148.8 | 0 | 0.32 | 0.26 | -0.13 | 0.26 | 0.32 | 0 |
| 7.99 | 7.99 | 7.99 | 8.13 | 7.99 | 7.99 | 7.99 | 150.6 | 0 | 0.31 | 0.26 | -0.13 | 0.26 | 0.31 | 0 |
| 8.00 | 8.00 | 8.00 | 8.15 | 8.00 | 8.00 | 8.00 | 152.4 | 0 | 0.31 | 0.25 | -0.12 | 0.25 | 0.31 | 0 |
| 8.00 | 8.00 | 8.00 | 8.15 | 8.00 | 8.00 | 8.00 | 154.2 | 0 | 0.31 | 0.25 | -0.12 | 0.25 | 0.31 | 0 |
| 7.99 | 7.99 | 7.99 | 8.13 | 7.99 | 7.99 | 7.99 | 156.0 | 0 | 0.31 | 0.26 | -0.13 | 0.26 | 0.31 | 0 |
| | | | | 0.00 | 0.00 | 0.00 | | 0 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0 |
| 7.99 | 7.99 | 7.99 | 8.13 | 7.99 | 7.99 | 7.99 | 156.0 | 0 | 0.31 | 0.26 | -0.13 | 0.26 | 0.31 | 0 |
| 7.96 | 7.96 | 7.96 | 8.10 | 7.96 | 7.96 | 7.96 | 157.8 | 0 | 0.32 | 0.26 | -0.13 | 0.26 | 0.32 | 0 |
| 7.90 | 7.90 | 7.90 | 8.05 | 7.90 | 7.90 | 7.90 | 159.6 | 0 | 0.32 | 0.26 | -0.13 | 0.26 | 0.32 | 0 |
| 7.84 | 7.84 | 7.84 | 7.98 | 7.84 | 7.84 | 7.84 | 161.4 | 0 | 0.33 | 0.27 | -0.13 | 0.27 | 0.33 | 0 |
| 7.75 | 7.75 | 7.75 | 7.89 | 7.75 | 7.75 | 7.75 | 163.2 | 0 | 0.34 | 0.28 | -0.14 | 0.28 | 0.34 | 0 |
| 7.65 | 7.65 | 7.65 | 7.79 | 7.65 | 7.65 | 7.65 | 165.0 | 0 | 0.36 | 0.29 | -0.14 | 0.29 | 0.36 | 0 |
| 7.53 | 7.53 | 7.53 | 7.67 | 7.53 | 7.53 | 7.53 | 166.8 | 0 | 0.37 | 0.31 | -0.15 | 0.31 | 0.37 | 0 |
| 7.39 | 7.39 | 7.39 | 7.52 | 7.39 | 7.39 | 7.39 | 168.6 | 0 | 0.39 | 0.32 | -0.16 | 0.32 | 0.39 | 0 |
| 7.22 | 7.22 | 7.22 | 7.35 | 7.22 | 7.22 | 7.22 | 170.4 | 0 | 0.42 | 0.34 | -0.17 | 0.34 | 0.42 | 0 |
| 7.04 | 7.04 | 7.04 | 7.16 | 7.04 | 7.04 | 7.04 | 172.2 | 0 | 0.44 | 0.36 | -0.18 | 0.36 | 0.44 | 0 |
| 6.82 | 6.82 | 6.82 | 6.95 | 6.82 | 6.82 | 6.82 | 174.0 | 0 | 0.47 | 0.38 | -0.19 | 0.38 | 0.47 | 0 |
| | | | | 0.00 | 0.00 | 0.00 | | 0 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0 |
| 6.82 | 6.82 | 6.82 | 6.95 | 6.82 | 6.82 | 6.82 | 174.0 | 0 | 0.48 | 0.39 | -0.19 | 0.39 | 0.48 | 0 |
| 6.59 | 6.59 | 6.59 | 6.71 | 6.59 | 6.59 | 6.59 | 175.8 | 0 | 0.50 | 0.41 | -0.20 | 0.41 | 0.50 | 0 |
| 6.32 | 6.32 | 6.32 | 6.44 | 6.32 | 6.32 | 6.32 | 177.6 | 0 | 0.54 | 0.44 | -0.22 | 0.44 | 0.54 | 0 |
| 6.03 | 6.03 | 6.03 | 6.14 | 6.03 | 6.03 | 6.03 | 179.4 | 0 | 0.58 | 0.47 | -0.23 | 0.47 | 0.58 | 0 |
| 5.71 | 5.71 | 5.71 | 5.81 | 5.71 | 5.71 | 5.71 | 181.2 | 0 | 0.62 | 0.51 | -0.25 | 0.51 | 0.62 | 0 |
| 5.35 | 5.35 | 5.35 | 5.45 | 5.35 | 5.35 | 5.35 | 183.0 | 0 | 0.67 | 0.55 | -0.27 | 0.55 | 0.67 | 0 |
| 4.96 | 4.96 | 4.96 | 5.05 | 4.96 | 4.96 | 4.96 | 184.8 | 0 | 0.73 | 0.59 | -0.29 | 0.59 | 0.73 | 0 |
| 4.52 | 4.52 | 4.52 | 4.60 | 4.52 | 4.52 | 4.52 | 186.6 | 0 | 0.79 | 0.64 | -0.32 | 0.64 | 0.79 | 0 |
| 4.05 | 4.05 | 4.05 | 4.12 | 4.05 | 4.05 | 4.05 | 188.4 | 0 | 0.85 | 0.70 | -0.34 | 0.70 | 0.85 | 0 |
| 3.52 | 3.52 | 3.52 | 3.59 | 3.52 | 3.52 | 3.52 | 190.2 | 0 | 0.92 | 0.75 | -0.37 | 0.75 | 0.92 | 0 |
| 2.95 | 2.95 | 2.95 | 3.01 | 2.95 | 2.95 | 2.95 | 192.0 | 0 | 1.00 | 0.81 | -0.40 | 0.81 | 1.00 | 0 |
| | | | | 0.00 | 0.00 | 0.00 | | 0 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0 |
| 2.95 | 2.95 | 2.95 | 3.01 | 2.95 | 2.95 | 2.95 | 192.0 | 0 | -0.95 | -0.77 | 0.38 | -0.77 | -0.95 | 0 |
| 2.29 | 2.29 | 2.29 | 2.33 | 2.29 | 2.29 | 2.29 | 194.1 | 0 | -0.86 | -0.70 | 0.34 | -0.70 | -0.86 | 0 |
| 1.69 | 1.69 | 1.69 | 1.72 | 1.69 | 1.69 | 1.69 | 196.2 | 0 | -0.78 | -0.64 | 0.31 | -0.64 | -0.78 | 0 |
| 1.14 | 1.14 | 1.14 | 1.16 | 1.14 | 1.14 | 1.14 | 198.3 | 0 | -0.71 | -0.58 | 0.28 | -0.58 | -0.71 | 0 |
| 0.65 | 0.65 | 0.65 | 0.66 | 0.65 | 0.65 | 0.65 | 200.4 | 0 | -0.64 | -0.52 | 0.26 | -0.52 | -0.64 | 0 |
| 0.21 | 0.21 | 0.21 | 0.21 | 0.21 | 0.21 | 0.21 | 202.5 | 0 | -0.58 | -0.47 | 0.23 | -0.47 | -0.58 | 0 |
| -0.20 | -0.20 | -0.20 | -0.20 | -0.20 | -0.20 | -0.20 | 204.6 | 0 | -0.52 | -0.43 | 0.21 | -0.43 | -0.52 | 0 |
| -0.56 | -0.56 | -0.56 | -0.57 | -0.56 | -0.56 | -0.56 | 206.7 | 0 | -0.47 | -0.38 | 0.19 | -0.38 | -0.47 | 0 |
| -0.89 | -0.89 | -0.89 | -0.90 | -0.89 | -0.89 | -0.89 | 208.8 | 0 | -0.43 | -0.35 | 0.17 | -0.35 | -0.43 | 0 |

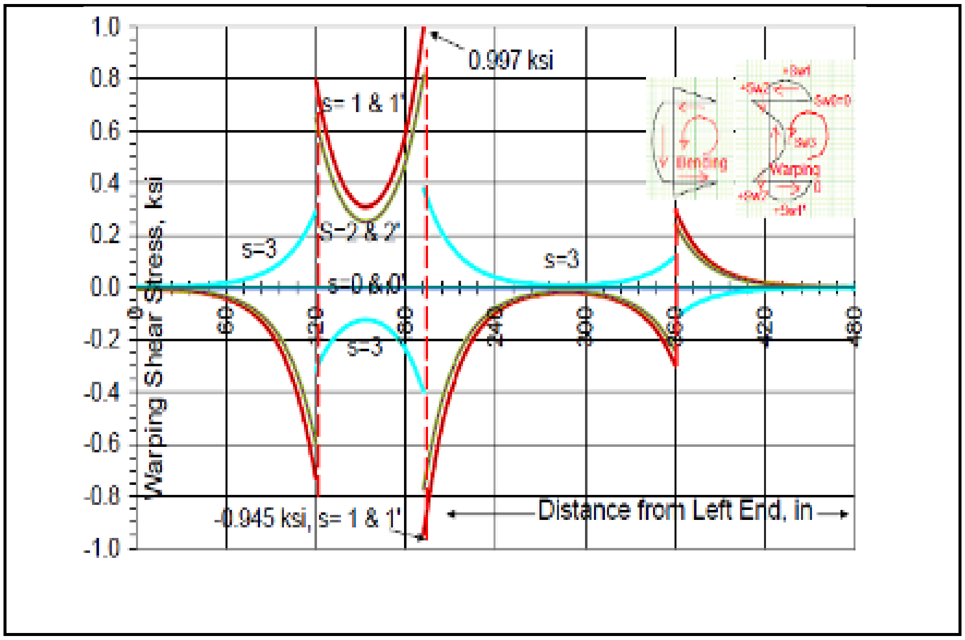
| 0 | 1 | 2 | 3 | 2' | 1' | 0' | z | 0 | 1 | 2 | 3 | 2' | 1' | 0' |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| -1.19 | -1.19 | -1.19 | -1.21 | -1.19 | -1.19 | -1.19 | 210.9 | 0 | -0.39 | -0.32 | 0.16 | -0.32 | -0.39 | 0 |
| -1.45 | -1.45 | -1.45 | -1.48 | -1.45 | -1.45 | -1.45 | 213.0 | 0 | -0.36 | -0.29 | 0.14 | -0.29 | -0.36 | 0 |
| 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 |  | 0 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0 |
| -1.45 | -1.45 | -1.45 | -1.48 | -1.45 | -1.45 | -1.45 | 213.0 | 0 | -0.35 | -0.28 | 0.14 | -0.28 | -0.35 | 0 |
| -1.70 | -1.70 | -1.70 | -1.73 | -1.70 | -1.70 | -1.70 | 215.1 | 0 | -0.32 | -0.26 | 0.13 | -0.26 | -0.32 | 0 |
| -1.92 | -1.92 | -1.92 | -1.95 | -1.92 | -1.92 | -1.92 | 217.2 | 0 | -0.29 | -0.23 | 0.11 | -0.23 | -0.29 | 0 |
| -2.12 | -2.12 | -2.12 | -2.15 | -2.12 | -2.12 | -2.12 | 219.3 | 0 | -0.26 | -0.21 | 0.10 | -0.21 | -0.26 | 0 |
| -2.30 | -2.30 | -2.30 | -2.34 | -2.30 | -2.30 | -2.30 | 221.4 | 0 | -0.23 | -0.19 | 0.09 | -0.19 | -0.23 | 0 |
| -2.46 | -2.46 | -2.46 | -2.50 | -2.46 | -2.46 | -2.46 | 223.5 | 0 | -0.21 | -0.17 | 0.08 | -0.17 | -0.21 | 0 |
| -2.61 | -2.61 | -2.61 | -2.65 | -2.61 | -2.61 | -2.61 | 225.6 | 0 | -0.19 | -0.16 | 0.08 | -0.16 | -0.19 | 0 |
| -2.74 | -2.74 | -2.74 | -2.79 | -2.74 | -2.74 | -2.74 | 227.7 | 0 | -0.17 | -0.14 | 0.07 | -0.14 | -0.17 | 0 |
| -2.86 | -2.86 | -2.86 | -2.91 | -2.86 | -2.86 | -2.86 | 229.8 | 0 | -0.16 | -0.13 | 0.06 | -0.13 | -0.16 | 0 |
| -2.97 | -2.97 | -2.97 | -3.02 | -2.97 | -2.97 | -2.97 | 231.9 | 0 | -0.14 | -0.12 | 0.06 | -0.12 | -0.14 | 0 |
| -3.06 | -3.06 | -3.06 | -3.12 | -3.06 | -3.06 | -3.06 | 234.0 | 0 | -0.13 | -0.11 | 0.05 | -0.11 | -0.13 | 0 |
| 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 |  | 0 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0 |
| -3.06 | -3.06 | -3.06 | -3.12 | -3.06 | -3.06 | -3.06 | 234.0 | 0 | -0.13 | -0.10 | 0.05 | -0.10 | -0.13 | 0 |
| -3.15 | -3.15 | -3.15 | -3.21 | -3.15 | -3.15 | -3.15 | 236.1 | 0 | -0.12 | -0.09 | 0.05 | -0.09 | -0.12 | 0 |
| -3.23 | -3.23 | -3.23 | -3.29 | -3.23 | -3.23 | -3.23 | 238.2 | 0 | -0.11 | -0.09 | 0.04 | -0.09 | -0.11 | 0 |
| -3.31 | -3.31 | -3.31 | -3.37 | -3.31 | -3.31 | -3.31 | 240.3 | 0 | -0.10 | -0.08 | 0.04 | -0.08 | -0.10 | 0 |
| -3.37 | -3.37 | -3.37 | -3.43 | -3.37 | -3.37 | -3.37 | 242.4 | 0 | -0.09 | -0.07 | 0.03 | -0.07 | -0.09 | 0 |
| -3.43 | -3.43 | -3.43 | -3.49 | -3.43 | -3.43 | -3.43 | 244.5 | 0 | -0.08 | -0.06 | 0.03 | -0.06 | -0.08 | 0 |
| -3.48 | -3.48 | -3.48 | -3.54 | -3.48 | -3.48 | -3.48 | 246.6 | 0 | -0.07 | -0.06 | 0.03 | -0.06 | -0.07 | 0 |
| -3.53 | -3.53 | -3.53 | -3.59 | -3.53 | -3.53 | -3.53 | 248.7 | 0 | -0.06 | -0.05 | 0.03 | -0.05 | -0.06 | 0 |
| -3.57 | -3.57 | -3.57 | -3.64 | -3.57 | -3.57 | -3.57 | 250.8 | 0 | -0.06 | -0.05 | 0.02 | -0.05 | -0.06 | 0 |
| -3.61 | -3.61 | -3.61 | -3.68 | -3.61 | -3.61 | -3.61 | 252.9 | 0 | -0.05 | -0.04 | 0.02 | -0.04 | -0.05 | 0 |
| -3.65 | -3.65 | -3.65 | -3.71 | -3.65 | -3.65 | -3.65 | 255.0 | 0 | -0.05 | -0.04 | 0.02 | -0.04 | -0.05 | 0 |
| 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 |  | 0 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0 |
| -3.65 | -3.65 | -3.65 | -3.71 | -3.65 | -3.65 | -3.65 | 255.0 | 0 | -0.05 | -0.04 | 0.02 | -0.04 | -0.05 | 0 |
| -3.68 | -3.68 | -3.68 | -3.74 | -3.68 | -3.68 | -3.68 | 257.1 | 0 | -0.04 | -0.04 | 0.02 | -0.04 | -0.04 | 0 |
| -3.71 | -3.71 | -3.71 | -3.77 | -3.71 | -3.71 | -3.71 | 259.2 | 0 | -0.04 | -0.03 | 0.02 | -0.03 | -0.04 | 0 |
| -3.73 | -3.73 | -3.73 | -3.80 | -3.73 | -3.73 | -3.73 | 261.3 | 0 | -0.04 | -0.03 | 0.01 | -0.03 | -0.04 | 0 |
| -3.75 | -3.75 | -3.75 | -3.82 | -3.75 | -3.75 | -3.75 | 263.4 | 0 | -0.03 | -0.03 | 0.01 | -0.03 | -0.03 | 0 |
| -3.77 | -3.77 | -3.77 | -3.84 | -3.77 | -3.77 | -3.77 | 265.5 | 0 | -0.03 | -0.03 | 0.01 | -0.03 | -0.03 | 0 |
| -3.79 | -3.79 | -3.79 | -3.86 | -3.79 | -3.79 | -3.79 | 267.6 | 0 | -0.03 | -0.02 | 0.01 | -0.02 | -0.03 | 0 |
| -3.80 | -3.80 | -3.80 | -3.87 | -3.80 | -3.80 | -3.80 | 269.7 | 0 | -0.03 | -0.02 | 0.01 | -0.02 | -0.03 | 0 |
| -3.82 | -3.82 | -3.82 | -3.89 | -3.82 | -3.82 | -3.82 | 271.8 | 0 | -0.03 | -0.02 | 0.01 | -0.02 | -0.03 | 0 |
| -3.83 | -3.83 | -3.83 | -3.90 | -3.83 | -3.83 | -3.83 | 273.9 | 0 | -0.02 | -0.02 | 0.01 | -0.02 | -0.02 | 0 |
| -3.84 | -3.84 | -3.84 | -3.91 | -3.84 | -3.84 | -3.84 | 276.0 | 0 | -0.02 | -0.02 | 0.01 | -0.02 | -0.02 | 0 |
| 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 |  | 0 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0 |
| -3.84 | -3.84 | -3.84 | -3.91 | -3.84 | -3.84 | -3.84 | 276.0 | 0 | -0.02 | -0.02 | 0.01 | -0.02 | -0.02 | 0 |
| -3.85 | -3.85 | -3.85 | -3.91 | -3.85 | -3.85 | -3.85 | 278.1 | 0 | -0.02 | -0.02 | 0.01 | -0.02 | -0.02 | 0 |
| -3.85 | -3.85 | -3.85 | -3.92 | -3.85 | -3.85 | -3.85 | 280.2 | 0 | -0.02 | -0.02 | 0.01 | -0.02 | -0.02 | 0 |
| -3.86 | -3.86 | -3.86 | -3.93 | -3.86 | -3.86 | -3.86 | 282.3 | 0 | -0.02 | -0.02 | 0.01 | -0.02 | -0.02 | 0 |
| -3.86 | -3.86 | -3.86 | -3.93 | -3.86 | -3.86 | -3.86 | 284.4 | 0 | -0.02 | -0.02 | 0.01 | -0.02 | -0.02 | 0 |
| -3.86 | -3.86 | -3.86 | -3.93 | -3.86 | -3.86 | -3.86 | 286.5 | 0 | -0.02 | -0.02 | 0.01 | -0.02 | -0.02 | 0 |

| 0 | 1 | 2 | 3 | 2' | 1' | 0' | z | 0 | 1 | 2 | 3 | 2' | 1' | 0' |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| -3.86 | -3.86 | -3.86 | -3.93 | -3.86 | -3.86 | -3.86 | 288.6 | 0 | -0.02 | -0.02 | 0.01 | -0.02 | -0.02 | 0 |
| -3.86 | -3.86 | -3.86 | -3.93 | -3.86 | -3.86 | -3.86 | 290.7 | 0 | -0.02 | -0.02 | 0.01 | -0.02 | -0.02 | 0 |
| -3.86 | -3.86 | -3.86 | -3.93 | -3.86 | -3.86 | -3.86 | 292.8 | 0 | -0.02 | -0.02 | 0.01 | -0.02 | -0.02 | 0 |
| -3.85 | -3.85 | -3.85 | -3.92 | -3.85 | -3.85 | -3.85 | 294.9 | 0 | -0.02 | -0.02 | 0.01 | -0.02 | -0.02 | 0 |
| -3.85 | -3.85 | -3.85 | -3.92 | -3.85 | -3.85 | -3.85 | 297.0 | 0 | -0.02 | -0.02 | 0.01 | -0.02 | -0.02 | 0 |
| 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | | 0 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0 |
| -3.85 | -3.85 | -3.85 | -3.92 | -3.85 | -3.85 | -3.85 | 297.0 | 0 | -0.02 | -0.02 | 0.01 | -0.02 | -0.02 | 0 |
| -3.84 | -3.84 | -3.84 | -3.91 | -3.84 | -3.84 | -3.84 | 299.1 | 0 | -0.02 | -0.02 | 0.01 | -0.02 | -0.02 | 0 |
| -3.83 | -3.83 | -3.83 | -3.90 | -3.83 | -3.83 | -3.83 | 301.2 | 0 | -0.02 | -0.02 | 0.01 | -0.02 | -0.02 | 0 |
| -3.82 | -3.82 | -3.82 | -3.89 | -3.82 | -3.82 | -3.82 | 303.3 | 0 | -0.02 | -0.02 | 0.01 | -0.02 | -0.02 | 0 |
| -3.81 | -3.81 | -3.81 | -3.88 | -3.81 | -3.81 | -3.81 | 305.4 | 0 | -0.03 | -0.02 | 0.01 | -0.02 | -0.03 | 0 |
| -3.80 | -3.80 | -3.80 | -3.86 | -3.80 | -3.80 | -3.80 | 307.5 | 0 | -0.03 | -0.02 | 0.01 | -0.02 | -0.03 | 0 |
| -3.78 | -3.78 | -3.78 | -3.85 | -3.78 | -3.78 | -3.78 | 309.6 | 0 | -0.03 | -0.02 | 0.01 | -0.02 | -0.03 | 0 |
| -3.76 | -3.76 | -3.76 | -3.83 | -3.76 | -3.76 | -3.76 | 311.7 | 0 | -0.03 | -0.03 | 0.01 | -0.03 | -0.03 | 0 |
| -3.74 | -3.74 | -3.74 | -3.81 | -3.74 | -3.74 | -3.74 | 313.8 | 0 | -0.04 | -0.03 | 0.01 | -0.03 | -0.04 | 0 |
| -3.72 | -3.72 | -3.72 | -3.78 | -3.72 | -3.72 | -3.72 | 315.9 | 0 | -0.04 | -0.03 | 0.02 | -0.03 | -0.04 | 0 |
| -3.69 | -3.69 | -3.69 | -3.76 | -3.69 | -3.69 | -3.69 | 318.0 | 0 | -0.04 | -0.03 | 0.02 | -0.03 | -0.04 | 0 |
| 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | | 0 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0 |
| -3.69 | -3.69 | -3.69 | -3.76 | -3.69 | -3.69 | -3.69 | 318.0 | 0 | -0.04 | -0.04 | 0.02 | -0.04 | -0.04 | 0 |
| -3.66 | -3.66 | -3.66 | -3.73 | -3.66 | -3.66 | -3.66 | 320.1 | 0 | -0.05 | -0.04 | 0.02 | -0.04 | -0.05 | 0 |
| -3.63 | -3.63 | -3.63 | -3.69 | -3.63 | -3.63 | -3.63 | 322.2 | 0 | -0.05 | -0.04 | 0.02 | -0.04 | -0.05 | 0 |
| -3.59 | -3.59 | -3.59 | -3.66 | -3.59 | -3.59 | -3.59 | 324.3 | 0 | -0.06 | -0.05 | 0.02 | -0.05 | -0.06 | 0 |
| -3.55 | -3.55 | -3.55 | -3.62 | -3.55 | -3.55 | -3.55 | 326.4 | 0 | -0.06 | -0.05 | 0.02 | -0.05 | -0.06 | 0 |
| -3.51 | -3.51 | -3.51 | -3.57 | -3.51 | -3.51 | -3.51 | 328.5 | 0 | -0.07 | -0.06 | 0.03 | -0.06 | -0.07 | 0 |
| -3.46 | -3.46 | -3.46 | -3.52 | -3.46 | -3.46 | -3.46 | 330.6 | 0 | -0.07 | -0.06 | 0.03 | -0.06 | -0.07 | 0 |
| -3.40 | -3.40 | -3.40 | -3.46 | -3.40 | -3.40 | -3.40 | 332.7 | 0 | -0.08 | -0.07 | 0.03 | -0.07 | -0.08 | 0 |
| -3.34 | -3.34 | -3.34 | -3.40 | -3.34 | -3.34 | -3.34 | 334.8 | 0 | -0.09 | -0.07 | 0.04 | -0.07 | -0.09 | 0 |
| -3.27 | -3.27 | -3.27 | -3.33 | -3.27 | -3.27 | -3.27 | 336.9 | 0 | -0.10 | -0.08 | 0.04 | -0.08 | -0.10 | 0 |
| -3.19 | -3.19 | -3.19 | -3.25 | -3.19 | -3.19 | -3.19 | 339.0 | 0 | -0.11 | -0.09 | 0.04 | -0.09 | -0.11 | 0 |
| 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | | 0 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0 |
| -3.19 | -3.19 | -3.19 | -3.25 | -3.19 | -3.19 | -3.19 | 339.0 | 0 | -0.11 | -0.09 | 0.05 | -0.09 | -0.11 | 0 |
| -3.11 | -3.11 | -3.11 | -3.17 | -3.11 | -3.11 | -3.11 | 341.1 | 0 | -0.12 | -0.10 | 0.05 | -0.10 | -0.12 | 0 |
| -3.02 | -3.02 | -3.02 | -3.07 | -3.02 | -3.02 | -3.02 | 343.2 | 0 | -0.13 | -0.11 | 0.05 | -0.11 | -0.13 | 0 |
| -2.91 | -2.91 | -2.91 | -2.97 | -2.91 | -2.91 | -2.91 | 345.3 | 0 | -0.15 | -0.12 | 0.06 | -0.12 | -0.15 | 0 |
| -2.80 | -2.80 | -2.80 | -2.85 | -2.80 | -2.80 | -2.80 | 347.4 | 0 | -0.16 | -0.13 | 0.07 | -0.13 | -0.16 | 0 |
| -2.67 | -2.67 | -2.67 | -2.72 | -2.67 | -2.67 | -2.67 | 349.5 | 0 | -0.18 | -0.15 | 0.07 | -0.15 | -0.18 | 0 |
| -2.53 | -2.53 | -2.53 | -2.58 | -2.53 | -2.53 | -2.53 | 351.6 | 0 | -0.20 | -0.16 | 0.08 | -0.16 | -0.20 | 0 |
| -2.38 | -2.38 | -2.38 | -2.42 | -2.38 | -2.38 | -2.38 | 353.7 | 0 | -0.22 | -0.18 | 0.09 | -0.18 | -0.22 | 0 |
| -2.21 | -2.21 | -2.21 | -2.25 | -2.21 | -2.21 | -2.21 | 355.8 | 0 | -0.25 | -0.20 | 0.10 | -0.20 | -0.25 | 0 |
| -2.02 | -2.02 | -2.02 | -2.05 | -2.02 | -2.02 | -2.02 | 357.9 | 0 | -0.27 | -0.22 | 0.11 | -0.22 | -0.27 | 0 |
| -1.81 | -1.81 | -1.81 | -1.84 | -1.81 | -1.81 | -1.81 | 360.0 | 0 | -0.30 | -0.24 | 0.12 | -0.24 | -0.30 | 0 |
| | | | | 0.00 | 0.00 | 0.00 | | 0 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0 |
| -1.81 | -1.81 | -1.81 | -1.84 | -1.81 | -1.81 | -1.81 | 360.0 | 0 | 0.30 | 0.24 | -0.12 | 0.24 | 0.30 | 0 |
| -1.61 | -1.61 | -1.61 | -1.64 | -1.61 | -1.61 | -1.61 | 362.0 | 0 | 0.27 | 0.22 | -0.11 | 0.22 | 0.27 | 0 |
| -1.43 | -1.43 | -1.43 | -1.45 | -1.43 | -1.43 | -1.43 | 364.0 | 0 | 0.25 | 0.20 | -0.10 | 0.20 | 0.25 | 0 |

| 0 | 1 | 2 | 3 | 2' | 1' | 0' | z | 0 | 1 | 2 | 3 | 2' | 1' | 0' |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| -1.26 | -1.26 | -1.26 | -1.28 | -1.26 | -1.26 | -1.26 | 366.0 | 0 | 0.23 | 0.18 | -0.09 | 0.18 | 0.23 | 0 |
| -1.11 | -1.11 | -1.11 | -1.13 | -1.11 | -1.11 | -1.11 | 368.0 | 0 | 0.20 | 0.17 | -0.08 | 0.17 | 0.20 | 0 |
| -0.98 | -0.98 | -0.98 | -0.99 | -0.98 | -0.98 | -0.98 | 370.0 | 0 | 0.19 | 0.15 | -0.07 | 0.15 | 0.19 | 0 |
| -0.85 | -0.85 | -0.85 | -0.87 | -0.85 | -0.85 | -0.85 | 372.0 | 0 | 0.17 | 0.14 | -0.07 | 0.14 | 0.17 | 0 |
| -0.74 | -0.74 | -0.74 | -0.75 | -0.74 | -0.74 | -0.74 | 374.0 | 0 | 0.15 | 0.13 | -0.06 | 0.13 | 0.15 | 0 |
| -0.64 | -0.64 | -0.64 | -0.65 | -0.64 | -0.64 | -0.64 | 376.0 | 0 | 0.14 | 0.11 | -0.06 | 0.11 | 0.14 | 0 |
| -0.54 | -0.54 | -0.54 | -0.55 | -0.54 | -0.54 | -0.54 | 378.0 | 0 | 0.13 | 0.10 | -0.05 | 0.10 | 0.13 | 0 |
| -0.46 | -0.46 | -0.46 | -0.47 | -0.46 | -0.46 | -0.46 | 380.0 | 0 | 0.12 | 0.10 | -0.05 | 0.10 | 0.12 | 0 |
| 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 |  | 0 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0 |
| -0.46 | -0.46 | -0.46 | -0.47 | -0.46 | -0.46 | -0.46 | 380.0 | 0 | 0.11 | 0.09 | -0.05 | 0.09 | 0.11 | 0 |
| -0.38 | -0.38 | -0.38 | -0.39 | -0.38 | -0.38 | -0.38 | 382.0 | 0 | 0.10 | 0.09 | -0.04 | 0.09 | 0.10 | 0 |
| -0.31 | -0.31 | -0.31 | -0.32 | -0.31 | -0.31 | -0.31 | 384.0 | 0 | 0.10 | 0.08 | -0.04 | 0.08 | 0.10 | 0 |
| -0.25 | -0.25 | -0.25 | -0.25 | -0.25 | -0.25 | -0.25 | 386.0 | 0 | 0.09 | 0.07 | -0.03 | 0.07 | 0.09 | 0 |
| -0.19 | -0.19 | -0.19 | -0.20 | -0.19 | -0.19 | -0.19 | 388.0 | 0 | 0.08 | 0.06 | -0.03 | 0.06 | 0.08 | 0 |
| -0.14 | -0.14 | -0.14 | -0.14 | -0.14 | -0.14 | -0.14 | 390.0 | 0 | 0.07 | 0.06 | -0.03 | 0.06 | 0.07 | 0 |
| -0.09 | -0.09 | -0.09 | -0.09 | -0.09 | -0.09 | -0.09 | 392.0 | 0 | 0.06 | 0.05 | -0.03 | 0.05 | 0.06 | 0 |
| -0.05 | -0.05 | -0.05 | -0.05 | -0.05 | -0.05 | -0.05 | 394.0 | 0 | 0.06 | 0.05 | -0.02 | 0.05 | 0.06 | 0 |
| -0.01 | -0.01 | -0.01 | -0.01 | -0.01 | -0.01 | -0.01 | 396.0 | 0 | 0.05 | 0.04 | -0.02 | 0.04 | 0.05 | 0 |
| 0.03 | 0.03 | 0.03 | 0.03 | 0.03 | 0.03 | 0.03 | 398.0 | 0 | 0.05 | 0.04 | -0.02 | 0.04 | 0.05 | 0 |
| 0.06 | 0.06 | 0.06 | 0.06 | 0.06 | 0.06 | 0.06 | 400.0 | 0 | 0.04 | 0.04 | -0.02 | 0.04 | 0.04 | 0 |
| 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 |  | 0 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0 |
| 0.06 | 0.06 | 0.06 | 0.06 | 0.06 | 0.06 | 0.06 | 400.0 | 0 | 0.04 | 0.04 | -0.02 | 0.04 | 0.04 | 0 |
| 0.09 | 0.09 | 0.09 | 0.09 | 0.09 | 0.09 | 0.09 | 402.0 | 0 | 0.04 | 0.03 | -0.02 | 0.03 | 0.04 | 0 |
| 0.11 | 0.11 | 0.11 | 0.12 | 0.11 | 0.11 | 0.11 | 404.0 | 0 | 0.04 | 0.03 | -0.01 | 0.03 | 0.04 | 0 |
| 0.14 | 0.14 | 0.14 | 0.14 | 0.14 | 0.14 | 0.14 | 406.0 | 0 | 0.03 | 0.03 | -0.01 | 0.03 | 0.03 | 0 |
| 0.16 | 0.16 | 0.16 | 0.16 | 0.16 | 0.16 | 0.16 | 408.0 | 0 | 0.03 | 0.02 | -0.01 | 0.02 | 0.03 | 0 |
| 0.18 | 0.18 | 0.18 | 0.18 | 0.18 | 0.18 | 0.18 | 410.0 | 0 | 0.03 | 0.02 | -0.01 | 0.02 | 0.03 | 0 |
| 0.20 | 0.20 | 0.20 | 0.20 | 0.20 | 0.20 | 0.20 | 412.0 | 0 | 0.02 | 0.02 | -0.01 | 0.02 | 0.02 | 0 |
| 0.22 | 0.22 | 0.22 | 0.22 | 0.22 | 0.22 | 0.22 | 414.0 | 0 | 0.02 | 0.02 | -0.01 | 0.02 | 0.02 | 0 |
| 0.23 | 0.23 | 0.23 | 0.23 | 0.23 | 0.23 | 0.23 | 416.0 | 0 | 0.02 | 0.02 | -0.01 | 0.02 | 0.02 | 0 |
| 0.24 | 0.24 | 0.24 | 0.25 | 0.24 | 0.24 | 0.24 | 418.0 | 0 | 0.02 | 0.02 | -0.01 | 0.02 | 0.02 | 0 |
| 0.26 | 0.26 | 0.26 | 0.26 | 0.26 | 0.26 | 0.26 | 420.0 | 0 | 0.02 | 0.01 | -0.01 | 0.01 | 0.02 | 0 |
| 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 |  | 0 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0 |
| 0.26 | 0.26 | 0.26 | 0.26 | 0.26 | 0.26 | 0.26 | 420.0 | 0 | 0.02 | 0.01 | -0.01 | 0.01 | 0.02 | 0 |
| 0.27 | 0.27 | 0.27 | 0.27 | 0.27 | 0.27 | 0.27 | 422.0 | 0 | 0.02 | 0.01 | -0.01 | 0.01 | 0.02 | 0 |
| 0.28 | 0.28 | 0.28 | 0.28 | 0.28 | 0.28 | 0.28 | 424.0 | 0 | 0.01 | 0.01 | -0.01 | 0.01 | 0.01 | 0 |
| 0.29 | 0.29 | 0.29 | 0.29 | 0.29 | 0.29 | 0.29 | 426.0 | 0 | 0.01 | 0.01 | -0.01 | 0.01 | 0.01 | 0 |
| 0.30 | 0.30 | 0.30 | 0.30 | 0.30 | 0.30 | 0.30 | 428.0 | 0 | 0.01 | 0.01 | 0.00 | 0.01 | 0.01 | 0 |
| 0.30 | 0.30 | 0.30 | 0.31 | 0.30 | 0.30 | 0.30 | 430.0 | 0 | 0.01 | 0.01 | 0.00 | 0.01 | 0.01 | 0 |
| 0.31 | 0.31 | 0.31 | 0.32 | 0.31 | 0.31 | 0.31 | 432.0 | 0 | 0.01 | 0.01 | 0.00 | 0.01 | 0.01 | 0 |
| 0.32 | 0.32 | 0.32 | 0.32 | 0.32 | 0.32 | 0.32 | 434.0 | 0 | 0.01 | 0.01 | 0.00 | 0.01 | 0.01 | 0 |
| 0.32 | 0.32 | 0.32 | 0.33 | 0.32 | 0.32 | 0.32 | 436.0 | 0 | 0.01 | 0.01 | 0.00 | 0.01 | 0.01 | 0 |
| 0.33 | 0.33 | 0.33 | 0.33 | 0.33 | 0.33 | 0.33 | 438.0 | 0 | 0.01 | 0.01 | 0.00 | 0.01 | 0.01 | 0 |
| 0.33 | 0.33 | 0.33 | 0.34 | 0.33 | 0.33 | 0.33 | 440.0 | 0 | 0.01 | 0.01 | 0.00 | 0.01 | 0.01 | 0 |
| 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 |  | 0 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0 |

| 0 | 1 | 2 | 3 | 2' | 1' | 0' | z | 0 | 1 | 2 | 3 | 2' | 1' | 0' |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| 0.33 | 0.33 | 0.33 | 0.34 | 0.33 | 0.33 | 0.33 | 440.0 | 0 | 0.01 | 0.01 | 0.00 | 0.01 | 0.01 | 0 |
| 0.34 | 0.34 | 0.34 | 0.34 | 0.34 | 0.34 | 0.34 | 442.0 | 0 | 0.01 | 0.00 | 0.00 | 0.00 | 0.01 | 0 |
| 0.34 | 0.34 | 0.34 | 0.35 | 0.34 | 0.34 | 0.34 | 444.0 | 0 | 0.01 | 0.00 | 0.00 | 0.00 | 0.01 | 0 |
| 0.34 | 0.34 | 0.34 | 0.35 | 0.34 | 0.34 | 0.34 | 446.0 | 0 | 0.01 | 0.00 | 0.00 | 0.00 | 0.01 | 0 |
| 0.35 | 0.35 | 0.35 | 0.35 | 0.35 | 0.35 | 0.35 | 448.0 | 0 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0 |
| 0.35 | 0.35 | 0.35 | 0.36 | 0.35 | 0.35 | 0.35 | 450.0 | 0 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0 |
| 0.35 | 0.35 | 0.35 | 0.36 | 0.35 | 0.35 | 0.35 | 452.0 | 0 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0 |
| 0.35 | 0.35 | 0.35 | 0.36 | 0.35 | 0.35 | 0.35 | 454.0 | 0 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0 |
| 0.36 | 0.36 | 0.36 | 0.36 | 0.36 | 0.36 | 0.36 | 456.0 | 0 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0 |
| 0.36 | 0.36 | 0.36 | 0.37 | 0.36 | 0.36 | 0.36 | 458.0 | 0 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0 |
| 0.36 | 0.36 | 0.36 | 0.37 | 0.36 | 0.36 | 0.36 | 460.0 | 0 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0 |
| 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | | 0 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0 |
| 0.36 | 0.36 | 0.36 | 0.37 | 0.36 | 0.36 | 0.36 | 460.0 | 0 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0 |
| 0.36 | 0.36 | 0.36 | 0.37 | 0.36 | 0.36 | 0.36 | 462.0 | 0 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0 |
| 0.36 | 0.36 | 0.36 | 0.37 | 0.36 | 0.36 | 0.36 | 464.0 | 0 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0 |
| 0.36 | 0.36 | 0.36 | 0.37 | 0.36 | 0.36 | 0.36 | 466.0 | 0 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0 |
| 0.36 | 0.36 | 0.36 | 0.37 | 0.36 | 0.36 | 0.36 | 468.0 | 0 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0 |
| 0.37 | 0.37 | 0.37 | 0.37 | 0.37 | 0.37 | 0.37 | 470.0 | 0 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0 |
| 0.37 | 0.37 | 0.37 | 0.37 | 0.37 | 0.37 | 0.37 | 472.0 | 0 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0 |
| 0.37 | 0.37 | 0.37 | 0.37 | 0.37 | 0.37 | 0.37 | 474.0 | 0 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0 |
| 0.37 | 0.37 | 0.37 | 0.37 | 0.37 | 0.37 | 0.37 | 476.0 | 0 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0 |
| 0.37 | 0.37 | 0.37 | 0.37 | 0.37 | 0.37 | 0.37 | 478.0 | 0 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0 |
| 0.37 | 0.37 | 0.37 | 0.37 | 0.37 | 0.37 | 0.37 | 480.0 | 0 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0 |






Pure and Warping Torsion Shear Stresses

Values for Bending Shear Stress

| s | 0 | 1 | 2 | 3 | 2' | 1' | 0' |
|---|---|---|---|---|---|---|---|
| t | 0.501 | 0.501 | 0.501 | 0.510 | 0.501 | 0.501 | 0.501 |
| Q | 0.000 | 5.479 | 7.830 | 17.000 | 7.830 | 5.479 | 0.000 |
| | Sx = 27 in^3 | | | | | **Ix** | 162.0 |

Table of Bending Stresses

| | Shear Stresses, V*Q/ I*t | | | | | | | Normal Stresses, M/ Sx | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| "s"= | 0 | 1 | 2 | 3 | 2' | 1' | 0' | 0 | 1 | 2 | 3 | 2' | 1' | 0' |
| z | | | | | | | | | | | | | | |
| 0.00 | 0 | 0.13 | 0.19 | 0.41 | 0.19 | 0.13 | 0 | 0.00 | 0.00 | 0.00 | 0 | 0.00 | 0.00 | 0.00 |
| 2.00 | 0 | 0.13 | 0.19 | 0.41 | 0.19 | 0.13 | 0 | 0.15 | 0.15 | 0.15 | 0 | -0.15 | -0.15 | -0.15 |
| 4.00 | 0 | 0.13 | 0.19 | 0.41 | 0.19 | 0.13 | 0 | 0.30 | 0.30 | 0.30 | 0 | -0.30 | -0.30 | -0.30 |
| 6.00 | 0 | 0.13 | 0.19 | 0.41 | 0.19 | 0.13 | 0 | 0.44 | 0.44 | 0.44 | 0 | -0.44 | -0.44 | -0.44 |
| 8.00 | 0 | 0.13 | 0.19 | 0.41 | 0.19 | 0.13 | 0 | 0.59 | 0.59 | 0.59 | 0 | -0.59 | -0.59 | -0.59 |
| 10.00 | 0 | 0.13 | 0.19 | 0.41 | 0.19 | 0.13 | 0 | 0.74 | 0.74 | 0.74 | 0 | -0.74 | -0.74 | -0.74 |
| 12.00 | 0 | 0.13 | 0.19 | 0.41 | 0.19 | 0.13 | 0 | 0.89 | 0.89 | 0.89 | 0 | -0.89 | -0.89 | -0.89 |
| 14.00 | 0 | 0.13 | 0.19 | 0.41 | 0.19 | 0.13 | 0 | 1.03 | 1.03 | 1.03 | 0 | -1.03 | -1.03 | -1.03 |
| 16.00 | 0 | 0.13 | 0.19 | 0.41 | 0.19 | 0.13 | 0 | 1.18 | 1.18 | 1.18 | 0 | -1.18 | -1.18 | -1.18 |
| 18.00 | 0 | 0.13 | 0.19 | 0.41 | 0.19 | 0.13 | 0 | 1.33 | 1.33 | 1.33 | 0 | -1.33 | -1.33 | -1.33 |
| 20.00 | 0 | 0.13 | 0.19 | 0.41 | 0.19 | 0.13 | 0 | 1.48 | 1.48 | 1.48 | 0 | -1.48 | -1.48 | -1.48 |
| | 0 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0 | 0.00 | 0.00 | 0.00 | 0 | 0.00 | 0.00 | 0.00 |
| 20.00 | 0 | 0.13 | 0.19 | 0.41 | 0.19 | 0.13 | 0 | 1.48 | 1.48 | 1.48 | 0 | -1.48 | -1.48 | -1.48 |
| 22.00 | 0 | 0.13 | 0.19 | 0.41 | 0.19 | 0.13 | 0 | 1.63 | 1.63 | 1.63 | 0 | -1.63 | -1.63 | -1.63 |
| 24.00 | 0 | 0.13 | 0.19 | 0.41 | 0.19 | 0.13 | 0 | 1.77 | 1.77 | 1.77 | 0 | -1.77 | -1.77 | -1.77 |

| | | | | | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| 26.0 | 0 | 0.13 | 0.19 | 0.41 | 0.19 | 0.13 | 0 | 1.92 | 1.92 | 1.92 | 0 | -1.92 | -1.92 | -1.92 |
| 28.0 | 0 | 0.13 | 0.19 | 0.41 | 0.19 | 0.13 | 0 | 2.07 | 2.07 | 2.07 | 0 | -2.07 | -2.07 | -2.07 |
| 30.0 | 0 | 0.13 | 0.19 | 0.41 | 0.19 | 0.13 | 0 | 2.22 | 2.22 | 2.22 | 0 | -2.22 | -2.22 | -2.22 |
| 32.0 | 0 | 0.13 | 0.19 | 0.41 | 0.19 | 0.13 | 0 | 2.36 | 2.36 | 2.36 | 0 | -2.36 | -2.36 | -2.36 |
| 34.0 | 0 | 0.13 | 0.19 | 0.41 | 0.19 | 0.13 | 0 | 2.51 | 2.51 | 2.51 | 0 | -2.51 | -2.51 | -2.51 |
| 36.0 | 0 | 0.13 | 0.19 | 0.41 | 0.19 | 0.13 | 0 | 2.66 | 2.66 | 2.66 | 0 | -2.66 | -2.66 | -2.66 |
| 38.0 | 0 | 0.13 | 0.19 | 0.41 | 0.19 | 0.13 | 0 | 2.81 | 2.81 | 2.81 | 0 | -2.81 | -2.81 | -2.81 |
| 40.0 | 0 | 0.13 | 0.19 | 0.41 | 0.19 | 0.13 | 0 | 2.96 | 2.96 | 2.96 | 0 | -2.96 | -2.96 | -2.96 |
| | 0 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0 | 0.00 | 0.00 | 0.00 | 0 | 0.00 | 0.00 | 0.00 |
| 40.0 | 0 | 0.13 | 0.19 | 0.41 | 0.19 | 0.13 | 0 | 2.96 | 2.96 | 2.96 | 0 | -2.96 | -2.96 | -2.96 |
| 42.0 | 0 | 0.13 | 0.19 | 0.41 | 0.19 | 0.13 | 0 | 3.10 | 3.10 | 3.10 | 0 | -3.10 | -3.10 | -3.10 |
| 44.0 | 0 | 0.13 | 0.19 | 0.41 | 0.19 | 0.13 | 0 | 3.25 | 3.25 | 3.25 | 0 | -3.25 | -3.25 | -3.25 |
| 46.0 | 0 | 0.13 | 0.19 | 0.41 | 0.19 | 0.13 | 0 | 3.40 | 3.40 | 3.40 | 0 | -3.40 | -3.40 | -3.40 |
| 48.0 | 0 | 0.13 | 0.19 | 0.41 | 0.19 | 0.13 | 0 | 3.55 | 3.55 | 3.55 | 0 | -3.55 | -3.55 | -3.55 |
| 50.0 | 0 | 0.13 | 0.19 | 0.41 | 0.19 | 0.13 | 0 | 3.69 | 3.69 | 3.69 | 0 | -3.69 | -3.69 | -3.69 |
| 52.0 | 0 | 0.13 | 0.19 | 0.41 | 0.19 | 0.13 | 0 | 3.84 | 3.84 | 3.84 | 0 | -3.84 | -3.84 | -3.84 |
| 54.0 | 0 | 0.13 | 0.19 | 0.41 | 0.19 | 0.13 | 0 | 3.99 | 3.99 | 3.99 | 0 | -3.99 | -3.99 | -3.99 |
| 56.0 | 0 | 0.13 | 0.19 | 0.41 | 0.19 | 0.13 | 0 | 4.14 | 4.14 | 4.14 | 0 | -4.14 | -4.14 | -4.14 |
| 58.0 | 0 | 0.13 | 0.19 | 0.41 | 0.19 | 0.13 | 0 | 4.29 | 4.29 | 4.29 | 0 | -4.29 | -4.29 | -4.29 |
| 60.0 | 0 | 0.13 | 0.19 | 0.41 | 0.19 | 0.13 | 0 | 4.43 | 4.43 | 4.43 | 0 | -4.43 | -4.43 | -4.43 |
| | 0 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0 | 0.00 | 0.00 | 0.00 | 0 | 0.00 | 0.00 | 0.00 |
| | Shear Stresses, V*Q/ I*t | | | | | | | Normal Stresses, M/ Sx | | | | | | |
| “s”= | 0 | 1 | 2 | 3 | 2' | 1' | 0' | 0 | 1 | 2 | 3 | 2' | 1' | 0' |
| z | | | | | | | | | | | | | | |
| 60.0 | 0 | 0.13 | 0.19 | 0.41 | 0.19 | 0.13 | 0 | 4.43 | 4.43 | 4.43 | 0 | -4.43 | -4.43 | -4.43 |
| 62.0 | 0 | 0.13 | 0.19 | 0.41 | 0.19 | 0.13 | 0 | 4.58 | 4.58 | 4.58 | 0 | -4.58 | -4.58 | -4.58 |
| 64.0 | 0 | 0.13 | 0.19 | 0.41 | 0.19 | 0.13 | 0 | 4.73 | 4.73 | 4.73 | 0 | -4.73 | -4.73 | -4.73 |
| 66.0 | 0 | 0.13 | 0.19 | 0.41 | 0.19 | 0.13 | 0 | 4.88 | 4.88 | 4.88 | 0 | -4.88 | -4.88 | -4.88 |
| 68.0 | 0 | 0.13 | 0.19 | 0.41 | 0.19 | 0.13 | 0 | 5.02 | 5.02 | 5.02 | 0 | -5.02 | -5.02 | -5.02 |
| 70.0 | 0 | 0.13 | 0.19 | 0.41 | 0.19 | 0.13 | 0 | 5.17 | 5.17 | 5.17 | 0 | -5.17 | -5.17 | -5.17 |
| 72.0 | 0 | 0.13 | 0.19 | 0.41 | 0.19 | 0.13 | 0 | 5.32 | 5.32 | 5.32 | 0 | -5.32 | -5.32 | -5.32 |
| 74.0 | 0 | 0.13 | 0.19 | 0.41 | 0.19 | 0.13 | 0 | 5.47 | 5.47 | 5.47 | 0 | -5.47 | -5.47 | -5.47 |
| 76.0 | 0 | 0.13 | 0.19 | 0.41 | 0.19 | 0.13 | 0 | 5.62 | 5.62 | 5.62 | 0 | -5.62 | -5.62 | -5.62 |
| 78.0 | 0 | 0.13 | 0.19 | 0.41 | 0.19 | 0.13 | 0 | 5.76 | 5.76 | 5.76 | 0 | -5.76 | -5.76 | -5.76 |
| 80.0 | 0 | 0.13 | 0.19 | 0.41 | 0.19 | 0.13 | 0 | 5.91 | 5.91 | 5.91 | 0 | -5.91 | -5.91 | -5.91 |
| | 0 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0 | 0.00 | 0.00 | 0.00 | 0 | 0.00 | 0.00 | 0.00 |
| 80.0 | 0 | 0.13 | 0.19 | 0.41 | 0.19 | 0.13 | 0 | 5.91 | 5.91 | 5.91 | 0 | -5.91 | -5.91 | -5.91 |
| 82.0 | 0 | 0.13 | 0.19 | 0.41 | 0.19 | 0.13 | 0 | 6.06 | 6.06 | 6.06 | 0 | -6.06 | -6.06 | -6.06 |
| 84.0 | 0 | 0.13 | 0.19 | 0.41 | 0.19 | 0.13 | 0 | 6.21 | 6.21 | 6.21 | 0 | -6.21 | -6.21 | -6.21 |
| 86.0 | 0 | 0.13 | 0.19 | 0.41 | 0.19 | 0.13 | 0 | 6.35 | 6.35 | 6.35 | 0 | -6.35 | -6.35 | -6.35 |
| 88.0 | 0 | 0.13 | 0.19 | 0.41 | 0.19 | 0.13 | 0 | 6.50 | 6.50 | 6.50 | 0 | -6.50 | -6.50 | -6.50 |
| 90.0 | 0 | 0.13 | 0.19 | 0.41 | 0.19 | 0.13 | 0 | 6.65 | 6.65 | 6.65 | 0 | -6.65 | -6.65 | -6.65 |
| 92.0 | 0 | 0.13 | 0.19 | 0.41 | 0.19 | 0.13 | 0 | 6.80 | 6.80 | 6.80 | 0 | -6.80 | -6.80 | -6.80 |
| 94.0 | 0 | 0.13 | 0.19 | 0.41 | 0.19 | 0.13 | 0 | 6.95 | 6.95 | 6.95 | 0 | -6.95 | -6.95 | -6.95 |

| | | | | | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| 96.0 | 0 | 0.13 | 0.19 | 0.41 | 0.19 | 0.13 | 0 | 7.09 | 7.09 | 7.09 | 0 | -7.09 | -7.09 | -7.09 |
| 98.0 | 0 | 0.13 | 0.19 | 0.41 | 0.19 | 0.13 | 0 | 7.24 | 7.24 | 7.24 | 0 | -7.24 | -7.24 | -7.24 |
| 100.0 | 0 | 0.13 | 0.19 | 0.41 | 0.19 | 0.13 | 0 | 7.39 | 7.39 | 7.39 | 0 | -7.39 | -7.39 | -7.39 |
| | 0 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0 | 0.00 | 0.00 | 0.00 | 0 | 0.00 | 0.00 | 0.00 |
| 100.0 | 0 | 0.13 | 0.19 | 0.41 | 0.19 | 0.13 | 0 | 7.39 | 7.39 | 7.39 | 0 | -7.39 | -7.39 | -7.39 |
| 102.0 | 0 | 0.13 | 0.19 | 0.41 | 0.19 | 0.13 | 0 | 7.54 | 7.54 | 7.54 | 0 | -7.54 | -7.54 | -7.54 |
| 104.0 | 0 | 0.13 | 0.19 | 0.41 | 0.19 | 0.13 | 0 | 7.68 | 7.68 | 7.68 | 0 | -7.68 | -7.68 | -7.68 |
| 106.0 | 0 | 0.13 | 0.19 | 0.41 | 0.19 | 0.13 | 0 | 7.83 | 7.83 | 7.83 | 0 | -7.83 | -7.83 | -7.83 |
| 108.0 | 0 | 0.13 | 0.19 | 0.41 | 0.19 | 0.13 | 0 | 7.98 | 7.98 | 7.98 | 0 | -7.98 | -7.98 | -7.98 |
| 110.0 | 0 | 0.13 | 0.19 | 0.41 | 0.19 | 0.13 | 0 | 8.13 | 8.13 | 8.13 | 0 | -8.13 | -8.13 | -8.13 |
| 112.0 | 0 | 0.13 | 0.19 | 0.41 | 0.19 | 0.13 | 0 | 8.28 | 8.28 | 8.28 | 0 | -8.28 | -8.28 | -8.28 |
| 114.0 | 0 | 0.13 | 0.19 | 0.41 | 0.19 | 0.13 | 0 | 8.42 | 8.42 | 8.42 | 0 | -8.42 | -8.42 | -8.42 |
| 116.0 | 0 | 0.13 | 0.19 | 0.41 | 0.19 | 0.13 | 0 | 8.57 | 8.57 | 8.57 | 0 | -8.57 | -8.57 | -8.57 |
| 118.0 | 0 | 0.13 | 0.19 | 0.41 | 0.19 | 0.13 | 0 | 8.72 | 8.72 | 8.72 | 0 | -8.72 | -8.72 | -8.72 |
| 120.0 | 0 | 0.13 | 0.19 | 0.41 | 0.19 | 0.13 | 0 | 8.87 | 8.87 | 8.87 | 0 | -8.87 | -8.87 | -8.87 |
| | | | | | 0.00 | 0.00 | 0 | | | | | 0.00 | 0.00 | 0.00 |
| 120.0 | 0 | -0.50 | -0.72 | -1.53 | -0.72 | -0.50 | 0 | 8.87 | 8.87 | 8.87 | 0 | -8.87 | -8.87 | -8.87 |
| 121.8 | 0 | -0.50 | -0.72 | -1.53 | -0.72 | -0.50 | 0 | 8.37 | 8.37 | 8.37 | 0 | -8.37 | -8.37 | -8.37 |
| 123.6 | 0 | -0.50 | -0.72 | -1.53 | -0.72 | -0.50 | 0 | 7.88 | 7.88 | 7.88 | 0 | -7.88 | -7.88 | -7.88 |
| 125.4 | 0 | -0.50 | -0.72 | -1.53 | -0.72 | -0.50 | 0 | 7.38 | 7.38 | 7.38 | 0 | -7.38 | -7.38 | -7.38 |
| 127.2 | 0 | -0.50 | -0.72 | -1.53 | -0.72 | -0.50 | 0 | 6.89 | 6.89 | 6.89 | 0 | -6.89 | -6.89 | -6.89 |
| 129.0 | 0 | -0.50 | -0.72 | -1.53 | -0.72 | -0.50 | 0 | 6.39 | 6.39 | 6.39 | 0 | -6.39 | -6.39 | -6.39 |
| 130.8 | 0 | -0.50 | -0.72 | -1.53 | -0.72 | -0.50 | 0 | 5.90 | 5.90 | 5.90 | 0 | -5.90 | -5.90 | -5.90 |
| | Shear Stresses, V*Q/ I*t | | | | | | | Normal Stresses, M/ Sx | | | | | | |
| “s”= | 0 | 1 | 2 | 3 | 2' | 1' | 0' | 0 | 1 | 2 | 3 | 2' | 1' | 0' |
| z | | | | | | | | | | | | | | |
| 132.6 | 0 | -0.50 | -0.72 | -1.53 | -0.72 | -0.50 | 0 | 5.40 | 5.40 | 5.40 | 0 | -5.40 | -5.40 | -5.40 |
| 134.4 | 0 | -0.50 | -0.72 | -1.53 | -0.72 | -0.50 | 0 | 4.91 | 4.91 | 4.91 | 0 | -4.91 | -4.91 | -4.91 |
| 136.2 | 0 | -0.50 | -0.72 | -1.53 | -0.72 | -0.50 | 0 | 4.41 | 4.41 | 4.41 | 0 | -4.41 | -4.41 | -4.41 |
| 138.0 | 0 | -0.50 | -0.72 | -1.53 | -0.72 | -0.50 | 0 | 3.92 | 3.92 | 3.92 | 0 | -3.92 | -3.92 | -3.92 |
| | | | | | 0.00 | 0.00 | 0 | | | | | 0.00 | 0.00 | 0.00 |
| 138.0 | 0 | -0.50 | -0.72 | -1.53 | -0.72 | -0.50 | 0 | 3.92 | 3.92 | 3.92 | 0 | -3.92 | -3.92 | -3.92 |
| 139.8 | 0 | -0.50 | -0.72 | -1.53 | -0.72 | -0.50 | 0 | 3.43 | 3.43 | 3.43 | 0 | -3.43 | -3.43 | -3.43 |
| 141.6 | 0 | -0.50 | -0.72 | -1.53 | -0.72 | -0.50 | 0 | 2.93 | 2.93 | 2.93 | 0 | -2.93 | -2.93 | -2.93 |
| 143.4 | 0 | -0.50 | -0.72 | -1.53 | -0.72 | -0.50 | 0 | 2.44 | 2.44 | 2.44 | 0 | -2.44 | -2.44 | -2.44 |
| 145.2 | 0 | -0.50 | -0.72 | -1.53 | -0.72 | -0.50 | 0 | 1.94 | 1.94 | 1.94 | 0 | -1.94 | -1.94 | -1.94 |
| 147.0 | 0 | -0.50 | -0.72 | -1.53 | -0.72 | -0.50 | 0 | 1.45 | 1.45 | 1.45 | 0 | -1.45 | -1.45 | -1.45 |
| 148.8 | 0 | -0.50 | -0.72 | -1.53 | -0.72 | -0.50 | 0 | 0.95 | 0.95 | 0.95 | 0 | -0.95 | -0.95 | -0.95 |
| 150.6 | 0 | -0.50 | -0.72 | -1.53 | -0.72 | -0.50 | 0 | 0.46 | 0.46 | 0.46 | 0 | -0.46 | -0.46 | -0.46 |
| 152.4 | 0 | -0.50 | -0.72 | -1.53 | -0.72 | -0.50 | 0 | -0.04 | -0.04 | -0.04 | 0 | 0.04 | 0.04 | 0.04 |
| 154.2 | 0 | -0.50 | -0.72 | -1.53 | -0.72 | -0.50 | 0 | -0.53 | -0.53 | -0.53 | 0 | 0.53 | 0.53 | 0.53 |
| 156.0 | 0 | -0.50 | -0.72 | -1.53 | -0.72 | -0.50 | 0 | -1.03 | -1.03 | -1.03 | 0 | 1.03 | 1.03 | 1.03 |
| | | | | | 0.00 | 0.00 | 0 | | | | | 0.00 | 0.00 | 0.00 |
| 156.0 | 0 | -0.50 | -0.72 | -1.53 | -0.72 | -0.50 | 0 | -1.03 | -1.03 | -1.03 | 0 | 1.03 | 1.03 | 1.03 |

| | | | | | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| 157.8 | 0 | -0.50 | -0.72 | -1.53 | -0.72 | -0.50 | 0 | -1.52 | -1.52 | -1.52 | 0 | 1.52 | 1.52 | 1.52 |
| 159.6 | 0 | -0.50 | -0.72 | -1.53 | -0.72 | -0.50 | 0 | -2.02 | -2.02 | -2.02 | 0 | 2.02 | 2.02 | 2.02 |
| 161.4 | 0 | -0.50 | -0.72 | -1.53 | -0.72 | -0.50 | 0 | -2.51 | -2.51 | -2.51 | 0 | 2.51 | 2.51 | 2.51 |
| 163.2 | 0 | -0.50 | -0.72 | -1.53 | -0.72 | -0.50 | 0 | -3.01 | -3.01 | -3.01 | 0 | 3.01 | 3.01 | 3.01 |
| 165.0 | 0 | -0.50 | -0.72 | -1.53 | -0.72 | -0.50 | 0 | -3.50 | -3.50 | -3.50 | 0 | 3.50 | 3.50 | 3.50 |
| 166.8 | 0 | -0.50 | -0.72 | -1.53 | -0.72 | -0.50 | 0 | -3.99 | -3.99 | -3.99 | 0 | 3.99 | 3.99 | 3.99 |
| 168.6 | 0 | -0.50 | -0.72 | -1.53 | -0.72 | -0.50 | 0 | -4.49 | -4.49 | -4.49 | 0 | 4.49 | 4.49 | 4.49 |
| 170.4 | 0 | -0.50 | -0.72 | -1.53 | -0.72 | -0.50 | 0 | -4.98 | -4.98 | -4.98 | 0 | 4.98 | 4.98 | 4.98 |
| 172.2 | 0 | -0.50 | -0.72 | -1.53 | -0.72 | -0.50 | 0 | -5.48 | -5.48 | -5.48 | 0 | 5.48 | 5.48 | 5.48 |
| 174.0 | 0 | -0.50 | -0.72 | -1.53 | -0.72 | -0.50 | 0 | -5.97 | -5.97 | -5.97 | 0 | 5.97 | 5.97 | 5.97 |
| | | | | | 0.00 | 0.00 | 0 | | | | | 0.00 | 0.00 | 0.00 |
| 174.0 | 0 | -0.50 | -0.72 | -1.53 | -0.72 | -0.50 | 0 | -5.97 | -5.97 | -5.97 | 0 | 5.97 | 5.97 | 5.97 |
| 175.8 | 0 | -0.50 | -0.72 | -1.53 | -0.72 | -0.50 | 0 | -6.47 | -6.47 | -6.47 | 0 | 6.47 | 6.47 | 6.47 |
| 177.6 | 0 | -0.50 | -0.72 | -1.53 | -0.72 | -0.50 | 0 | -6.96 | -6.96 | -6.96 | 0 | 6.96 | 6.96 | 6.96 |
| 179.4 | 0 | -0.50 | -0.72 | -1.53 | -0.72 | -0.50 | 0 | -7.46 | -7.46 | -7.46 | 0 | 7.46 | 7.46 | 7.46 |
| 181.2 | 0 | -0.50 | -0.72 | -1.53 | -0.72 | -0.50 | 0 | -7.95 | -7.95 | -7.95 | 0 | 7.95 | 7.95 | 7.95 |
| 183.0 | 0 | -0.50 | -0.72 | -1.53 | -0.72 | -0.50 | 0 | -8.45 | -8.45 | -8.45 | 0 | 8.45 | 8.45 | 8.45 |
| 184.8 | 0 | -0.50 | -0.72 | -1.53 | -0.72 | -0.50 | 0 | -8.94 | -8.94 | -8.94 | 0 | 8.94 | 8.94 | 8.94 |
| 186.6 | 0 | -0.50 | -0.72 | -1.53 | -0.72 | -0.50 | 0 | -9.44 | -9.44 | -9.44 | 0 | 9.44 | 9.44 | 9.44 |
| 188.4 | 0 | -0.50 | -0.72 | -1.53 | -0.72 | -0.50 | 0 | -9.93 | -9.93 | -9.93 | 0 | 9.93 | 9.93 | 9.93 |
| 190.2 | 0 | -0.50 | -0.72 | -1.53 | -0.72 | -0.50 | 0 | -10.43 | -10.43 | -10.43 | 0 | 10.43 | 10.43 | 10.43 |
| 192.0 | 0 | -0.50 | -0.72 | -1.53 | -0.72 | -0.50 | 0 | -10.92 | -10.92 | -10.92 | 0 | 10.92 | 10.92 | 10.92 |
| | | | | | 0.00 | 0.00 | 0 | | | | | 0.00 | 0.00 | 0.00 |
| 192.0 | 0 | 0.17 | 0.25 | 0.53 | 0.25 | 0.17 | 0 | -10.92 | -10.92 | -10.92 | 0 | 10.92 | 10.92 | 10.92 |
| 194.1 | 0 | 0.17 | 0.25 | 0.53 | 0.25 | 0.17 | 0 | -10.72 | -10.72 | -10.72 | 0 | 10.72 | 10.72 | 10.72 |
| | Shear Stresses, V*Q/ I*t | | | | | | | Normal Stresses, M/ Sx | | | | | | |
| “s”= | 0 | 1 | 2 | 3 | 2' | 1' | 0' | 0 | 1 | 2 | 3 | 2' | 1' | 0' |
| Z | | | | | | | | | | | | | | |
| 196.2 | 0 | 0.17 | 0.25 | 0.53 | 0.25 | 0.17 | 0 | -10.52 | -10.52 | -10.52 | 0 | 10.52 | 10.52 | 10.52 |
| 198.3 | 0 | 0.17 | 0.25 | 0.53 | 0.25 | 0.17 | 0 | -10.32 | -10.32 | -10.32 | 0 | 10.32 | 10.32 | 10.32 |
| 200.4 | 0 | 0.17 | 0.25 | 0.53 | 0.25 | 0.17 | 0 | -10.12 | -10.12 | -10.12 | 0 | 10.12 | 10.12 | 10.12 |
| 202.5 | 0 | 0.17 | 0.25 | 0.53 | 0.25 | 0.17 | 0 | -9.92 | -9.92 | -9.92 | 0 | 9.92 | 9.92 | 9.92 |
| 204.6 | 0 | 0.17 | 0.25 | 0.53 | 0.25 | 0.17 | 0 | -9.72 | -9.72 | -9.72 | 0 | 9.72 | 9.72 | 9.72 |
| 206.7 | 0 | 0.17 | 0.25 | 0.53 | 0.25 | 0.17 | 0 | -9.52 | -9.52 | -9.52 | 0 | 9.52 | 9.52 | 9.52 |
| 208.8 | 0 | 0.17 | 0.25 | 0.53 | 0.25 | 0.17 | 0 | -9.31 | -9.31 | -9.31 | 0 | 9.31 | 9.31 | 9.31 |
| 210.9 | 0 | 0.17 | 0.25 | 0.53 | 0.25 | 0.17 | 0 | -9.11 | -9.11 | -9.11 | 0 | 9.11 | 9.11 | 9.11 |
| 213.0 | 0 | 0.17 | 0.25 | 0.53 | 0.25 | 0.17 | 0 | -8.91 | -8.91 | -8.91 | 0 | 8.91 | 8.91 | 8.91 |
| | 0 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0 | 0.00 | 0.00 | 0.00 | 0 | 0.00 | 0.00 | 0.00 |
| 213.0 | 0 | 0.17 | 0.25 | 0.53 | 0.25 | 0.17 | 0 | -8.91 | -8.91 | -8.91 | 0 | 8.91 | 8.91 | 8.91 |
| 215.1 | 0 | 0.17 | 0.25 | 0.53 | 0.25 | 0.17 | 0 | -8.71 | -8.71 | -8.71 | 0 | 8.71 | 8.71 | 8.71 |
| 217.2 | 0 | 0.17 | 0.25 | 0.53 | 0.25 | 0.17 | 0 | -8.51 | -8.51 | -8.51 | 0 | 8.51 | 8.51 | 8.51 |
| 219.3 | 0 | 0.17 | 0.25 | 0.53 | 0.25 | 0.17 | 0 | -8.31 | -8.31 | -8.31 | 0 | 8.31 | 8.31 | 8.31 |
| 221.4 | 0 | 0.17 | 0.25 | 0.53 | 0.25 | 0.17 | 0 | -8.11 | -8.11 | -8.11 | 0 | 8.11 | 8.11 | 8.11 |
| 223.5 | 0 | 0.17 | 0.25 | 0.53 | 0.25 | 0.17 | 0 | -7.91 | -7.91 | -7.91 | 0 | 7.91 | 7.91 | 7.91 |

| | | | | | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| 225.6 | 0 | 0.17 | 0.25 | 0.53 | 0.25 | 0.17 | 0 | -7.71 | -7.71 | -7.71 | 0 | 7.71 | 7.71 | 7.71 |
| 227.7 | 0 | 0.17 | 0.25 | 0.53 | 0.25 | 0.17 | 0 | -7.51 | -7.51 | -7.51 | 0 | 7.51 | 7.51 | 7.51 |
| 229.8 | 0 | 0.17 | 0.25 | 0.53 | 0.25 | 0.17 | 0 | -7.31 | -7.31 | -7.31 | 0 | 7.31 | 7.31 | 7.31 |
| 231.9 | 0 | 0.17 | 0.25 | 0.53 | 0.25 | 0.17 | 0 | -7.11 | -7.11 | -7.11 | 0 | 7.11 | 7.11 | 7.11 |
| 234.0 | 0 | 0.17 | 0.25 | 0.53 | 0.25 | 0.17 | 0 | -6.91 | -6.91 | -6.91 | 0 | 6.91 | 6.91 | 6.91 |
| | 0 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0 | 0.00 | 0.00 | 0.00 | 0 | 0.00 | 0.00 | 0.00 |
| 234.0 | 0 | 0.17 | 0.25 | 0.53 | 0.25 | 0.17 | 0 | -6.91 | -6.91 | -6.91 | 0 | 6.91 | 6.91 | 6.91 |
| 236.1 | 0 | 0.17 | 0.25 | 0.53 | 0.25 | 0.17 | 0 | -6.71 | -6.71 | -6.71 | 0 | 6.71 | 6.71 | 6.71 |
| 238.2 | 0 | 0.17 | 0.25 | 0.53 | 0.25 | 0.17 | 0 | -6.51 | -6.51 | -6.51 | 0 | 6.51 | 6.51 | 6.51 |
| 240.3 | 0 | 0.17 | 0.25 | 0.53 | 0.25 | 0.17 | 0 | -6.30 | -6.30 | -6.30 | 0 | 6.30 | 6.30 | 6.30 |
| 242.4 | 0 | 0.17 | 0.25 | 0.53 | 0.25 | 0.17 | 0 | -6.10 | -6.10 | -6.10 | 0 | 6.10 | 6.10 | 6.10 |
| 244.5 | 0 | 0.17 | 0.25 | 0.53 | 0.25 | 0.17 | 0 | -5.90 | -5.90 | -5.90 | 0 | 5.90 | 5.90 | 5.90 |
| 246.6 | 0 | 0.17 | 0.25 | 0.53 | 0.25 | 0.17 | 0 | -5.70 | -5.70 | -5.70 | 0 | 5.70 | 5.70 | 5.70 |
| 248.7 | 0 | 0.17 | 0.25 | 0.53 | 0.25 | 0.17 | 0 | -5.50 | -5.50 | -5.50 | 0 | 5.50 | 5.50 | 5.50 |
| 250.8 | 0 | 0.17 | 0.25 | 0.53 | 0.25 | 0.17 | 0 | -5.30 | -5.30 | -5.30 | 0 | 5.30 | 5.30 | 5.30 |
| 252.9 | 0 | 0.17 | 0.25 | 0.53 | 0.25 | 0.17 | 0 | -5.10 | -5.10 | -5.10 | 0 | 5.10 | 5.10 | 5.10 |
| 255.0 | 0 | 0.17 | 0.25 | 0.53 | 0.25 | 0.17 | 0 | -4.90 | -4.90 | -4.90 | 0 | 4.90 | 4.90 | 4.90 |
| | 0 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0 | 0.00 | 0.00 | 0.00 | 0 | 0.00 | 0.00 | 0.00 |
| 255.0 | 0 | 0.17 | 0.25 | 0.53 | 0.25 | 0.17 | 0 | -4.90 | -4.90 | -4.90 | 0 | 4.90 | 4.90 | 4.90 |
| 257.1 | 0 | 0.17 | 0.25 | 0.53 | 0.25 | 0.17 | 0 | -4.70 | -4.70 | -4.70 | 0 | 4.70 | 4.70 | 4.70 |
| 259.2 | 0 | 0.17 | 0.25 | 0.53 | 0.25 | 0.17 | 0 | -4.50 | -4.50 | -4.50 | 0 | 4.50 | 4.50 | 4.50 |
| 261.3 | 0 | 0.17 | 0.25 | 0.53 | 0.25 | 0.17 | 0 | -4.30 | -4.30 | -4.30 | 0 | 4.30 | 4.30 | 4.30 |
| 263.4 | 0 | 0.17 | 0.25 | 0.53 | 0.25 | 0.17 | 0 | -4.10 | -4.10 | -4.10 | 0 | 4.10 | 4.10 | 4.10 |
| 265.5 | 0 | 0.17 | 0.25 | 0.53 | 0.25 | 0.17 | 0 | -3.90 | -3.90 | -3.90 | 0 | 3.90 | 3.90 | 3.90 |
| 267.6 | 0 | 0.17 | 0.25 | 0.53 | 0.25 | 0.17 | 0 | -3.70 | -3.70 | -3.70 | 0 | 3.70 | 3.70 | 3.70 |
| 269.7 | 0 | 0.17 | 0.25 | 0.53 | 0.25 | 0.17 | 0 | -3.50 | -3.50 | -3.50 | 0 | 3.50 | 3.50 | 3.50 |
| 271.8 | 0 | 0.17 | 0.25 | 0.53 | 0.25 | 0.17 | 0 | -3.29 | -3.29 | -3.29 | 0 | 3.29 | 3.29 | 3.29 |
| | 0 | 1 | 2 | 3 | 2' | 1' | 0' | 0 | 1 | 2 | 3 | 2' | 1' | 0' |
| z | | | | | | | | | | | | | | |
| 273.9 | 0 | 0.17 | 0.25 | 0.53 | 0.25 | 0.17 | 0 | -3.09 | -3.09 | -3.09 | 0 | 3.09 | 3.09 | 3.09 |
| 276.0 | 0 | 0.17 | 0.25 | 0.53 | 0.25 | 0.17 | 0 | -2.89 | -2.89 | -2.89 | 0 | 2.89 | 2.89 | 2.89 |
| | 0 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0 | 0.00 | 0.00 | 0.00 | 0 | 0.00 | 0.00 | 0.00 |
| 276.0 | 0 | 0.17 | 0.25 | 0.53 | 0.25 | 0.17 | 0 | -2.89 | -2.89 | -2.89 | 0 | 2.89 | 2.89 | 2.89 |
| 278.1 | 0 | 0.17 | 0.25 | 0.53 | 0.25 | 0.17 | 0 | -2.69 | -2.69 | -2.69 | 0 | 2.69 | 2.69 | 2.69 |
| 280.2 | 0 | 0.17 | 0.25 | 0.53 | 0.25 | 0.17 | 0 | -2.49 | -2.49 | -2.49 | 0 | 2.49 | 2.49 | 2.49 |
| 282.3 | 0 | 0.17 | 0.25 | 0.53 | 0.25 | 0.17 | 0 | -2.29 | -2.29 | -2.29 | 0 | 2.29 | 2.29 | 2.29 |
| 284.4 | 0 | 0.17 | 0.25 | 0.53 | 0.25 | 0.17 | 0 | -2.09 | -2.09 | -2.09 | 0 | 2.09 | 2.09 | 2.09 |
| 286.5 | 0 | 0.17 | 0.25 | 0.53 | 0.25 | 0.17 | 0 | -1.89 | -1.89 | -1.89 | 0 | 1.89 | 1.89 | 1.89 |
| 288.6 | 0 | 0.17 | 0.25 | 0.53 | 0.25 | 0.17 | 0 | -1.69 | -1.69 | -1.69 | 0 | 1.69 | 1.69 | 1.69 |
| 290.7 | 0 | 0.17 | 0.25 | 0.53 | 0.25 | 0.17 | 0 | -1.49 | -1.49 | -1.49 | 0 | 1.49 | 1.49 | 1.49 |
| 292.8 | 0 | 0.17 | 0.25 | 0.53 | 0.25 | 0.17 | 0 | -1.29 | -1.29 | -1.29 | 0 | 1.29 | 1.29 | 1.29 |
| 294.9 | 0 | 0.17 | 0.25 | 0.53 | 0.25 | 0.17 | 0 | -1.09 | -1.09 | -1.09 | 0 | 1.09 | 1.09 | 1.09 |
| 297.0 | 0 | 0.17 | 0.25 | 0.53 | 0.25 | 0.17 | 0 | -0.89 | -0.89 | -0.89 | 0 | 0.89 | 0.89 | 0.89 |
| | 0 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0 | 0.00 | 0.00 | 0.00 | 0 | 0.00 | 0.00 | 0.00 |

| | | | | | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| 297.0 | 0 | 0.17 | 0.25 | 0.53 | 0.25 | 0.17 | 0 | -0.89 | -0.89 | -0.89 | 0 | 0.89 | 0.89 | 0.89 |
| 299.1 | 0 | 0.17 | 0.25 | 0.53 | 0.25 | 0.17 | 0 | -0.69 | -0.69 | -0.69 | 0 | 0.69 | 0.69 | 0.69 |
| 301.2 | 0 | 0.17 | 0.25 | 0.53 | 0.25 | 0.17 | 0 | -0.49 | -0.49 | -0.49 | 0 | 0.49 | 0.49 | 0.49 |
| 303.3 | 0 | 0.17 | 0.25 | 0.53 | 0.25 | 0.17 | 0 | -0.28 | -0.28 | -0.28 | 0 | 0.28 | 0.28 | 0.28 |
| 305.4 | 0 | 0.17 | 0.25 | 0.53 | 0.25 | 0.17 | 0 | -0.08 | -0.08 | -0.08 | 0 | 0.08 | 0.08 | 0.08 |
| 307.5 | 0 | 0.17 | 0.25 | 0.53 | 0.25 | 0.17 | 0 | 0.12 | 0.12 | 0.12 | 0 | -0.12 | -0.12 | -0.12 |
| 309.6 | 0 | 0.17 | 0.25 | 0.53 | 0.25 | 0.17 | 0 | 0.32 | 0.32 | 0.32 | 0 | -0.32 | -0.32 | -0.32 |
| 311.7 | 0 | 0.17 | 0.25 | 0.53 | 0.25 | 0.17 | 0 | 0.52 | 0.52 | 0.52 | 0 | -0.52 | -0.52 | -0.52 |
| 313.8 | 0 | 0.17 | 0.25 | 0.53 | 0.25 | 0.17 | 0 | 0.72 | 0.72 | 0.72 | 0 | -0.72 | -0.72 | -0.72 |
| 315.9 | 0 | 0.17 | 0.25 | 0.53 | 0.25 | 0.17 | 0 | 0.92 | 0.92 | 0.92 | 0 | -0.92 | -0.92 | -0.92 |
| 318.0 | 0 | 0.17 | 0.25 | 0.53 | 0.25 | 0.17 | 0 | 1.12 | 1.12 | 1.12 | 0 | -1.12 | -1.12 | -1.12 |
| | 0 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0 | 0.00 | 0.00 | 0.00 | 0 | 0.00 | 0.00 | 0.00 |
| 318.0 | 0 | 0.17 | 0.25 | 0.53 | 0.25 | 0.17 | 0 | 1.12 | 1.12 | 1.12 | 0 | -1.12 | -1.12 | -1.12 |
| 320.1 | 0 | 0.17 | 0.25 | 0.53 | 0.25 | 0.17 | 0 | 1.32 | 1.32 | 1.32 | 0 | -1.32 | -1.32 | -1.32 |
| 322.2 | 0 | 0.17 | 0.25 | 0.53 | 0.25 | 0.17 | 0 | 1.52 | 1.52 | 1.52 | 0 | -1.52 | -1.52 | -1.52 |
| 324.3 | 0 | 0.17 | 0.25 | 0.53 | 0.25 | 0.17 | 0 | 1.72 | 1.72 | 1.72 | 0 | -1.72 | -1.72 | -1.72 |
| 326.4 | 0 | 0.17 | 0.25 | 0.53 | 0.25 | 0.17 | 0 | 1.92 | 1.92 | 1.92 | 0 | -1.92 | -1.92 | -1.92 |
| 328.5 | 0 | 0.17 | 0.25 | 0.53 | 0.25 | 0.17 | 0 | 2.12 | 2.12 | 2.12 | 0 | -2.12 | -2.12 | -2.12 |
| 330.6 | 0 | 0.17 | 0.25 | 0.53 | 0.25 | 0.17 | 0 | 2.32 | 2.32 | 2.32 | 0 | -2.32 | -2.32 | -2.32 |
| 332.7 | 0 | 0.17 | 0.25 | 0.53 | 0.25 | 0.17 | 0 | 2.52 | 2.52 | 2.52 | 0 | -2.52 | -2.52 | -2.52 |
| 334.8 | 0 | 0.17 | 0.25 | 0.53 | 0.25 | 0.17 | 0 | 2.73 | 2.73 | 2.73 | 0 | -2.73 | -2.73 | -2.73 |
| 336.9 | 0 | 0.17 | 0.25 | 0.53 | 0.25 | 0.17 | 0 | 2.93 | 2.93 | 2.93 | 0 | -2.93 | -2.93 | -2.93 |
| 339.0 | 0 | 0.17 | 0.25 | 0.53 | 0.25 | 0.17 | 0 | 3.13 | 3.13 | 3.13 | 0 | -3.13 | -3.13 | -3.13 |
| | 0 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0 | 0.00 | 0.00 | 0.00 | 0 | 0.00 | 0.00 | 0.00 |
| 339.0 | 0 | 0.17 | 0.25 | 0.53 | 0.25 | 0.17 | 0 | 3.13 | 3.13 | 3.13 | 0 | -3.13 | -3.13 | -3.13 |
| 341.1 | 0 | 0.17 | 0.25 | 0.53 | 0.25 | 0.17 | 0 | 3.33 | 3.33 | 3.33 | 0 | -3.33 | -3.33 | -3.33 |
| 343.2 | 0 | 0.17 | 0.25 | 0.53 | 0.25 | 0.17 | 0 | 3.53 | 3.53 | 3.53 | 0 | -3.53 | -3.53 | -3.53 |
| 345.3 | 0 | 0.17 | 0.25 | 0.53 | 0.25 | 0.17 | 0 | 3.73 | 3.73 | 3.73 | 0 | -3.73 | -3.73 | -3.73 |
| 347.4 | 0 | 0.17 | 0.25 | 0.53 | 0.25 | 0.17 | 0 | 3.93 | 3.93 | 3.93 | 0 | -3.93 | -3.93 | -3.93 |
| | 0 | 1 | 2 | 3 | 2' | 1' | 0' | 0 | 1 | 2 | 3 | 2' | 1' | 0' |
| | | | | | | | | | | | | | | |
| 349.5 | 0 | 0.17 | 0.25 | 0.53 | 0.25 | 0.17 | 0 | 4.13 | 4.13 | 4.13 | 0 | -4.13 | -4.13 | -4.13 |
| 351.6 | 0 | 0.17 | 0.25 | 0.53 | 0.25 | 0.17 | 0 | 4.33 | 4.33 | 4.33 | 0 | -4.33 | -4.33 | -4.33 |
| 353.7 | 0 | 0.17 | 0.25 | 0.53 | 0.25 | 0.17 | 0 | 4.53 | 4.53 | 4.53 | 0 | -4.53 | -4.53 | -4.53 |
| 355.8 | 0 | 0.17 | 0.25 | 0.53 | 0.25 | 0.17 | 0 | 4.73 | 4.73 | 4.73 | 0 | -4.73 | -4.73 | -4.73 |
| 357.9 | 0 | 0.17 | 0.25 | 0.53 | 0.25 | 0.17 | 0 | 4.93 | 4.93 | 4.93 | 0 | -4.93 | -4.93 | -4.93 |
| 360.0 | 0 | 0.17 | 0.25 | 0.53 | 0.25 | 0.17 | 0 | 5.13 | 5.13 | 5.13 | 0 | -5.13 | -5.13 | -5.13 |
| | | | | | 0.00 | 0.00 | 0 | | | | | 0.00 | 0.00 | 0.00 |
| 360.0 | 0 | -0.08 | -0.11 | -0.24 | -0.11 | -0.08 | 0 | 5.13 | 5.13 | 5.13 | 0 | -5.13 | -5.13 | -5.13 |
| 362.0 | 0 | -0.08 | -0.11 | -0.24 | -0.11 | -0.08 | 0 | 5.05 | 5.05 | 5.05 | 0 | -5.05 | -5.05 | -5.05 |
| 364.0 | 0 | -0.08 | -0.11 | -0.24 | -0.11 | -0.08 | 0 | 4.96 | 4.96 | 4.96 | 0 | -4.96 | -4.96 | -4.96 |
| 366.0 | 0 | -0.08 | -0.11 | -0.24 | -0.11 | -0.08 | 0 | 4.88 | 4.88 | 4.88 | 0 | -4.88 | -4.88 | -4.88 |
| 368.0 | 0 | -0.08 | -0.11 | -0.24 | -0.11 | -0.08 | 0 | 4.79 | 4.79 | 4.79 | 0 | -4.79 | -4.79 | -4.79 |
| 370.0 | 0 | -0.08 | -0.11 | -0.24 | -0.11 | -0.08 | 0 | 4.71 | 4.71 | 4.71 | 0 | -4.71 | -4.71 | -4.71 |

| | | | | | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| 372.0 | 0 | -0.08 | -0.11 | -0.24 | -0.11 | -0.08 | 0 | 4.62 | 4.62 | 4.62 | 0 | -4.62 | -4.62 | -4.62 |
| 374.0 | 0 | -0.08 | -0.11 | -0.24 | -0.11 | -0.08 | 0 | 4.53 | 4.53 | 4.53 | 0 | -4.53 | -4.53 | -4.53 |
| 376.0 | 0 | -0.08 | -0.11 | -0.24 | -0.11 | -0.08 | 0 | 4.45 | 4.45 | 4.45 | 0 | -4.45 | -4.45 | -4.45 |
| 378.0 | 0 | -0.08 | -0.11 | -0.24 | -0.11 | -0.08 | 0 | 4.36 | 4.36 | 4.36 | 0 | -4.36 | -4.36 | -4.36 |
| 380.0 | 0 | -0.08 | -0.11 | -0.24 | -0.11 | -0.08 | 0 | 4.28 | 4.28 | 4.28 | 0 | -4.28 | -4.28 | -4.28 |
| | 0 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0 | 0.00 | 0.00 | 0.00 | 0 | 0.00 | 0.00 | 0.00 |
| 380.0 | 0 | -0.08 | -0.11 | -0.24 | -0.11 | -0.08 | 0 | 4.28 | 4.28 | 4.28 | 0 | -4.28 | -4.28 | -4.28 |
| 382.0 | 0 | -0.08 | -0.11 | -0.24 | -0.11 | -0.08 | 0 | 4.19 | 4.19 | 4.19 | 0 | -4.19 | -4.19 | -4.19 |
| 384.0 | 0 | -0.08 | -0.11 | -0.24 | -0.11 | -0.08 | 0 | 4.11 | 4.11 | 4.11 | 0 | -4.11 | -4.11 | -4.11 |
| 386.0 | 0 | -0.08 | -0.11 | -0.24 | -0.11 | -0.08 | 0 | 4.02 | 4.02 | 4.02 | 0 | -4.02 | -4.02 | -4.02 |
| 388.0 | 0 | -0.08 | -0.11 | -0.24 | -0.11 | -0.08 | 0 | 3.94 | 3.94 | 3.94 | 0 | -3.94 | -3.94 | -3.94 |
| 390.0 | 0 | -0.08 | -0.11 | -0.24 | -0.11 | -0.08 | 0 | 3.85 | 3.85 | 3.85 | 0 | -3.85 | -3.85 | -3.85 |
| 392.0 | 0 | -0.08 | -0.11 | -0.24 | -0.11 | -0.08 | 0 | 3.76 | 3.76 | 3.76 | 0 | -3.76 | -3.76 | -3.76 |
| 394.0 | 0 | -0.08 | -0.11 | -0.24 | -0.11 | -0.08 | 0 | 3.68 | 3.68 | 3.68 | 0 | -3.68 | -3.68 | -3.68 |
| 396.0 | 0 | -0.08 | -0.11 | -0.24 | -0.11 | -0.08 | 0 | 3.59 | 3.59 | 3.59 | 0 | -3.59 | -3.59 | -3.59 |
| 398.0 | 0 | -0.08 | -0.11 | -0.24 | -0.11 | -0.08 | 0 | 3.51 | 3.51 | 3.51 | 0 | -3.51 | -3.51 | -3.51 |
| 400.0 | 0 | -0.08 | -0.11 | -0.24 | -0.11 | -0.08 | 0 | 3.42 | 3.42 | 3.42 | 0 | -3.42 | -3.42 | -3.42 |
| | 0 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0 | 0.00 | 0.00 | 0.00 | 0 | 0.00 | 0.00 | 0.00 |
| 400.0 | 0 | -0.08 | -0.11 | -0.24 | -0.11 | -0.08 | 0 | 3.42 | 3.42 | 3.42 | 0 | -3.42 | -3.42 | -3.42 |
| 402.0 | 0 | -0.08 | -0.11 | -0.24 | -0.11 | -0.08 | 0 | 3.34 | 3.34 | 3.34 | 0 | -3.34 | -3.34 | -3.34 |
| 404.0 | 0 | -0.08 | -0.11 | -0.24 | -0.11 | -0.08 | 0 | 3.25 | 3.25 | 3.25 | 0 | -3.25 | -3.25 | -3.25 |
| 406.0 | 0 | -0.08 | -0.11 | -0.24 | -0.11 | -0.08 | 0 | 3.17 | 3.17 | 3.17 | 0 | -3.17 | -3.17 | -3.17 |
| 408.0 | 0 | -0.08 | -0.11 | -0.24 | -0.11 | -0.08 | 0 | 3.08 | 3.08 | 3.08 | 0 | -3.08 | -3.08 | -3.08 |
| 410.0 | 0 | -0.08 | -0.11 | -0.24 | -0.11 | -0.08 | 0 | 2.99 | 2.99 | 2.99 | 0 | -2.99 | -2.99 | -2.99 |
| 412.0 | 0 | -0.08 | -0.11 | -0.24 | -0.11 | -0.08 | 0 | 2.91 | 2.91 | 2.91 | 0 | -2.91 | -2.91 | -2.91 |
| 414.0 | 0 | -0.08 | -0.11 | -0.24 | -0.11 | -0.08 | 0 | 2.82 | 2.82 | 2.82 | 0 | -2.82 | -2.82 | -2.82 |
| 416.0 | 0 | -0.08 | -0.11 | -0.24 | -0.11 | -0.08 | 0 | 2.74 | 2.74 | 2.74 | 0 | -2.74 | -2.74 | -2.74 |
| 418.0 | 0 | -0.08 | -0.11 | -0.24 | -0.11 | -0.08 | 0 | 2.65 | 2.65 | 2.65 | 0 | -2.65 | -2.65 | -2.65 |
| 420.0 | 0 | -0.08 | -0.11 | -0.24 | -0.11 | -0.08 | 0 | 2.57 | 2.57 | 2.57 | 0 | -2.57 | -2.57 | -2.57 |
| | 0 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0 | 0.00 | 0.00 | 0.00 | 0 | 0.00 | 0.00 | 0.00 |
| 420.0 | 0 | -0.08 | -0.11 | -0.24 | -0.11 | -0.08 | 0 | 2.57 | 2.57 | 2.57 | 0 | -2.57 | -2.57 | -2.57 |
| | 0 | 1 | 2 | 3 | 2' | 1' | 0' | 0 | 1 | 2 | 3 | 2' | 1' | 0' |
| Z | | | | | | | | | | | | | | |
| 422.0 | 0 | -0.08 | -0.11 | -0.24 | -0.11 | -0.08 | 0 | 2.48 | 2.48 | 2.48 | 0 | -2.48 | -2.48 | -2.48 |
| 424.0 | 0 | -0.08 | -0.11 | -0.24 | -0.11 | -0.08 | 0 | 2.40 | 2.40 | 2.40 | 0 | -2.40 | -2.40 | -2.40 |
| 426.0 | 0 | -0.08 | -0.11 | -0.24 | -0.11 | -0.08 | 0 | 2.31 | 2.31 | 2.31 | 0 | -2.31 | -2.31 | -2.31 |
| 428.0 | 0 | -0.08 | -0.11 | -0.24 | -0.11 | -0.08 | 0 | 2.22 | 2.22 | 2.22 | 0 | -2.22 | -2.22 | -2.22 |
| 430.0 | 0 | -0.08 | -0.11 | -0.24 | -0.11 | -0.08 | 0 | 2.14 | 2.14 | 2.14 | 0 | -2.14 | -2.14 | -2.14 |
| 432.0 | 0 | -0.08 | -0.11 | -0.24 | -0.11 | -0.08 | 0 | 2.05 | 2.05 | 2.05 | 0 | -2.05 | -2.05 | -2.05 |
| 434.0 | 0 | -0.08 | -0.11 | -0.24 | -0.11 | -0.08 | 0 | 1.97 | 1.97 | 1.97 | 0 | -1.97 | -1.97 | -1.97 |
| 436.0 | 0 | -0.08 | -0.11 | -0.24 | -0.11 | -0.08 | 0 | 1.88 | 1.88 | 1.88 | 0 | -1.88 | -1.88 | -1.88 |
| 438.0 | 0 | -0.08 | -0.11 | -0.24 | -0.11 | -0.08 | 0 | 1.80 | 1.80 | 1.80 | 0 | -1.80 | -1.80 | -1.80 |
| 440.0 | 0 | -0.08 | -0.11 | -0.24 | -0.11 | -0.08 | 0 | 1.71 | 1.71 | 1.71 | 0 | -1.71 | -1.71 | -1.71 |
| | 0 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0 | 0.00 | 0.00 | 0.00 | 0 | 0.00 | 0.00 | 0.00 |

| | | | | | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| 440.0 | 0 | -0.08 | -0.11 | -0.24 | -0.11 | -0.08 | 0 | 1.71 | 1.71 | 1.71 | 0 | -1.71 | -1.71 | -1.71 |
| 442.0 | 0 | -0.08 | -0.11 | -0.24 | -0.11 | -0.08 | 0 | 1.63 | 1.63 | 1.63 | 0 | -1.63 | -1.63 | -1.63 |
| 444.0 | 0 | -0.08 | -0.11 | -0.24 | -0.11 | -0.08 | 0 | 1.54 | 1.54 | 1.54 | 0 | -1.54 | -1.54 | -1.54 |
| 446.0 | 0 | -0.08 | -0.11 | -0.24 | -0.11 | -0.08 | 0 | 1.45 | 1.45 | 1.45 | 0 | -1.45 | -1.45 | -1.45 |
| 448.0 | 0 | -0.08 | -0.11 | -0.24 | -0.11 | -0.08 | 0 | 1.37 | 1.37 | 1.37 | 0 | -1.37 | -1.37 | -1.37 |
| 450.0 | 0 | -0.08 | -0.11 | -0.24 | -0.11 | -0.08 | 0 | 1.28 | 1.28 | 1.28 | 0 | -1.28 | -1.28 | -1.28 |
| 452.0 | 0 | -0.08 | -0.11 | -0.24 | -0.11 | -0.08 | 0 | 1.20 | 1.20 | 1.20 | 0 | -1.20 | -1.20 | -1.20 |
| 454.0 | 0 | -0.08 | -0.11 | -0.24 | -0.11 | -0.08 | 0 | 1.11 | 1.11 | 1.11 | 0 | -1.11 | -1.11 | -1.11 |
| 456.0 | 0 | -0.08 | -0.11 | -0.24 | -0.11 | -0.08 | 0 | 1.03 | 1.03 | 1.03 | 0 | -1.03 | -1.03 | -1.03 |
| 458.0 | 0 | -0.08 | -0.11 | -0.24 | -0.11 | -0.08 | 0 | 0.94 | 0.94 | 0.94 | 0 | -0.94 | -0.94 | -0.94 |
| 460.0 | 0 | -0.08 | -0.11 | -0.24 | -0.11 | -0.08 | 0 | 0.86 | 0.86 | 0.86 | 0 | -0.86 | -0.86 | -0.86 |
| | 0 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0 | 0.00 | 0.00 | 0.00 | 0 | 0.00 | 0.00 | 0.00 |
| 460.0 | 0 | -0.08 | -0.11 | -0.24 | -0.11 | -0.08 | 0 | 0.86 | 0.86 | 0.86 | 0 | -0.86 | -0.86 | -0.86 |
| 462.0 | 0 | -0.08 | -0.11 | -0.24 | -0.11 | -0.08 | 0 | 0.77 | 0.77 | 0.77 | 0 | -0.77 | -0.77 | -0.77 |
| 464.0 | 0 | -0.08 | -0.11 | -0.24 | -0.11 | -0.08 | 0 | 0.68 | 0.68 | 0.68 | 0 | -0.68 | -0.68 | -0.68 |
| 466.0 | 0 | -0.08 | -0.11 | -0.24 | -0.11 | -0.08 | 0 | 0.60 | 0.60 | 0.60 | 0 | -0.60 | -0.60 | -0.60 |
| 468.0 | 0 | -0.08 | -0.11 | -0.24 | -0.11 | -0.08 | 0 | 0.51 | 0.51 | 0.51 | 0 | -0.51 | -0.51 | -0.51 |
| 470.0 | 0 | -0.08 | -0.11 | -0.24 | -0.11 | -0.08 | 0 | 0.43 | 0.43 | 0.43 | 0 | -0.43 | -0.43 | -0.43 |
| 472.0 | 0 | -0.08 | -0.11 | -0.24 | -0.11 | -0.08 | 0 | 0.34 | 0.34 | 0.34 | 0 | -0.34 | -0.34 | -0.34 |
| 474.0 | 0 | -0.08 | -0.11 | -0.24 | -0.11 | -0.08 | 0 | 0.26 | 0.26 | 0.26 | 0 | -0.26 | -0.26 | -0.26 |
| 476.0 | 0 | -0.08 | -0.11 | -0.24 | -0.11 | -0.08 | 0 | 0.17 | 0.17 | 0.17 | 0 | -0.17 | -0.17 | -0.17 |
| 478.0 | 0 | -0.08 | -0.11 | -0.24 | -0.11 | -0.08 | 0 | 0.09 | 0.09 | 0.09 | 0 | -0.09 | -0.09 | -0.09 |
| 480.0 | 0 | -0.08 | -0.11 | -0.24 | -0.11 | -0.08 | 0 | 0.00 | 0.00 | 0.00 | 0 | 0.00 | 0.00 | 0.00 |

Table of Combined Normal Stresses

| Z | | | | | | | |
|---|---|---|---|---|---|---|---|
| 0.0 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 |
| 2.0 | 0.139 | 0.148 | 0.152 | 0.000 | -0.152 | -0.148 | -0.139 |
| 4.0 | 0.277 | 0.296 | 0.304 | 0.000 | -0.304 | -0.296 | -0.277 |
| 6.0 | 0.415 | 0.443 | 0.455 | 0.000 | -0.455 | -0.443 | -0.415 |
| 8.0 | 0.553 | 0.591 | 0.607 | 0.000 | -0.607 | -0.591 | -0.553 |
| 10.0 | 0.691 | 0.739 | 0.759 | 0.000 | -0.759 | -0.739 | -0.691 |
| 12.0 | 0.828 | 0.887 | 0.912 | 0.000 | -0.912 | -0.887 | -0.828 |
| 14.0 | 0.965 | 1.034 | 1.064 | 0.000 | -1.064 | -1.034 | -0.965 |
| 16.0 | 1.101 | 1.182 | 1.217 | 0.000 | -1.217 | -1.182 | -1.101 |
| 18.0 | 1.236 | 1.330 | 1.370 | 0.000 | -1.370 | -1.330 | -1.236 |
| 20.0 | 1.371 | 1.478 | 1.524 | 0.000 | -1.524 | -1.478 | -1.371 |
| | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 |
| 20.0 | 1.371 | 1.478 | 1.524 | 0.000 | -1.524 | -1.478 | -1.371 |
| 22.0 | 1.504 | 1.626 | 1.678 | 0.000 | -1.678 | -1.626 | -1.504 |
| 24.0 | 1.637 | 1.773 | 1.832 | 0.000 | -1.832 | -1.773 | -1.637 |
| 26.0 | 1.768 | 1.921 | 1.987 | 0.000 | -1.987 | -1.921 | -1.768 |
| 28.0 | 1.898 | 2.069 | 2.142 | 0.000 | -2.142 | -2.069 | -1.898 |
| 30.0 | 2.026 | 2.217 | 2.299 | 0.000 | -2.299 | -2.217 | -2.026 |

| | | | | | | | |
|---|---|---|---|---|---|---|---|
| 32.0 | 2.152 | 2.364 | 2.456 | 0.000 | -2.456 | -2.364 | -2.152 |
| 34.0 | 2.277 | 2.512 | 2.613 | 0.000 | -2.613 | -2.512 | -2.277 |
| 36.0 | 2.399 | 2.660 | 2.772 | 0.000 | -2.772 | -2.660 | -2.399 |
| 38.0 | 2.519 | 2.808 | 2.932 | 0.000 | -2.932 | -2.808 | -2.519 |
| 40.0 | 2.636 | 2.956 | 3.093 | 0.000 | -3.093 | -2.956 | -2.636 |
| | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 |
| 40.0 | 2.636 | 2.956 | 3.093 | 0.000 | -3.093 | -2.956 | -2.636 |
| 42.0 | 2.750 | 3.103 | 3.255 | 0.000 | -3.255 | -3.103 | -2.750 |
| 44.0 | 2.861 | 3.251 | 3.418 | 0.000 | -3.418 | -3.251 | -2.861 |
| 46.0 | 2.969 | 3.399 | 3.583 | 0.000 | -3.583 | -3.399 | -2.969 |
| 48.0 | 3.073 | 3.547 | 3.750 | 0.000 | -3.750 | -3.547 | -3.073 |
| 50.0 | 3.172 | 3.694 | 3.919 | 0.000 | -3.919 | -3.694 | -3.172 |
| 52.0 | 3.266 | 3.842 | 4.089 | 0.000 | -4.089 | -3.842 | -3.266 |
| 54.0 | 3.356 | 3.990 | 4.262 | 0.000 | -4.262 | -3.990 | -3.356 |
| 56.0 | 3.439 | 4.138 | 4.438 | 0.000 | -4.438 | -4.138 | -3.439 |
| 58.0 | 3.516 | 4.286 | 4.616 | 0.000 | -4.616 | -4.286 | -3.516 |
| 60.0 | 3.586 | 4.433 | 4.797 | 0.000 | -4.797 | -4.433 | -3.586 |
| | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 |
| 60.0 | 3.586 | 4.433 | 4.797 | 0.000 | -4.797 | -4.433 | -3.586 |
| 62.0 | 3.648 | 4.581 | 4.982 | 0.000 | -4.982 | -4.581 | -3.648 |
| 64.0 | 3.701 | 4.729 | 5.170 | 0.000 | -5.170 | -4.729 | -3.701 |
| 66.0 | 3.746 | 4.877 | 5.362 | 0.000 | -5.362 | -4.877 | -3.746 |
| 68.0 | 3.780 | 5.024 | 5.558 | 0.000 | -5.558 | -5.024 | -3.780 |
| 70.0 | 3.802 | 5.172 | 5.760 | 0.000 | -5.760 | -5.172 | -3.802 |
| 72.0 | 3.812 | 5.320 | 5.967 | 0.000 | -5.967 | -5.320 | -3.812 |
| 74.0 | 3.808 | 5.468 | 6.180 | 0.000 | -6.180 | -5.468 | -3.808 |
| 76.0 | 3.788 | 5.616 | 6.400 | 0.000 | -6.400 | -5.616 | -3.788 |
| 78.0 | 3.753 | 5.763 | 6.626 | 0.000 | -6.626 | -5.763 | -3.753 |
| 80.0 | 3.699 | 5.911 | 6.860 | 0.000 | -6.860 | -5.911 | -3.699 |
| | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 |
| 80.0 | 3.700 | 5.911 | 6.860 | 0.000 | -6.860 | -5.911 | -3.700 |
| 82.0 | 3.624 | 6.059 | 7.104 | 0.000 | -7.104 | -6.059 | -3.624 |
| 84.0 | 3.527 | 6.207 | 7.357 | 0.000 | -7.357 | -6.207 | -3.527 |
| 86.0 | 3.406 | 6.354 | 7.620 | 0.000 | -7.620 | -6.354 | -3.406 |
| 88.0 | 3.257 | 6.502 | 7.894 | 0.000 | -7.894 | -6.502 | -3.257 |
| 90.0 | 3.081 | 6.650 | 8.182 | 0.000 | -8.182 | -6.650 | -3.081 |
| 92.0 | 2.869 | 6.798 | 8.484 | 0.000 | -8.484 | -6.798 | -2.869 |
| 94.0 | 2.619 | 6.946 | 8.802 | 0.000 | -8.802 | -6.946 | -2.619 |
| 96.0 | 2.333 | 7.093 | 9.136 | 0.000 | -9.136 | -7.093 | -2.333 |
| 98.0 | 2.006 | 7.241 | 9.487 | 0.000 | -9.487 | -7.241 | -2.006 |
| 100.0 | 1.628 | 7.389 | 9.861 | 0.000 | -9.861 | -7.389 | -1.628 |
| | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 |
| 100.0 | 1.631 | 7.389 | 9.859 | 0.000 | -9.859 | -7.389 | -1.631 |

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|---|---|---|---|---|---|---|---|
| 102.0 | 1.195 | 7.537 | 10.258 | 0.000 | -10.258 | -7.537 | -1.195 |
| 104.0 | 0.705 | 7.684 | 10.679 | 0.000 | -10.679 | -7.684 | -0.705 |
| 106.0 | 0.154 | 7.832 | 11.127 | 0.000 | -11.127 | -7.832 | -0.154 |
| 108.0 | -0.469 | 7.980 | 11.605 | 0.000 | -11.605 | -7.980 | 0.469 |
| 110.0 | -1.169 | 8.128 | 12.117 | 0.000 | -12.117 | -8.128 | 1.169 |
| 112.0 | -1.954 | 8.276 | 12.665 | 0.000 | -12.665 | -8.276 | 1.954 |
| 114.0 | -2.835 | 8.423 | 13.254 | 0.000 | -13.254 | -8.423 | 2.835 |
| 116.0 | -3.820 | 8.571 | 13.888 | 0.000 | -13.888 | -8.571 | 3.820 |
| 118.0 | -4.914 | 8.719 | 14.568 | 0.000 | -14.568 | -8.719 | 4.914 |
| 120.0 | -6.130 | 8.867 | 15.301 | 0.000 | -15.301 | -8.867 | 6.130 |
| | | | | | 0.000 | 0.000 | 0.000 |
| 120.0 | -6.134 | 8.867 | 15.303 | 0.000 | -15.303 | -8.867 | 6.134 |
| 121.8 | -5.285 | 8.372 | 14.232 | 0.000 | -14.232 | -8.372 | 5.285 |
| 123.6 | -4.534 | 7.877 | 13.203 | 0.000 | -13.203 | -7.877 | 4.534 |
| 125.4 | -3.872 | 7.383 | 12.212 | 0.000 | -12.212 | -7.383 | 3.872 |
| 127.2 | -3.294 | 6.888 | 11.257 | 0.000 | -11.257 | -6.888 | 3.294 |
| 129.0 | -2.795 | 6.393 | 10.336 | 0.000 | -10.336 | -6.393 | 2.795 |
| 130.8 | -2.363 | 5.899 | 9.444 | 0.000 | -9.444 | -5.899 | 2.363 |
| 132.6 | -1.993 | 5.404 | 8.578 | 0.000 | -8.578 | -5.404 | 1.993 |
| 134.4 | -1.676 | 4.909 | 7.735 | 0.000 | -7.735 | -4.909 | 1.676 |
| 136.2 | -1.411 | 4.415 | 6.914 | 0.000 | -6.914 | -4.415 | 1.411 |
| 138.0 | -1.183 | 3.920 | 6.110 | 0.000 | -6.110 | -3.920 | 1.183 |
| | | | | | 0.000 | 0.000 | 0.000 |
| 138.0 | -1.186 | 3.920 | 6.111 | 0.000 | -6.111 | -3.920 | 1.186 |
| 139.8 | -0.999 | 3.425 | 5.324 | 0.000 | -5.324 | -3.425 | 0.999 |
| 141.6 | -0.846 | 2.931 | 4.551 | 0.000 | -4.551 | -2.931 | 0.846 |
| 143.4 | -0.722 | 2.436 | 3.791 | 0.000 | -3.791 | -2.436 | 0.722 |
| 145.2 | -0.620 | 1.941 | 3.040 | 0.000 | -3.040 | -1.941 | 0.620 |
| 147.0 | -0.537 | 1.447 | 2.298 | 0.000 | -2.298 | -1.447 | 0.537 |
| 148.8 | -0.469 | 0.952 | 1.562 | 0.000 | -1.562 | -0.952 | 0.469 |
| 150.6 | -0.412 | 0.457 | 0.830 | 0.000 | -0.830 | -0.457 | 0.412 |
| 152.4 | -0.361 | -0.037 | 0.102 | 0.000 | -0.102 | 0.037 | 0.361 |
| 154.2 | -0.312 | -0.532 | -0.626 | 0.000 | 0.626 | 0.532 | 0.312 |
| 156.0 | -0.261 | -1.027 | -1.355 | 0.000 | 1.355 | 1.027 | 0.261 |
| | | | | | 0.000 | 0.000 | 0.000 |
| 156.0 | -0.263 | -1.027 | -1.354 | 0.000 | 1.354 | 1.027 | 0.263 |
| 157.8 | -0.206 | -1.521 | -2.086 | 0.000 | 2.086 | 1.521 | 0.206 |
| 159.6 | -0.140 | -2.016 | -2.821 | 0.000 | 2.821 | 2.016 | 0.140 |
| 161.4 | -0.061 | -2.511 | -3.562 | 0.000 | 3.562 | 2.511 | 0.061 |
| 163.2 | 0.037 | -3.005 | -4.311 | 0.000 | 4.311 | 3.005 | -0.037 |
| 165.0 | 0.158 | -3.500 | -5.069 | 0.000 | 5.069 | 3.500 | -0.158 |
| 166.8 | 0.304 | -3.995 | -5.839 | 0.000 | 5.839 | 3.995 | -0.304 |
| 168.6 | 0.485 | -4.489 | -6.624 | 0.000 | 6.624 | 4.489 | -0.485 |

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|---|---|---|---|---|---|---|---|
| 170.4 | 0.703 | -4.984 | -7.424 | 0.000 | 7.424 | 4.984 | -0.703 |
| 172.2 | 0.958 | -5.479 | -8.240 | 0.000 | 8.240 | 5.479 | -0.958 |
| 174.0 | 1.264 | -5.973 | -9.079 | 0.000 | 9.079 | 5.973 | -1.264 |
| | | | | | 0.000 | 0.000 | 0.000 |
| 174.0 | 1.261 | -5.973 | -9.077 | 0.000 | 9.077 | 5.973 | -1.261 |
| 175.8 | 1.628 | -6.468 | -9.942 | 0.000 | 9.942 | 6.468 | -1.628 |
| 177.6 | 2.046 | -6.963 | -10.828 | 0.000 | 10.828 | 6.963 | -2.046 |
| 179.4 | 2.532 | -7.457 | -11.743 | 0.000 | 11.743 | 7.457 | -2.532 |
| 181.2 | 3.092 | -7.952 | -12.691 | 0.000 | 12.691 | 7.952 | -3.092 |
| 183.0 | 3.734 | -8.447 | -13.673 | 0.000 | 13.673 | 8.447 | -3.734 |
| 184.8 | 4.468 | -8.941 | -14.695 | 0.000 | 14.695 | 8.941 | -4.468 |
| 186.6 | 5.303 | -9.436 | -15.760 | 0.000 | 15.760 | 9.436 | -5.303 |
| 188.4 | 6.247 | -9.931 | -16.872 | 0.000 | 16.872 | 9.931 | -6.247 |
| 190.2 | 7.306 | -10.425 | -18.033 | 0.000 | 18.033 | 10.425 | -7.306 |
| 192.0 | 8.491 | -10.920 | -19.249 | 0.000 | 19.249 | 10.920 | -8.491 |
| | | | | | 0.000 | 0.000 | 0.000 |
| 192.0 | 8.491 | -10.920 | -19.249 | 0.000 | 19.249 | 10.920 | -8.491 |
| 194.1 | 6.846 | -10.719 | -18.256 | 0.000 | 18.256 | 10.719 | -6.846 |
| 196.2 | 5.367 | -10.519 | -17.335 | 0.000 | 17.335 | 10.519 | -5.367 |
| 198.3 | 4.048 | -10.318 | -16.482 | 0.000 | 16.482 | 10.318 | -4.048 |
| 200.4 | 2.871 | -10.117 | -15.690 | 0.000 | 15.690 | 10.117 | -2.871 |
| 202.5 | 1.830 | -9.917 | -14.957 | 0.000 | 14.957 | 9.917 | -1.830 |
| 204.6 | 0.907 | -9.716 | -14.274 | 0.000 | 14.274 | 9.716 | -0.907 |
| 206.7 | 0.094 | -9.515 | -13.638 | 0.000 | 13.638 | 9.515 | -0.094 |
| 208.8 | -0.622 | -9.315 | -13.044 | 0.000 | 13.044 | 9.315 | 0.622 |
| 210.9 | -1.252 | -9.114 | -12.487 | 0.000 | 12.487 | 9.114 | 1.252 |
| 213.0 | -1.815 | -8.913 | -11.959 | 0.000 | 11.959 | 8.913 | 1.815 |
| | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 |
| 213.0 | -1.808 | -8.913 | -11.962 | 0.000 | 11.962 | 8.913 | 1.808 |
| 215.1 | -2.286 | -8.713 | -11.470 | 0.000 | 11.470 | 8.713 | 2.286 |
| 217.2 | -2.700 | -8.512 | -11.006 | 0.000 | 11.006 | 8.512 | 2.700 |
| 219.3 | -3.056 | -8.311 | -10.566 | 0.000 | 10.566 | 8.311 | 3.056 |
| 221.4 | -3.360 | -8.111 | -10.149 | 0.000 | 10.149 | 8.111 | 3.360 |
| 223.5 | -3.618 | -7.910 | -9.752 | 0.000 | 9.752 | 7.910 | 3.618 |
| 225.6 | -3.828 | -7.709 | -9.375 | 0.000 | 9.375 | 7.709 | 3.828 |
| 227.7 | -4.000 | -7.509 | -9.014 | 0.000 | 9.014 | 7.509 | 4.000 |
| 229.8 | -4.136 | -7.308 | -8.669 | 0.000 | 8.669 | 7.308 | 4.136 |
| 231.9 | -4.241 | -7.107 | -8.337 | 0.000 | 8.337 | 7.107 | 4.241 |
| 234.0 | -4.320 | -6.907 | -8.017 | 0.000 | 8.017 | 6.907 | 4.320 |
| | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 |
| 234.0 | -4.318 | -6.907 | -8.017 | 0.000 | 8.017 | 6.907 | 4.318 |
| 236.1 | -4.367 | -6.706 | -7.710 | 0.000 | 7.710 | 6.706 | 4.367 |
| 238.2 | -4.393 | -6.505 | -7.412 | 0.000 | 7.412 | 6.505 | 4.393 |

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|---|---|---|---|---|---|---|---|
| 240.3 | -4.398 | -6.305 | -7.123 | 0.000 | 7.123 | 6.305 | 4.398 |
| 242.4 | -4.384 | -6.104 | -6.842 | 0.000 | 6.842 | 6.104 | 4.384 |
| 244.5 | -4.352 | -5.903 | -6.569 | 0.000 | 6.569 | 5.903 | 4.352 |
| 246.6 | -4.305 | -5.703 | -6.302 | 0.000 | 6.302 | 5.703 | 4.305 |
| 248.7 | -4.243 | -5.502 | -6.042 | 0.000 | 6.042 | 5.502 | 4.243 |
| 250.8 | -4.168 | -5.301 | -5.788 | 0.000 | 5.788 | 5.301 | 4.168 |
| 252.9 | -4.082 | -5.101 | -5.538 | 0.000 | 5.538 | 5.101 | 4.082 |
| 255.0 | -3.988 | -4.900 | -5.291 | 0.000 | 5.291 | 4.900 | 3.988 |
| | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 |
| 255.0 | -3.987 | -4.900 | -5.292 | 0.000 | 5.292 | 4.900 | 3.987 |
| 257.1 | -3.881 | -4.699 | -5.050 | 0.000 | 5.050 | 4.699 | 3.881 |
| 259.2 | -3.768 | -4.499 | -4.812 | 0.000 | 4.812 | 4.499 | 3.768 |
| 261.3 | -3.647 | -4.298 | -4.577 | 0.000 | 4.577 | 4.298 | 3.647 |
| 263.4 | -3.520 | -4.097 | -4.345 | 0.000 | 4.345 | 4.097 | 3.520 |
| 265.5 | -3.386 | -3.897 | -4.116 | 0.000 | 4.116 | 3.897 | 3.386 |
| 267.6 | -3.248 | -3.696 | -3.888 | 0.000 | 3.888 | 3.696 | 3.248 |
| 269.7 | -3.105 | -3.495 | -3.663 | 0.000 | 3.663 | 3.495 | 3.105 |
| 271.8 | -2.958 | -3.295 | -3.439 | 0.000 | 3.439 | 3.295 | 2.958 |
| 273.9 | -2.807 | -3.094 | -3.217 | 0.000 | 3.217 | 3.094 | 2.807 |
| 276.0 | -2.655 | -2.893 | -2.996 | 0.000 | 2.996 | 2.893 | 2.655 |
| | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 |
| 276.0 | -2.654 | -2.893 | -2.996 | 0.000 | 2.996 | 2.893 | 2.654 |
| 278.1 | -2.499 | -2.693 | -2.776 | 0.000 | 2.776 | 2.693 | 2.499 |
| 280.2 | -2.341 | -2.492 | -2.557 | 0.000 | 2.557 | 2.492 | 2.341 |
| 282.3 | -2.182 | -2.291 | -2.338 | 0.000 | 2.338 | 2.291 | 2.182 |
| 284.4 | -2.022 | -2.091 | -2.120 | 0.000 | 2.120 | 2.091 | 2.022 |
| 286.5 | -1.861 | -1.890 | -1.903 | 0.000 | 1.903 | 1.890 | 1.861 |
| 288.6 | -1.699 | -1.689 | -1.685 | 0.000 | 1.685 | 1.689 | 1.699 |
| 290.7 | -1.538 | -1.489 | -1.467 | 0.000 | 1.467 | 1.489 | 1.538 |
| 292.8 | -1.378 | -1.288 | -1.250 | 0.000 | 1.250 | 1.288 | 1.378 |
| 294.9 | -1.218 | -1.087 | -1.031 | 0.000 | 1.031 | 1.087 | 1.218 |
| 297.0 | -1.059 | -0.887 | -0.813 | 0.000 | 0.813 | 0.887 | 1.059 |
| | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 |
| 297.0 | -1.059 | -0.887 | -0.813 | 0.000 | 0.813 | 0.887 | 1.059 |
| 299.1 | -0.903 | -0.686 | -0.593 | 0.000 | 0.593 | 0.686 | 0.903 |
| 301.2 | -0.748 | -0.485 | -0.372 | 0.000 | 0.372 | 0.485 | 0.748 |
| 303.3 | -0.596 | -0.285 | -0.151 | 0.000 | 0.151 | 0.285 | 0.596 |
| 305.4 | -0.448 | -0.084 | 0.072 | 0.000 | -0.072 | 0.084 | 0.448 |
| 307.5 | -0.303 | 0.117 | 0.297 | 0.000 | -0.297 | -0.117 | 0.303 |
| 309.6 | -0.162 | 0.317 | 0.523 | 0.000 | -0.523 | -0.317 | 0.162 |
| 311.7 | -0.026 | 0.518 | 0.751 | 0.000 | -0.751 | -0.518 | 0.026 |
| 313.8 | 0.105 | 0.719 | 0.982 | 0.000 | -0.982 | -0.719 | -0.105 |
| 315.9 | 0.229 | 0.919 | 1.216 | 0.000 | -1.216 | -0.919 | -0.229 |

| | | | | | | | | |
|---|---|---|---|---|---|---|---|---|
| 318.0 | 0.346 | 1.120 | 1.452 | 0.000 | -1.452 | -1.120 | -0.346 |
| | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 |
| 318.0 | 0.347 | 1.120 | 1.452 | 0.000 | -1.452 | -1.120 | -0.347 |
| 320.1 | 0.455 | 1.321 | 1.692 | 0.000 | -1.692 | -1.321 | -0.455 |
| 322.2 | 0.556 | 1.521 | 1.936 | 0.000 | -1.936 | -1.521 | -0.556 |
| 324.3 | 0.647 | 1.722 | 2.183 | 0.000 | -2.183 | -1.722 | -0.647 |
| 326.4 | 0.727 | 1.923 | 2.436 | 0.000 | -2.436 | -1.923 | -0.727 |
| 328.5 | 0.796 | 2.123 | 2.693 | 0.000 | -2.693 | -2.123 | -0.796 |
| 330.6 | 0.850 | 2.324 | 2.956 | 0.000 | -2.956 | -2.324 | -0.850 |
| 332.7 | 0.889 | 2.525 | 3.226 | 0.000 | -3.226 | -2.525 | -0.889 |
| 334.8 | 0.912 | 2.725 | 3.503 | 0.000 | -3.503 | -2.725 | -0.912 |
| 336.9 | 0.917 | 2.926 | 3.788 | 0.000 | -3.788 | -2.926 | -0.917 |
| 339.0 | 0.903 | 3.127 | 4.081 | 0.000 | -4.081 | -3.127 | -0.903 |
| | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 |
| 339.0 | 0.904 | 3.127 | 4.080 | 0.000 | -4.080 | -3.127 | -0.904 |
| 341.1 | 0.864 | 3.327 | 4.384 | 0.000 | -4.384 | -3.327 | -0.864 |
| 343.2 | 0.801 | 3.528 | 4.698 | 0.000 | -4.698 | -3.528 | -0.801 |
| 345.3 | 0.711 | 3.729 | 5.023 | 0.000 | -5.023 | -3.729 | -0.711 |
| 347.4 | 0.591 | 3.929 | 5.362 | 0.000 | -5.362 | -3.929 | -0.591 |
| 349.5 | 0.435 | 4.130 | 5.715 | 0.000 | -5.715 | -4.130 | -0.435 |
| 351.6 | 0.245 | 4.331 | 6.083 | 0.000 | -6.083 | -4.331 | -0.245 |
| 353.7 | 0.012 | 4.531 | 6.470 | 0.000 | -6.470 | -4.531 | -0.012 |
| 355.8 | -0.269 | 4.732 | 6.878 | 0.000 | -6.878 | -4.732 | 0.269 |
| 357.9 | -0.598 | 4.933 | 7.306 | 0.000 | -7.306 | -4.933 | 0.598 |
| 360.0 | -0.981 | 5.133 | 7.757 | 0.000 | -7.757 | -5.133 | 0.981 |
| | | | | | 0.000 | 0.000 | 0.000 |
| 360.0 | -0.981 | 5.133 | 7.757 | 0.000 | -7.757 | -5.133 | 0.981 |
| 362.0 | -0.510 | 5.048 | 7.432 | 0.000 | -7.432 | -5.048 | 0.510 |
| 364.0 | -0.090 | 4.962 | 7.130 | 0.000 | -7.130 | -4.962 | 0.090 |
| 366.0 | 0.286 | 4.877 | 6.846 | 0.000 | -6.846 | -4.877 | -0.286 |
| 368.0 | 0.621 | 4.791 | 6.580 | 0.000 | -6.580 | -4.791 | -0.621 |
| 370.0 | 0.916 | 4.706 | 6.332 | 0.000 | -6.332 | -4.706 | -0.916 |
| 372.0 | 1.176 | 4.620 | 6.098 | 0.000 | -6.098 | -4.620 | -1.176 |
| 374.0 | 1.404 | 4.534 | 5.878 | 0.000 | -5.878 | -4.534 | -1.404 |
| 376.0 | 1.604 | 4.449 | 5.670 | 0.000 | -5.670 | -4.449 | -1.604 |
| 378.0 | 1.778 | 4.363 | 5.473 | 0.000 | -5.473 | -4.363 | -1.778 |
| 380.0 | 1.931 | 4.278 | 5.285 | 0.000 | -5.285 | -4.278 | -1.931 |
| 380.0 | 1.929 | 4.278 | 5.285 | 0.000 | -5.285 | -4.278 | -1.929 |
| 382.0 | 2.057 | 4.192 | 5.108 | 0.000 | -5.108 | -4.192 | -2.057 |
| 384.0 | 2.167 | 4.107 | 4.939 | 0.000 | -4.939 | -4.107 | -2.167 |
| 386.0 | 2.258 | 4.021 | 4.777 | 0.000 | -4.777 | -4.021 | -2.258 |
| 388.0 | 2.334 | 3.936 | 4.623 | 0.000 | -4.623 | -3.936 | -2.334 |
| 390.0 | 2.395 | 3.850 | 4.474 | 0.000 | -4.474 | -3.850 | -2.395 |

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|---|---|---|---|---|---|---|---|
| 392.0 | 2.442 | 3.764 | 4.332 | 0.000 | -4.332 | -3.764 | -2.442 |
| 394.0 | 2.477 | 3.679 | 4.195 | 0.000 | -4.195 | -3.679 | -2.477 |
| 396.0 | 2.501 | 3.593 | 4.062 | 0.000 | -4.062 | -3.593 | -2.501 |
| 398.0 | 2.515 | 3.508 | 3.934 | 0.000 | -3.934 | -3.508 | -2.515 |
| 400.0 | 2.521 | 3.422 | 3.809 | 0.000 | -3.809 | -3.422 | -2.521 |
| 400.0 | 2.521 | 3.422 | 3.809 | 0.000 | -3.809 | -3.422 | -2.521 |
| 402.0 | 2.517 | 3.337 | 3.688 | 0.000 | -3.688 | -3.337 | -2.517 |
| 404.0 | 2.506 | 3.251 | 3.571 | 0.000 | -3.571 | -3.251 | -2.506 |
| 406.0 | 2.489 | 3.166 | 3.456 | 0.000 | -3.456 | -3.166 | -2.489 |
| 408.0 | 2.465 | 3.080 | 3.344 | 0.000 | -3.344 | -3.080 | -2.465 |
| 410.0 | 2.436 | 2.994 | 3.234 | 0.000 | -3.234 | -2.994 | -2.436 |
| 412.0 | 2.402 | 2.909 | 3.127 | 0.000 | -3.127 | -2.909 | -2.402 |
| 414.0 | 2.362 | 2.823 | 3.021 | 0.000 | -3.021 | -2.823 | -2.362 |
| 416.0 | 2.319 | 2.738 | 2.918 | 0.000 | -2.918 | -2.738 | -2.319 |
| 418.0 | 2.272 | 2.652 | 2.815 | 0.000 | -2.815 | -2.652 | -2.272 |
| 420.0 | 2.222 | 2.567 | 2.715 | 0.000 | -2.715 | -2.567 | -2.222 |
| | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 |
| 420.0 | 2.221 | 2.567 | 2.715 | 0.000 | -2.715 | -2.567 | -2.221 |
| 422.0 | 2.167 | 2.481 | 2.616 | 0.000 | -2.616 | -2.481 | -2.167 |
| 424.0 | 2.111 | 2.396 | 2.518 | 0.000 | -2.518 | -2.396 | -2.111 |
| 426.0 | 2.051 | 2.310 | 2.421 | 0.000 | -2.421 | -2.310 | -2.051 |
| 428.0 | 1.990 | 2.224 | 2.325 | 0.000 | -2.325 | -2.224 | -1.990 |
| 430.0 | 1.926 | 2.139 | 2.230 | 0.000 | -2.230 | -2.139 | -1.926 |
| 432.0 | 1.860 | 2.053 | 2.136 | 0.000 | -2.136 | -2.053 | -1.860 |
| 434.0 | 1.793 | 1.968 | 2.043 | 0.000 | -2.043 | -1.968 | -1.793 |
| 436.0 | 1.723 | 1.882 | 1.950 | 0.000 | -1.950 | -1.882 | -1.723 |
| 438.0 | 1.653 | 1.797 | 1.858 | 0.000 | -1.858 | -1.797 | -1.653 |
| 440.0 | 1.581 | 1.711 | 1.767 | 0.000 | -1.767 | -1.711 | -1.581 |
| | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 |
| 440.0 | 1.581 | 1.711 | 1.767 | 0.000 | -1.767 | -1.711 | -1.581 |
| 442.0 | 1.508 | 1.626 | 1.676 | 0.000 | -1.676 | -1.626 | -1.508 |
| 444.0 | 1.434 | 1.540 | 1.586 | 0.000 | -1.586 | -1.540 | -1.434 |
| 446.0 | 1.358 | 1.454 | 1.496 | 0.000 | -1.496 | -1.454 | -1.358 |
| 448.0 | 1.282 | 1.369 | 1.406 | 0.000 | -1.406 | -1.369 | -1.282 |
| 450.0 | 1.205 | 1.283 | 1.317 | 0.000 | -1.317 | -1.283 | -1.205 |
| 452.0 | 1.128 | 1.198 | 1.228 | 0.000 | -1.228 | -1.198 | -1.128 |
| 454.0 | 1.050 | 1.112 | 1.139 | 0.000 | -1.139 | -1.112 | -1.050 |
| 456.0 | 0.971 | 1.027 | 1.051 | 0.000 | -1.051 | -1.027 | -0.971 |
| 458.0 | 0.892 | 0.941 | 0.962 | 0.000 | -0.962 | -0.941 | -0.892 |
| 460.0 | 0.812 | 0.856 | 0.874 | 0.000 | -0.874 | -0.856 | -0.812 |
| | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 |
| 460.0 | 0.812 | 0.856 | 0.874 | 0.000 | -0.874 | -0.856 | -0.812 |
| 462.0 | 0.732 | 0.770 | 0.786 | 0.000 | -0.786 | -0.770 | -0.732 |

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|---|---|---|---|---|---|---|---|
| 464.0 | 0.651 | 0.684 | 0.699 | 0.000 | -0.699 | -0.684 | -0.651 |
| 466.0 | 0.571 | 0.599 | 0.611 | 0.000 | -0.611 | -0.599 | -0.571 |
| 468.0 | 0.490 | 0.513 | 0.524 | 0.000 | -0.524 | -0.513 | -0.490 |
| 470.0 | 0.408 | 0.428 | 0.436 | 0.000 | -0.436 | -0.428 | -0.408 |
| 472.0 | 0.327 | 0.342 | 0.349 | 0.000 | -0.349 | -0.342 | -0.327 |
| 474.0 | 0.245 | 0.257 | 0.262 | 0.000 | -0.262 | -0.257 | -0.245 |
| 476.0 | 0.164 | 0.171 | 0.174 | 0.000 | -0.174 | -0.171 | -0.164 |
| 478.0 | 0.082 | 0.086 | 0.087 | 0.000 | -0.087 | -0.086 | -0.082 |
| 480.0 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 |

Total Shear with Positive St. Vt. Contribution

| z | Combined Shear Stress with Positive St. Vt. | | | | | | |
|---|---|---|---|---|---|---|---|
| | | | | | | | |
| 0.0 | -0.901 | -0.771 | -0.713 | -0.505 | -0.713 | -0.771 | -0.901 |
| 2.0 | -0.901 | -0.771 | -0.712 | -0.505 | -0.712 | -0.771 | -0.901 |
| 4.0 | -0.900 | -0.770 | -0.712 | -0.504 | -0.712 | -0.770 | -0.900 |
| 6.0 | -0.900 | -0.770 | -0.711 | -0.503 | -0.711 | -0.770 | -0.900 |
| 8.0 | -0.898 | -0.769 | -0.710 | -0.502 | -0.710 | -0.769 | -0.898 |
| 10.0 | -0.897 | -0.768 | -0.709 | -0.501 | -0.709 | -0.768 | -0.897 |
| 12.0 | -0.895 | -0.766 | -0.707 | -0.498 | -0.707 | -0.766 | -0.895 |
| 14.0 | -0.893 | -0.764 | -0.706 | -0.496 | -0.706 | -0.764 | -0.893 |
| 16.0 | -0.890 | -0.762 | -0.703 | -0.493 | -0.703 | -0.762 | -0.890 |
| 18.0 | -0.887 | -0.759 | -0.700 | -0.490 | -0.700 | -0.759 | -0.887 |
| 20.0 | -0.884 | -0.756 | -0.697 | -0.487 | -0.697 | -0.756 | -0.884 |
| | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 |
| 20.0 | -0.884 | -0.756 | -0.697 | -0.486 | -0.697 | -0.756 | -0.884 |
| 22.0 | -0.880 | -0.753 | -0.694 | -0.483 | -0.694 | -0.753 | -0.880 |
| 24.0 | -0.876 | -0.749 | -0.690 | -0.478 | -0.690 | -0.749 | -0.876 |
| 26.0 | -0.871 | -0.745 | -0.685 | -0.472 | -0.685 | -0.745 | -0.871 |
| 28.0 | -0.865 | -0.740 | -0.681 | -0.467 | -0.681 | -0.740 | -0.865 |
| 30.0 | -0.859 | -0.735 | -0.675 | -0.460 | -0.675 | -0.735 | -0.859 |
| 32.0 | -0.852 | -0.729 | -0.669 | -0.452 | -0.669 | -0.729 | -0.852 |
| 34.0 | -0.844 | -0.722 | -0.662 | -0.444 | -0.662 | -0.722 | -0.844 |
| 36.0 | -0.836 | -0.715 | -0.654 | -0.435 | -0.654 | -0.715 | -0.836 |
| 38.0 | -0.826 | -0.707 | -0.646 | -0.425 | -0.646 | -0.707 | -0.826 |
| 40.0 | -0.816 | -0.698 | -0.637 | -0.414 | -0.637 | -0.698 | -0.816 |
| | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 |
| 40.0 | -0.816 | -0.698 | -0.637 | -0.413 | -0.637 | -0.698 | -0.816 |
| 42.0 | -0.805 | -0.688 | -0.627 | -0.402 | -0.627 | -0.688 | -0.805 |
| 44.0 | -0.792 | -0.677 | -0.616 | -0.388 | -0.616 | -0.677 | -0.792 |
| 46.0 | -0.778 | -0.665 | -0.603 | -0.373 | -0.603 | -0.665 | -0.778 |
| 48.0 | -0.762 | -0.651 | -0.589 | -0.356 | -0.589 | -0.651 | -0.762 |

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|---|---|---|---|---|---|---|---|
| 50.0 | -0.746 | -0.637 | -0.574 | -0.338 | -0.574 | -0.637 | -0.746 |
| 52.0 | -0.726 | -0.620 | -0.557 | -0.318 | -0.557 | -0.620 | -0.726 |
| 54.0 | -0.706 | -0.603 | -0.539 | -0.296 | -0.539 | -0.603 | -0.706 |
| 56.0 | -0.683 | -0.583 | -0.519 | -0.271 | -0.519 | -0.583 | -0.683 |
| 58.0 | -0.658 | -0.561 | -0.496 | -0.244 | -0.496 | -0.561 | -0.658 |
| 60.0 | -0.630 | -0.537 | -0.471 | -0.214 | -0.471 | -0.537 | -0.630 |
| | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 |
| 60.0 | -0.630 | -0.538 | -0.472 | -0.214 | -0.472 | -0.538 | -0.630 |
| 62.0 | -0.600 | -0.511 | -0.445 | -0.182 | -0.445 | -0.511 | -0.600 |
| 64.0 | -0.566 | -0.482 | -0.415 | -0.146 | -0.415 | -0.482 | -0.566 |
| 66.0 | -0.529 | -0.450 | -0.382 | -0.106 | -0.382 | -0.450 | -0.529 |
| 68.0 | -0.489 | -0.415 | -0.346 | -0.062 | -0.346 | -0.415 | -0.489 |
| 70.0 | -0.444 | -0.377 | -0.306 | -0.014 | -0.306 | -0.377 | -0.444 |
| 72.0 | -0.395 | -0.334 | -0.263 | 0.039 | -0.263 | -0.334 | -0.395 |
| 74.0 | -0.340 | -0.287 | -0.215 | 0.097 | -0.215 | -0.287 | -0.340 |
| 76.0 | -0.281 | -0.236 | -0.162 | 0.161 | -0.162 | -0.236 | -0.281 |
| 78.0 | -0.215 | -0.179 | -0.103 | 0.231 | -0.103 | -0.179 | -0.215 |
| 80.0 | -0.143 | -0.116 | -0.038 | 0.308 | -0.038 | -0.116 | -0.143 |
| | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 |
| 80.0 | -0.143 | -0.118 | -0.040 | 0.309 | -0.040 | -0.118 | -0.143 |
| 82.0 | -0.063 | -0.049 | 0.031 | 0.394 | 0.031 | -0.049 | -0.063 |
| 84.0 | 0.024 | 0.028 | 0.109 | 0.488 | 0.109 | 0.028 | 0.024 |
| 86.0 | 0.121 | 0.111 | 0.195 | 0.591 | 0.195 | 0.111 | 0.121 |
| 88.0 | 0.227 | 0.202 | 0.289 | 0.705 | 0.289 | 0.202 | 0.227 |
| 90.0 | 0.343 | 0.303 | 0.393 | 0.830 | 0.393 | 0.303 | 0.343 |
| 92.0 | 0.472 | 0.413 | 0.506 | 0.968 | 0.506 | 0.413 | 0.472 |
| 94.0 | 0.613 | 0.535 | 0.632 | 1.120 | 0.632 | 0.535 | 0.613 |
| 96.0 | 0.768 | 0.669 | 0.770 | 1.286 | 0.770 | 0.669 | 0.768 |
| 98.0 | 0.940 | 0.818 | 0.923 | 1.470 | 0.923 | 0.818 | 0.940 |
| 100.0 | 1.127 | 0.981 | 1.091 | 1.671 | 1.091 | 0.981 | 1.127 |
| | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 |
| 100.0 | 1.127 | 0.975 | 1.085 | 1.673 | 1.085 | 0.975 | 1.127 |
| 102.0 | 1.335 | 1.157 | 1.272 | 1.895 | 1.272 | 1.157 | 1.335 |
| 104.0 | 1.563 | 1.355 | 1.476 | 2.138 | 1.476 | 1.355 | 1.563 |
| 106.0 | 1.814 | 1.573 | 1.699 | 2.407 | 1.699 | 1.573 | 1.814 |
| 108.0 | 2.090 | 1.811 | 1.944 | 2.704 | 1.944 | 1.811 | 2.090 |
| 110.0 | 2.393 | 2.072 | 2.213 | 3.029 | 2.213 | 2.072 | 2.393 |
| 112.0 | 2.727 | 2.359 | 2.509 | 3.388 | 2.509 | 2.359 | 2.727 |
| 114.0 | 3.095 | 2.677 | 2.836 | 3.783 | 2.836 | 2.677 | 3.095 |
| 116.0 | 3.500 | 3.027 | 3.196 | 4.217 | 3.196 | 3.027 | 3.500 |
| 118.0 | 3.945 | 3.413 | 3.593 | 4.694 | 3.593 | 3.413 | 3.945 |
| 120.0 | 4.436 | 3.840 | 4.032 | 5.219 | 4.032 | 3.840 | 4.436 |

| | | | | | | | |
|---|---|---|---|---|---|---|---|
| | | | | | 0.000 | 0.000 | 0.000 |
| 120.0 | 4.436 | 4.730 | 4.369 | 2.671 | 4.369 | 4.730 | 4.436 |
| 121.8 | 4.877 | 5.113 | 4.763 | 3.143 | 4.763 | 5.113 | 4.877 |
| 123.6 | 5.279 | 5.461 | 5.121 | 3.573 | 5.121 | 5.461 | 5.279 |
| 125.4 | 5.644 | 5.776 | 5.445 | 3.965 | 5.445 | 5.776 | 5.644 |
| 127.2 | 5.974 | 6.061 | 5.739 | 4.319 | 5.739 | 6.061 | 5.974 |
| 129.0 | 6.271 | 6.316 | 6.001 | 4.638 | 6.001 | 6.316 | 6.271 |
| 130.8 | 6.538 | 6.546 | 6.238 | 4.925 | 6.238 | 6.546 | 6.538 |
| 132.6 | 6.782 | 6.757 | 6.455 | 5.187 | 6.455 | 6.757 | 6.782 |
| 134.4 | 6.997 | 6.943 | 6.647 | 5.417 | 6.647 | 6.943 | 6.997 |
| 136.2 | 7.189 | 7.110 | 6.818 | 5.622 | 6.818 | 7.110 | 7.189 |
| 138.0 | 7.357 | 7.258 | 6.969 | 5.802 | 6.969 | 7.258 | 7.357 |
| | | | | | 0.000 | 0.000 | 0.000 |
| 138.0 | 7.357 | 7.254 | 6.966 | 5.804 | 6.966 | 7.254 | 7.357 |
| 139.8 | 7.503 | 7.380 | 7.096 | 5.960 | 7.096 | 7.380 | 7.503 |
| 141.6 | 7.631 | 7.490 | 7.210 | 6.096 | 7.210 | 7.490 | 7.631 |
| 143.4 | 7.735 | 7.580 | 7.302 | 6.209 | 7.302 | 7.580 | 7.735 |
| 145.2 | 7.828 | 7.661 | 7.385 | 6.308 | 7.385 | 7.661 | 7.828 |
| 147.0 | 7.898 | 7.721 | 7.447 | 6.383 | 7.447 | 7.721 | 7.898 |
| 148.8 | 7.950 | 7.766 | 7.493 | 6.439 | 7.493 | 7.766 | 7.950 |
| 150.6 | 7.985 | 7.796 | 7.524 | 6.477 | 7.524 | 7.796 | 7.985 |
| 152.4 | 8.003 | 7.811 | 7.540 | 6.496 | 7.540 | 7.811 | 8.003 |
| 154.2 | 8.003 | 7.811 | 7.540 | 6.496 | 7.540 | 7.811 | 8.003 |
| 156.0 | 7.985 | 7.796 | 7.524 | 6.477 | 7.524 | 7.796 | 7.985 |
| | | | | | 0.000 | 0.000 | 0.000 |
| 156.0 | 7.985 | 7.797 | 7.525 | 6.477 | 7.525 | 7.797 | 7.985 |
| 157.8 | 7.956 | 7.772 | 7.499 | 6.446 | 7.499 | 7.772 | 7.956 |
| 159.6 | 7.904 | 7.726 | 7.452 | 6.390 | 7.452 | 7.726 | 7.904 |
| 161.4 | 7.840 | 7.671 | 7.395 | 6.321 | 7.395 | 7.671 | 7.840 |
| 163.2 | 7.753 | 7.595 | 7.317 | 6.228 | 7.317 | 7.595 | 7.753 |
| 165.0 | 7.654 | 7.511 | 7.230 | 6.121 | 7.230 | 7.511 | 7.654 |
| 166.8 | 7.532 | 7.406 | 7.122 | 5.990 | 7.122 | 7.406 | 7.532 |
| 168.6 | 7.387 | 7.280 | 6.993 | 5.835 | 6.993 | 7.280 | 7.387 |
| 170.4 | 7.224 | 7.140 | 6.848 | 5.660 | 6.848 | 7.140 | 7.224 |
| 172.2 | 7.038 | 6.978 | 6.683 | 5.461 | 6.683 | 6.978 | 7.038 |
| 174.0 | 6.823 | 6.791 | 6.490 | 5.231 | 6.490 | 6.791 | 6.823 |
| | | | | | 0.000 | 0.000 | 0.000 |
| 174.0 | 6.823 | 6.797 | 6.495 | 5.228 | 6.495 | 6.797 | 6.823 |
| 175.8 | 6.590 | 6.594 | 6.287 | 4.980 | 6.287 | 6.594 | 6.590 |
| 177.6 | 6.323 | 6.361 | 6.048 | 4.694 | 6.048 | 6.361 | 6.323 |
| 179.4 | 6.032 | 6.110 | 5.789 | 4.382 | 5.789 | 6.110 | 6.032 |
| 181.2 | 5.709 | 5.831 | 5.502 | 4.035 | 5.502 | 5.831 | 5.709 |

| | | | | | | | |
|---|---|---|---|---|---|---|---|
| 183.0 | 5.351 | 5.523 | 5.185 | 3.651 | 5.185 | 5.523 | 5.351 |
| 184.8 | 4.957 | 5.183 | 4.835 | 3.228 | 4.835 | 5.183 | 4.957 |
| 186.6 | 4.523 | 4.810 | 4.451 | 2.762 | 4.451 | 4.810 | 4.523 |
| 188.4 | 4.047 | 4.398 | 4.027 | 2.251 | 4.027 | 4.398 | 4.047 |
| 190.2 | 3.524 | 3.945 | 3.561 | 1.692 | 3.561 | 3.945 | 3.524 |
| 192.0 | 2.952 | 3.448 | 3.051 | 1.079 | 3.051 | 3.448 | 2.952 |
| | | | | | 0.000 | 0.000 | 0.000 |
| 192.0 | 2.952 | 2.181 | 2.429 | 3.915 | 2.429 | 2.181 | 2.952 |
| 194.1 | 2.287 | 1.602 | 1.834 | 3.204 | 1.834 | 1.602 | 2.287 |
| 196.2 | 1.687 | 1.081 | 1.298 | 2.560 | 1.298 | 1.081 | 1.687 |
| 198.3 | 1.143 | 0.611 | 0.815 | 1.977 | 0.815 | 0.611 | 1.143 |
| 200.4 | 0.651 | 0.187 | 0.378 | 1.449 | 0.378 | 0.187 | 0.651 |
| 202.5 | 0.207 | -0.196 | -0.015 | 0.972 | -0.015 | -0.196 | 0.207 |
| 204.6 | -0.195 | -0.542 | -0.372 | 0.540 | -0.372 | -0.542 | -0.195 |
| 206.7 | -0.559 | -0.855 | -0.694 | 0.150 | -0.694 | -0.855 | -0.559 |
| 208.8 | -0.888 | -1.140 | -0.987 | -0.202 | -0.987 | -1.140 | -0.888 |
| 210.9 | -1.186 | -1.399 | -1.254 | -0.521 | -1.254 | -1.399 | -1.186 |
| 213.0 | -1.454 | -1.635 | -1.496 | -0.807 | -1.496 | -1.635 | -1.454 |
| | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 |
| 213.0 | -1.454 | -1.626 | -1.488 | -0.811 | -1.488 | -1.626 | -1.454 |
| 215.1 | -1.698 | -1.838 | -1.706 | -1.071 | -1.706 | -1.838 | -1.698 |
| 217.2 | -1.917 | -2.029 | -1.902 | -1.306 | -1.902 | -2.029 | -1.917 |
| 219.3 | -2.116 | -2.201 | -2.079 | -1.519 | -2.079 | -2.201 | -2.116 |
| 221.4 | -2.296 | -2.356 | -2.238 | -1.712 | -2.238 | -2.356 | -2.296 |
| 223.5 | -2.458 | -2.496 | -2.382 | -1.887 | -2.382 | -2.496 | -2.458 |
| 225.6 | -2.605 | -2.622 | -2.512 | -2.045 | -2.512 | -2.622 | -2.605 |
| 227.7 | -2.738 | -2.736 | -2.630 | -2.187 | -2.630 | -2.736 | -2.738 |
| 229.8 | -2.858 | -2.841 | -2.737 | -2.316 | -2.737 | -2.841 | -2.858 |
| 231.9 | -2.967 | -2.935 | -2.834 | -2.432 | -2.834 | -2.935 | -2.967 |
| 234.0 | -3.064 | -3.021 | -2.922 | -2.536 | -2.922 | -3.021 | -3.064 |
| | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 |
| 234.0 | -3.064 | -3.018 | -2.920 | -2.538 | -2.920 | -3.018 | -3.064 |
| 236.1 | -3.153 | -3.095 | -2.999 | -2.633 | -2.999 | -3.095 | -3.153 |
| 238.2 | -3.233 | -3.164 | -3.070 | -2.718 | -3.070 | -3.164 | -3.233 |
| 240.3 | -3.306 | -3.227 | -3.135 | -2.796 | -3.135 | -3.227 | -3.306 |
| 242.4 | -3.371 | -3.283 | -3.193 | -2.866 | -3.193 | -3.283 | -3.371 |
| 244.5 | -3.429 | -3.334 | -3.245 | -2.929 | -3.245 | -3.334 | -3.429 |
| 246.6 | -3.482 | -3.379 | -3.292 | -2.986 | -3.292 | -3.379 | -3.482 |
| 248.7 | -3.530 | -3.420 | -3.334 | -3.037 | -3.334 | -3.420 | -3.530 |
| 250.8 | -3.573 | -3.458 | -3.372 | -3.083 | -3.372 | -3.458 | -3.573 |
| 252.9 | -3.612 | -3.492 | -3.407 | -3.124 | -3.407 | -3.492 | -3.612 |
| 255.0 | -3.646 | -3.522 | -3.438 | -3.161 | -3.438 | -3.522 | -3.646 |

| | | | | | | | |
|---|---|---|---|---|---|---|---|
| | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 |
| 255.0 | -3.646 | -3.520 | -3.437 | -3.161 | -3.437 | -3.520 | -3.646 |
| 257.1 | -3.678 | -3.548 | -3.465 | -3.195 | -3.465 | -3.548 | -3.678 |
| 259.2 | -3.705 | -3.572 | -3.490 | -3.225 | -3.490 | -3.572 | -3.705 |
| 261.3 | -3.730 | -3.594 | -3.512 | -3.252 | -3.512 | -3.594 | -3.730 |
| 263.4 | -3.753 | -3.613 | -3.532 | -3.275 | -3.532 | -3.613 | -3.753 |
| 265.5 | -3.772 | -3.629 | -3.549 | -3.296 | -3.549 | -3.629 | -3.772 |
| 267.6 | -3.789 | -3.644 | -3.564 | -3.315 | -3.564 | -3.644 | -3.789 |
| 269.7 | -3.804 | -3.657 | -3.578 | -3.331 | -3.578 | -3.657 | -3.804 |
| 271.8 | -3.817 | -3.668 | -3.589 | -3.345 | -3.589 | -3.668 | -3.817 |
| 273.9 | -3.829 | -3.678 | -3.599 | -3.357 | -3.599 | -3.678 | -3.829 |
| 276.0 | -3.838 | -3.687 | -3.608 | -3.367 | -3.608 | -3.687 | -3.838 |
| | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 |
| 276.0 | -3.838 | -3.686 | -3.607 | -3.367 | -3.607 | -3.686 | -3.838 |
| 278.1 | -3.846 | -3.693 | -3.614 | -3.375 | -3.614 | -3.693 | -3.846 |
| 280.2 | -3.852 | -3.698 | -3.620 | -3.382 | -3.620 | -3.698 | -3.852 |
| 282.3 | -3.857 | -3.702 | -3.624 | -3.387 | -3.624 | -3.702 | -3.857 |
| 284.4 | -3.859 | -3.705 | -3.627 | -3.390 | -3.627 | -3.705 | -3.859 |
| 286.5 | -3.861 | -3.706 | -3.628 | -3.392 | -3.628 | -3.706 | -3.861 |
| 288.6 | -3.862 | -3.707 | -3.629 | -3.393 | -3.629 | -3.707 | -3.862 |
| 290.7 | -3.861 | -3.706 | -3.628 | -3.391 | -3.628 | -3.706 | -3.861 |
| 292.8 | -3.858 | -3.704 | -3.626 | -3.389 | -3.626 | -3.704 | -3.858 |
| 294.9 | -3.854 | -3.700 | -3.622 | -3.385 | -3.622 | -3.700 | -3.854 |
| 297.0 | -3.849 | -3.696 | -3.617 | -3.379 | -3.617 | -3.696 | -3.849 |
| | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 |
| 297.0 | -3.849 | -3.696 | -3.617 | -3.379 | -3.617 | -3.696 | -3.849 |
| 299.1 | -3.842 | -3.690 | -3.611 | -3.371 | -3.611 | -3.690 | -3.842 |
| 301.2 | -3.833 | -3.682 | -3.603 | -3.362 | -3.603 | -3.682 | -3.833 |
| 303.3 | -3.823 | -3.673 | -3.594 | -3.351 | -3.594 | -3.673 | -3.823 |
| 305.4 | -3.811 | -3.663 | -3.583 | -3.338 | -3.583 | -3.663 | -3.811 |
| 307.5 | -3.797 | -3.651 | -3.571 | -3.323 | -3.571 | -3.651 | -3.797 |
| 309.6 | -3.780 | -3.637 | -3.556 | -3.305 | -3.556 | -3.637 | -3.780 |
| 311.7 | -3.762 | -3.621 | -3.540 | -3.286 | -3.540 | -3.621 | -3.762 |
| 313.8 | -3.742 | -3.603 | -3.522 | -3.264 | -3.522 | -3.603 | -3.742 |
| 315.9 | -3.718 | -3.582 | -3.501 | -3.238 | -3.501 | -3.582 | -3.718 |
| 318.0 | -3.692 | -3.560 | -3.477 | -3.210 | -3.477 | -3.560 | -3.692 |
| | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 |
| 318.0 | -3.692 | -3.561 | -3.478 | -3.210 | -3.478 | -3.561 | -3.692 |
| 320.1 | -3.662 | -3.535 | -3.451 | -3.178 | -3.451 | -3.535 | -3.662 |
| 322.2 | -3.629 | -3.506 | -3.422 | -3.143 | -3.422 | -3.506 | -3.629 |
| 324.3 | -3.593 | -3.475 | -3.390 | -3.104 | -3.390 | -3.475 | -3.593 |
| 326.4 | -3.552 | -3.439 | -3.353 | -3.060 | -3.353 | -3.439 | -3.552 |

| | | | | | | | |
|---|---|---|---|---|---|---|---|
| 328.5 | -3.507 | -3.400 | -3.313 | -3.012 | -3.313 | -3.400 | -3.507 |
| 330.6 | -3.456 | -3.357 | -3.268 | -2.957 | -3.268 | -3.357 | -3.456 |
| 332.7 | -3.400 | -3.309 | -3.219 | -2.898 | -3.219 | -3.309 | -3.400 |
| 334.8 | -3.338 | -3.255 | -3.164 | -2.831 | -3.164 | -3.255 | -3.338 |
| 336.9 | -3.270 | -3.196 | -3.103 | -2.757 | -3.103 | -3.196 | -3.270 |
| 339.0 | -3.193 | -3.129 | -3.034 | -2.676 | -3.034 | -3.129 | -3.193 |
| | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 |
| 339.0 | -3.193 | -3.132 | -3.037 | -2.675 | -3.037 | -3.132 | -3.193 |
| 341.1 | -3.109 | -3.058 | -2.961 | -2.585 | -2.961 | -3.058 | -3.109 |
| 343.2 | -3.016 | -2.977 | -2.878 | -2.485 | -2.878 | -2.977 | -3.016 |
| 345.3 | -2.913 | -2.887 | -2.785 | -2.375 | -2.785 | -2.887 | -2.913 |
| 347.4 | -2.799 | -2.789 | -2.684 | -2.252 | -2.684 | -2.789 | -2.799 |
| 349.5 | -2.672 | -2.680 | -2.572 | -2.116 | -2.572 | -2.680 | -2.672 |
| 351.6 | -2.533 | -2.560 | -2.449 | -1.967 | -2.449 | -2.560 | -2.533 |
| 353.7 | -2.378 | -2.426 | -2.311 | -1.800 | -2.311 | -2.426 | -2.378 |
| 355.8 | -2.207 | -2.279 | -2.159 | -1.617 | -2.159 | -2.279 | -2.207 |
| 357.9 | -2.017 | -2.114 | -1.990 | -1.414 | -1.990 | -2.114 | -2.017 |
| 360.0 | -1.808 | -1.932 | -1.803 | -1.190 | -1.803 | -1.932 | -1.808 |
| | | | | | 0.000 | 0.000 | 0.000 |
| 360.0 | -1.808 | -1.588 | -1.676 | -2.197 | -1.676 | -1.588 | -1.808 |
| 362.0 | -1.609 | -1.415 | -1.498 | -1.984 | -1.498 | -1.415 | -1.609 |
| 364.0 | -1.427 | -1.257 | -1.336 | -1.789 | -1.336 | -1.257 | -1.427 |
| 366.0 | -1.262 | -1.114 | -1.189 | -1.612 | -1.189 | -1.114 | -1.262 |
| 368.0 | -1.112 | -0.985 | -1.056 | -1.451 | -1.056 | -0.985 | -1.112 |
| 370.0 | -0.976 | -0.868 | -0.935 | -1.305 | -0.935 | -0.868 | -0.976 |
| 372.0 | -0.852 | -0.761 | -0.826 | -1.172 | -0.826 | -0.761 | -0.852 |
| 374.0 | -0.739 | -0.664 | -0.725 | -1.052 | -0.725 | -0.664 | -0.739 |
| 376.0 | -0.637 | -0.575 | -0.634 | -0.942 | -0.634 | -0.575 | -0.637 |
| 378.0 | -0.544 | -0.495 | -0.552 | -0.843 | -0.552 | -0.495 | -0.544 |
| 380.0 | -0.460 | -0.421 | -0.475 | -0.752 | -0.475 | -0.421 | -0.460 |
| | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 |
| 380.0 | -0.460 | -0.423 | -0.478 | -0.751 | -0.478 | -0.423 | -0.460 |
| 382.0 | -0.383 | -0.356 | -0.409 | -0.669 | -0.409 | -0.356 | -0.383 |
| 384.0 | -0.313 | -0.296 | -0.347 | -0.595 | -0.347 | -0.296 | -0.313 |
| 386.0 | -0.250 | -0.241 | -0.290 | -0.527 | -0.290 | -0.241 | -0.250 |
| 388.0 | -0.192 | -0.192 | -0.239 | -0.465 | -0.239 | -0.192 | -0.192 |
| 390.0 | -0.140 | -0.146 | -0.193 | -0.409 | -0.193 | -0.146 | -0.140 |
| 392.0 | -0.092 | -0.106 | -0.151 | -0.358 | -0.151 | -0.106 | -0.092 |
| 394.0 | -0.049 | -0.068 | -0.113 | -0.311 | -0.113 | -0.068 | -0.049 |
| 396.0 | -0.010 | -0.034 | -0.078 | -0.269 | -0.078 | -0.034 | -0.010 |
| 398.0 | 0.026 | -0.003 | -0.046 | -0.231 | -0.046 | -0.003 | 0.026 |
| 400.0 | 0.058 | 0.025 | -0.017 | -0.196 | -0.017 | 0.025 | 0.058 |

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|---|---|---|---|---|---|---|---|
| | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 |
| 400.0 | 0.058 | 0.024 | -0.017 | -0.196 | -0.017 | 0.024 | 0.058 |
| 402.0 | 0.088 | 0.050 | 0.009 | -0.165 | 0.009 | 0.050 | 0.088 |
| 404.0 | 0.114 | 0.073 | 0.033 | -0.136 | 0.033 | 0.073 | 0.114 |
| 406.0 | 0.139 | 0.094 | 0.055 | -0.110 | 0.055 | 0.094 | 0.139 |
| 408.0 | 0.161 | 0.113 | 0.074 | -0.086 | 0.074 | 0.113 | 0.161 |
| 410.0 | 0.181 | 0.130 | 0.092 | -0.064 | 0.092 | 0.130 | 0.181 |
| 412.0 | 0.199 | 0.146 | 0.108 | -0.045 | 0.108 | 0.146 | 0.199 |
| 414.0 | 0.216 | 0.160 | 0.123 | -0.027 | 0.123 | 0.160 | 0.216 |
| 416.0 | 0.231 | 0.173 | 0.136 | -0.011 | 0.136 | 0.173 | 0.231 |
| 418.0 | 0.244 | 0.185 | 0.148 | 0.004 | 0.148 | 0.185 | 0.244 |
| 420.0 | 0.257 | 0.196 | 0.160 | 0.017 | 0.160 | 0.196 | 0.257 |
| | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 |
| 420.0 | 0.257 | 0.196 | 0.159 | 0.017 | 0.159 | 0.196 | 0.257 |
| 422.0 | 0.268 | 0.206 | 0.169 | 0.029 | 0.169 | 0.206 | 0.268 |
| 424.0 | 0.278 | 0.215 | 0.179 | 0.040 | 0.179 | 0.215 | 0.278 |
| 426.0 | 0.288 | 0.223 | 0.187 | 0.050 | 0.187 | 0.223 | 0.288 |
| 428.0 | 0.296 | 0.230 | 0.194 | 0.059 | 0.194 | 0.230 | 0.296 |
| 430.0 | 0.304 | 0.236 | 0.201 | 0.067 | 0.201 | 0.236 | 0.304 |
| 432.0 | 0.311 | 0.242 | 0.207 | 0.075 | 0.207 | 0.242 | 0.311 |
| 434.0 | 0.317 | 0.248 | 0.213 | 0.082 | 0.213 | 0.248 | 0.317 |
| 436.0 | 0.323 | 0.253 | 0.218 | 0.088 | 0.218 | 0.253 | 0.323 |
| 438.0 | 0.328 | 0.257 | 0.223 | 0.093 | 0.223 | 0.257 | 0.328 |
| 440.0 | 0.333 | 0.262 | 0.227 | 0.098 | 0.227 | 0.262 | 0.333 |
| | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 |
| 440.0 | 0.333 | 0.261 | 0.227 | 0.098 | 0.227 | 0.261 | 0.333 |
| 442.0 | 0.337 | 0.265 | 0.230 | 0.103 | 0.230 | 0.265 | 0.337 |
| 444.0 | 0.341 | 0.268 | 0.234 | 0.107 | 0.234 | 0.268 | 0.341 |
| 446.0 | 0.344 | 0.271 | 0.237 | 0.111 | 0.237 | 0.271 | 0.344 |
| 448.0 | 0.347 | 0.274 | 0.240 | 0.114 | 0.240 | 0.274 | 0.347 |
| 450.0 | 0.350 | 0.277 | 0.242 | 0.117 | 0.242 | 0.277 | 0.350 |
| 452.0 | 0.353 | 0.279 | 0.244 | 0.120 | 0.244 | 0.279 | 0.353 |
| 454.0 | 0.355 | 0.281 | 0.246 | 0.122 | 0.246 | 0.281 | 0.355 |
| 456.0 | 0.357 | 0.282 | 0.248 | 0.124 | 0.248 | 0.282 | 0.357 |
| 458.0 | 0.359 | 0.284 | 0.250 | 0.126 | 0.250 | 0.284 | 0.359 |
| 460.0 | 0.360 | 0.285 | 0.251 | 0.128 | 0.251 | 0.285 | 0.360 |
| | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 |
| 460.0 | 0.360 | 0.285 | 0.251 | 0.128 | 0.251 | 0.285 | 0.360 |
| 462.0 | 0.362 | 0.286 | 0.253 | 0.130 | 0.253 | 0.286 | 0.362 |
| 464.0 | 0.363 | 0.288 | 0.254 | 0.131 | 0.254 | 0.288 | 0.363 |
| 466.0 | 0.364 | 0.288 | 0.255 | 0.132 | 0.255 | 0.288 | 0.364 |
| 468.0 | 0.365 | 0.289 | 0.255 | 0.133 | 0.255 | 0.289 | 0.365 |

| | | | | | | | |
|---|---|---|---|---|---|---|---|
| 470.0 | 0.366 | 0.290 | 0.256 | 0.134 | 0.256 | 0.290 | 0.366 |
| 472.0 | 0.366 | 0.290 | 0.257 | 0.134 | 0.257 | 0.290 | 0.366 |
| 474.0 | 0.367 | 0.291 | 0.257 | 0.135 | 0.257 | 0.291 | 0.367 |
| 476.0 | 0.367 | 0.291 | 0.257 | 0.135 | 0.257 | 0.291 | 0.367 |
| 478.0 | 0.367 | 0.291 | 0.257 | 0.135 | 0.257 | 0.291 | 0.367 |
| 480.0 | 0.367 | 0.291 | 0.258 | 0.136 | 0.258 | 0.291 | 0.367 |

Total Shear with Negative St. Vt. Contribution

| z | Combined Shear Stress with Negative St. Vt. | | | | | | |
|---|---|---|---|---|---|---|---|
| | | | | | | | |
| 0.0 | 0.901 | 1.031 | 1.090 | 1.330 | 1.090 | 1.031 | 0.901 |
| 2.0 | 0.901 | 1.031 | 1.089 | 1.329 | 1.089 | 1.031 | 0.901 |
| 4.0 | 0.900 | 1.030 | 1.089 | 1.329 | 1.089 | 1.030 | 0.900 |
| 6.0 | 0.900 | 1.029 | 1.088 | 1.328 | 1.088 | 1.029 | 0.900 |
| 8.0 | 0.898 | 1.028 | 1.087 | 1.327 | 1.087 | 1.028 | 0.898 |
| 10.0 | 0.897 | 1.027 | 1.085 | 1.326 | 1.085 | 1.027 | 0.897 |
| 12.0 | 0.895 | 1.024 | 1.083 | 1.324 | 1.083 | 1.024 | 0.895 |
| 14.0 | 0.893 | 1.022 | 1.081 | 1.322 | 1.081 | 1.022 | 0.893 |
| 16.0 | 0.890 | 1.019 | 1.078 | 1.319 | 1.078 | 1.019 | 0.890 |
| 18.0 | 0.887 | 1.016 | 1.075 | 1.317 | 1.075 | 1.016 | 0.887 |
| 20.0 | 0.884 | 1.012 | 1.071 | 1.313 | 1.071 | 1.012 | 0.884 |
| | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 |
| 20.0 | 0.884 | 1.011 | 1.071 | 1.313 | 1.071 | 1.011 | 0.884 |
| 22.0 | 0.880 | 1.008 | 1.067 | 1.310 | 1.067 | 1.008 | 0.880 |
| 24.0 | 0.876 | 1.002 | 1.062 | 1.305 | 1.062 | 1.002 | 0.876 |
| 26.0 | 0.871 | 0.996 | 1.056 | 1.300 | 1.056 | 0.996 | 0.871 |
| 28.0 | 0.865 | 0.990 | 1.050 | 1.295 | 1.050 | 0.990 | 0.865 |
| 30.0 | 0.859 | 0.983 | 1.043 | 1.289 | 1.043 | 0.983 | 0.859 |
| 32.0 | 0.852 | 0.975 | 1.035 | 1.282 | 1.035 | 0.975 | 0.852 |
| 34.0 | 0.844 | 0.967 | 1.027 | 1.275 | 1.027 | 0.967 | 0.844 |
| 36.0 | 0.836 | 0.957 | 1.017 | 1.267 | 1.017 | 0.957 | 0.836 |
| 38.0 | 0.826 | 0.946 | 1.007 | 1.258 | 1.007 | 0.946 | 0.826 |
| 40.0 | 0.816 | 0.934 | 0.995 | 1.248 | 0.995 | 0.934 | 0.816 |
| | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 |
| 40.0 | 0.816 | 0.934 | 0.995 | 1.248 | 0.995 | 0.934 | 0.816 |
| 42.0 | 0.805 | 0.922 | 0.983 | 1.237 | 0.983 | 0.922 | 0.805 |
| 44.0 | 0.792 | 0.907 | 0.969 | 1.225 | 0.969 | 0.907 | 0.792 |
| 46.0 | 0.778 | 0.891 | 0.953 | 1.211 | 0.953 | 0.891 | 0.778 |
| 48.0 | 0.762 | 0.873 | 0.936 | 1.196 | 0.936 | 0.873 | 0.762 |
| 50.0 | 0.746 | 0.854 | 0.917 | 1.180 | 0.917 | 0.854 | 0.746 |
| 52.0 | 0.726 | 0.832 | 0.895 | 1.161 | 0.895 | 0.832 | 0.726 |
| 54.0 | 0.706 | 0.809 | 0.873 | 1.142 | 0.873 | 0.809 | 0.706 |
| 56.0 | 0.683 | 0.783 | 0.847 | 1.119 | 0.847 | 0.783 | 0.683 |
| 58.0 | 0.658 | 0.755 | 0.819 | 1.095 | 0.819 | 0.755 | 0.658 |

| | | | | | | | |
|---|---|---|---|---|---|---|---|
| 60.0 | 0.630 | 0.723 | 0.789 | 1.068 | 0.789 | 0.723 | 0.630 |
| | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 |
| 60.0 | 0.630 | 0.722 | 0.788 | 1.069 | 0.788 | 0.722 | 0.630 |
| 62.0 | 0.600 | 0.688 | 0.754 | 1.040 | 0.754 | 0.688 | 0.600 |
| 64.0 | 0.566 | 0.650 | 0.717 | 1.007 | 0.717 | 0.650 | 0.566 |
| 66.0 | 0.529 | 0.608 | 0.676 | 0.972 | 0.676 | 0.608 | 0.529 |
| 68.0 | 0.489 | 0.562 | 0.631 | 0.932 | 0.631 | 0.562 | 0.489 |
| 70.0 | 0.444 | 0.511 | 0.581 | 0.889 | 0.581 | 0.511 | 0.444 |
| 72.0 | 0.395 | 0.455 | 0.526 | 0.842 | 0.526 | 0.455 | 0.395 |
| 74.0 | 0.340 | 0.393 | 0.466 | 0.790 | 0.466 | 0.393 | 0.340 |
| 76.0 | 0.281 | 0.326 | 0.400 | 0.732 | 0.400 | 0.326 | 0.281 |
| 78.0 | 0.215 | 0.251 | 0.327 | 0.669 | 0.327 | 0.251 | 0.215 |
| 80.0 | 0.143 | 0.170 | 0.247 | 0.599 | 0.247 | 0.170 | 0.143 |
| | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 |
| 80.0 | 0.143 | 0.167 | 0.245 | 0.600 | 0.245 | 0.167 | 0.143 |
| 82.0 | 0.063 | 0.078 | 0.158 | 0.523 | 0.158 | 0.078 | 0.063 |
| 84.0 | -0.024 | -0.021 | 0.061 | 0.438 | 0.061 | -0.021 | -0.024 |
| 86.0 | -0.121 | -0.130 | -0.046 | 0.345 | -0.046 | -0.130 | -0.121 |
| 88.0 | -0.227 | -0.251 | -0.164 | 0.243 | -0.164 | -0.251 | -0.227 |
| 90.0 | -0.343 | -0.384 | -0.294 | 0.131 | -0.294 | -0.384 | -0.343 |
| 92.0 | -0.472 | -0.530 | -0.437 | 0.008 | -0.437 | -0.530 | -0.472 |
| 94.0 | -0.613 | -0.691 | -0.594 | -0.129 | -0.594 | -0.691 | -0.613 |
| 96.0 | -0.768 | -0.867 | -0.767 | -0.278 | -0.767 | -0.867 | -0.768 |
| 98.0 | -0.940 | -1.061 | -0.957 | -0.443 | -0.957 | -1.061 | -0.940 |
| 100.0 | -1.127 | -1.273 | -1.164 | -0.625 | -1.164 | -1.273 | -1.127 |
| | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 |
| 100.0 | -1.127 | -1.280 | -1.170 | -0.622 | -1.170 | -1.280 | -1.127 |
| 102.0 | -1.335 | -1.513 | -1.398 | -0.823 | -1.398 | -1.513 | -1.335 |
| 104.0 | -1.563 | -1.770 | -1.650 | -1.043 | -1.650 | -1.770 | -1.563 |
| 106.0 | -1.814 | -2.055 | -1.928 | -1.285 | -1.928 | -2.055 | -1.814 |
| 108.0 | -2.090 | -2.369 | -2.236 | -1.551 | -2.236 | -2.369 | -2.090 |
| 110.0 | -2.393 | -2.715 | -2.573 | -1.843 | -2.573 | -2.715 | -2.393 |
| 112.0 | -2.727 | -3.095 | -2.946 | -2.165 | -2.946 | -3.095 | -2.727 |
| 114.0 | -3.095 | -3.514 | -3.355 | -2.519 | -3.355 | -3.514 | -3.095 |
| 116.0 | -3.500 | -3.974 | -3.805 | -2.909 | -3.805 | -3.974 | -3.500 |
| 118.0 | -3.945 | -4.478 | -4.298 | -3.339 | -4.298 | -4.478 | -3.945 |
| 120.0 | -4.436 | -5.032 | -4.840 | -3.813 | -4.840 | -5.032 | -4.436 |
| | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 |
| 120.0 | -4.436 | -4.142 | -4.503 | -6.361 | -4.503 | -4.142 | -4.436 |
| 121.8 | -4.877 | -4.641 | -4.991 | -6.787 | -4.991 | -4.641 | -4.877 |
| 123.6 | -5.279 | -5.097 | -5.437 | -7.174 | -5.437 | -5.097 | -5.279 |
| 125.4 | -5.644 | -5.511 | -5.842 | -7.525 | -5.842 | -5.511 | -5.644 |
| 127.2 | -5.974 | -5.887 | -6.210 | -7.844 | -6.210 | -5.887 | -5.974 |
| 129.0 | -6.271 | -6.225 | -6.540 | -8.129 | -6.540 | -6.225 | -6.271 |

| | | | | | | | |
|---|---|---|---|---|---|---|---|
| 130.8 | -6.538 | -6.530 | -6.838 | -8.386 | -6.838 | -6.530 | -6.538 |
| 132.6 | -6.782 | -6.807 | -7.109 | -8.621 | -7.109 | -6.807 | -6.782 |
| 134.4 | -6.997 | -7.051 | -7.348 | -8.829 | -7.348 | -7.051 | -6.997 |
| 136.2 | -7.189 | -7.268 | -7.560 | -9.014 | -7.560 | -7.268 | -7.189 |
| 138.0 | -7.357 | -7.457 | -7.746 | -9.177 | -7.746 | -7.457 | -7.357 |
| | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 |
| 138.0 | -7.357 | -7.461 | -7.749 | -9.175 | -7.749 | -7.461 | -7.357 |
| 139.8 | -7.503 | -7.626 | -7.910 | -9.316 | -7.910 | -7.626 | -7.503 |
| 141.6 | -7.631 | -7.771 | -8.052 | -9.439 | -8.052 | -7.771 | -7.631 |
| 143.4 | -7.735 | -7.890 | -8.168 | -9.540 | -8.168 | -7.890 | -7.735 |
| 145.2 | -7.828 | -7.995 | -8.271 | -9.629 | -8.271 | -7.995 | -7.828 |
| 147.0 | -7.898 | -8.075 | -8.349 | -9.696 | -8.349 | -8.075 | -7.898 |
| 148.8 | -7.950 | -8.134 | -8.407 | -9.747 | -8.407 | -8.134 | -7.950 |
| 150.6 | -7.985 | -8.174 | -8.446 | -9.780 | -8.446 | -8.174 | -7.985 |
| 152.4 | -8.003 | -8.194 | -8.465 | -9.797 | -8.465 | -8.194 | -8.003 |
| 154.2 | -8.003 | -8.194 | -8.465 | -9.797 | -8.465 | -8.194 | -8.003 |
| 156.0 | -7.985 | -8.174 | -8.446 | -9.780 | -8.446 | -8.174 | -7.985 |
| | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 |
| 156.0 | -7.985 | -8.173 | -8.445 | -9.781 | -8.445 | -8.173 | -7.985 |
| 157.8 | -7.956 | -8.141 | -8.413 | -9.752 | -8.413 | -8.141 | -7.956 |
| 159.6 | -7.904 | -8.082 | -8.356 | -9.702 | -8.356 | -8.082 | -7.904 |
| 161.4 | -7.840 | -8.009 | -8.285 | -9.640 | -8.285 | -8.009 | -7.840 |
| 163.2 | -7.753 | -7.910 | -8.188 | -9.556 | -8.188 | -7.910 | -7.753 |
| 165.0 | -7.654 | -7.797 | -8.078 | -9.461 | -8.078 | -7.797 | -7.654 |
| 166.8 | -7.532 | -7.658 | -7.942 | -9.344 | -7.942 | -7.658 | -7.532 |
| 168.6 | -7.387 | -7.493 | -7.780 | -9.204 | -7.780 | -7.493 | -7.387 |
| 170.4 | -7.224 | -7.308 | -7.599 | -9.047 | -7.599 | -7.308 | -7.224 |
| 172.2 | -7.038 | -7.097 | -7.393 | -8.868 | -7.393 | -7.097 | -7.038 |
| 174.0 | -6.823 | -6.854 | -7.155 | -8.660 | -7.155 | -6.854 | -6.823 |
| | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 |
| 174.0 | -6.823 | -6.849 | -7.150 | -8.662 | -7.150 | -6.849 | -6.823 |
| 175.8 | -6.590 | -6.587 | -6.894 | -8.437 | -6.894 | -6.587 | -6.590 |
| 177.6 | -6.323 | -6.285 | -6.598 | -8.179 | -6.598 | -6.285 | -6.323 |
| 179.4 | -6.032 | -5.955 | -6.276 | -7.899 | -6.276 | -5.955 | -6.032 |
| 181.2 | -5.709 | -5.587 | -5.916 | -7.588 | -5.916 | -5.587 | -5.709 |
| 183.0 | -5.351 | -5.179 | -5.517 | -7.243 | -5.517 | -5.179 | -5.351 |
| 184.8 | -4.957 | -4.730 | -5.078 | -6.864 | -5.078 | -4.730 | -4.957 |
| 186.6 | -4.523 | -4.237 | -4.596 | -6.446 | -4.596 | -4.237 | -4.523 |
| 188.4 | -4.047 | -3.696 | -4.066 | -5.987 | -4.066 | -3.696 | -4.047 |
| 190.2 | -3.524 | -3.103 | -3.487 | -5.483 | -3.487 | -3.103 | -3.524 |
| 192.0 | -2.952 | -2.457 | -2.854 | -4.931 | -2.854 | -2.457 | -2.952 |
| | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 |
| 192.0 | -2.952 | -3.723 | -3.476 | -2.096 | -3.476 | -3.723 | -2.952 |
| 194.1 | -2.287 | -2.973 | -2.741 | -1.453 | -2.741 | -2.973 | -2.287 |

| | | | | | | | |
|---|---|---|---|---|---|---|---|
| 196.2 | -1.687 | -2.292 | -2.075 | -0.874 | -2.075 | -2.292 | -1.687 |
| 198.3 | -1.143 | -1.675 | -1.471 | -0.349 | -1.471 | -1.675 | -1.143 |
| 200.4 | -0.651 | -1.115 | -0.924 | 0.124 | -0.924 | -1.115 | -0.651 |
| 202.5 | -0.207 | -0.609 | -0.429 | 0.551 | -0.429 | -0.609 | -0.207 |
| 204.6 | 0.195 | -0.151 | 0.019 | 0.938 | 0.019 | -0.151 | 0.195 |
| 206.7 | 0.559 | 0.263 | 0.424 | 1.288 | 0.424 | 0.263 | 0.559 |
| 208.8 | 0.888 | 0.636 | 0.789 | 1.605 | 0.789 | 0.636 | 0.888 |
| 210.9 | 1.186 | 0.972 | 1.118 | 1.893 | 1.118 | 0.972 | 1.186 |
| 213.0 | 1.454 | 1.273 | 1.413 | 2.153 | 1.413 | 1.273 | 1.454 |
| | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 |
| 213.0 | 1.454 | 1.282 | 1.420 | 2.150 | 1.420 | 1.282 | 1.454 |
| 215.1 | 1.698 | 1.557 | 1.689 | 2.385 | 1.689 | 1.557 | 1.698 |
| 217.2 | 1.917 | 1.806 | 1.933 | 2.597 | 1.933 | 1.806 | 1.917 |
| 219.3 | 2.116 | 2.031 | 2.153 | 2.789 | 2.153 | 2.031 | 2.116 |
| 221.4 | 2.296 | 2.236 | 2.353 | 2.961 | 2.353 | 2.236 | 2.296 |
| 223.5 | 2.458 | 2.421 | 2.534 | 3.118 | 2.534 | 2.421 | 2.458 |
| 225.6 | 2.605 | 2.588 | 2.698 | 3.260 | 2.698 | 2.588 | 2.605 |
| 227.7 | 2.738 | 2.739 | 2.846 | 3.387 | 2.846 | 2.739 | 2.738 |
| 229.8 | 2.858 | 2.876 | 2.979 | 3.503 | 2.979 | 2.876 | 2.858 |
| 231.9 | 2.967 | 2.998 | 3.099 | 3.608 | 3.099 | 2.998 | 2.967 |
| 234.0 | 3.064 | 3.108 | 3.206 | 3.703 | 3.206 | 3.108 | 3.064 |
| | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 |
| 234.0 | 3.064 | 3.111 | 3.209 | 3.701 | 3.209 | 3.111 | 3.064 |
| 236.1 | 3.153 | 3.211 | 3.307 | 3.787 | 3.307 | 3.211 | 3.153 |
| 238.2 | 3.233 | 3.302 | 3.396 | 3.864 | 3.396 | 3.302 | 3.233 |
| 240.3 | 3.306 | 3.384 | 3.476 | 3.934 | 3.476 | 3.384 | 3.306 |
| 242.4 | 3.371 | 3.458 | 3.549 | 3.997 | 3.549 | 3.458 | 3.371 |
| 244.5 | 3.429 | 3.525 | 3.614 | 4.053 | 3.614 | 3.525 | 3.429 |
| 246.6 | 3.482 | 3.585 | 3.673 | 4.104 | 3.673 | 3.585 | 3.482 |
| 248.7 | 3.530 | 3.640 | 3.726 | 4.150 | 3.726 | 3.640 | 3.530 |
| 250.8 | 3.573 | 3.688 | 3.774 | 4.192 | 3.774 | 3.688 | 3.573 |
| 252.9 | 3.612 | 3.732 | 3.817 | 4.229 | 3.817 | 3.732 | 3.612 |
| 255.0 | 3.646 | 3.771 | 3.855 | 4.262 | 3.855 | 3.771 | 3.646 |
| | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 |
| 255.0 | 3.646 | 3.772 | 3.856 | 4.262 | 3.856 | 3.772 | 3.646 |
| 257.1 | 3.678 | 3.807 | 3.890 | 4.292 | 3.890 | 3.807 | 3.678 |
| 259.2 | 3.705 | 3.839 | 3.921 | 4.319 | 3.921 | 3.839 | 3.705 |
| 261.3 | 3.730 | 3.867 | 3.949 | 4.343 | 3.949 | 3.867 | 3.730 |
| 263.4 | 3.753 | 3.892 | 3.973 | 4.365 | 3.973 | 3.892 | 3.753 |
| 265.5 | 3.772 | 3.914 | 3.995 | 4.383 | 3.995 | 3.914 | 3.772 |
| 267.6 | 3.789 | 3.934 | 4.014 | 4.400 | 4.014 | 3.934 | 3.789 |
| 269.7 | 3.804 | 3.951 | 4.031 | 4.414 | 4.031 | 3.951 | 3.804 |
| 271.8 | 3.817 | 3.966 | 4.045 | 4.427 | 4.045 | 3.966 | 3.817 |
| 273.9 | 3.829 | 3.979 | 4.058 | 4.438 | 4.058 | 3.979 | 3.829 |

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|---|---|---|---|---|---|---|---|
| 276.0 | 3.838 | 3.989 | 4.068 | 4.447 | 4.068 | 3.989 | 3.838 |
| | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 |
| 276.0 | 3.838 | 3.990 | 4.069 | 4.447 | 4.069 | 3.990 | 3.838 |
| 278.1 | 3.846 | 3.998 | 4.077 | 4.454 | 4.077 | 3.998 | 3.846 |
| 280.2 | 3.852 | 4.005 | 4.084 | 4.460 | 4.084 | 4.005 | 3.852 |
| 282.3 | 3.857 | 4.011 | 4.089 | 4.465 | 4.089 | 4.011 | 3.857 |
| 284.4 | 3.859 | 4.014 | 4.092 | 4.468 | 4.092 | 4.014 | 3.859 |
| 286.5 | 3.861 | 4.016 | 4.094 | 4.469 | 4.094 | 4.016 | 3.861 |
| 288.6 | 3.862 | 4.017 | 4.095 | 4.470 | 4.095 | 4.017 | 3.862 |
| 290.7 | 3.861 | 4.015 | 4.094 | 4.469 | 4.094 | 4.015 | 3.861 |
| 292.8 | 3.858 | 4.013 | 4.091 | 4.466 | 4.091 | 4.013 | 3.858 |
| 294.9 | 3.854 | 4.008 | 4.087 | 4.462 | 4.087 | 4.008 | 3.854 |
| 297.0 | 3.849 | 4.002 | 4.081 | 4.457 | 4.081 | 4.002 | 3.849 |
| | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 |
| 297.0 | 3.849 | 4.002 | 4.081 | 4.458 | 4.081 | 4.002 | 3.849 |
| 299.1 | 3.842 | 3.994 | 4.073 | 4.451 | 4.073 | 3.994 | 3.842 |
| 301.2 | 3.833 | 3.984 | 4.063 | 4.442 | 4.063 | 3.984 | 3.833 |
| 303.3 | 3.823 | 3.973 | 4.052 | 4.432 | 4.052 | 3.973 | 3.823 |
| 305.4 | 3.811 | 3.959 | 4.038 | 4.420 | 4.038 | 3.959 | 3.811 |
| 307.5 | 3.797 | 3.943 | 4.023 | 4.407 | 4.023 | 3.943 | 3.797 |
| 309.6 | 3.780 | 3.924 | 4.005 | 4.391 | 4.005 | 3.924 | 3.780 |
| 311.7 | 3.762 | 3.904 | 3.984 | 4.374 | 3.984 | 3.904 | 3.762 |
| 313.8 | 3.742 | 3.880 | 3.961 | 4.354 | 3.961 | 3.880 | 3.742 |
| 315.9 | 3.718 | 3.853 | 3.935 | 4.331 | 3.935 | 3.853 | 3.718 |
| 318.0 | 3.692 | 3.823 | 3.906 | 4.306 | 3.906 | 3.823 | 3.692 |
| | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 |
| 318.0 | 3.692 | 3.822 | 3.905 | 4.306 | 3.905 | 3.822 | 3.692 |
| 320.1 | 3.662 | 3.789 | 3.873 | 4.277 | 3.873 | 3.789 | 3.662 |
| 322.2 | 3.629 | 3.752 | 3.837 | 4.246 | 3.837 | 3.752 | 3.629 |
| 324.3 | 3.593 | 3.711 | 3.796 | 4.211 | 3.796 | 3.711 | 3.593 |
| 326.4 | 3.552 | 3.665 | 3.751 | 4.171 | 3.751 | 3.665 | 3.552 |
| 328.5 | 3.507 | 3.613 | 3.700 | 4.128 | 3.700 | 3.613 | 3.507 |
| 330.6 | 3.456 | 3.555 | 3.644 | 4.079 | 3.644 | 3.555 | 3.456 |
| 332.7 | 3.400 | 3.492 | 3.582 | 4.025 | 3.582 | 3.492 | 3.400 |
| 334.8 | 3.338 | 3.421 | 3.513 | 3.965 | 3.513 | 3.421 | 3.338 |
| 336.9 | 3.270 | 3.344 | 3.437 | 3.899 | 3.437 | 3.344 | 3.270 |
| 339.0 | 3.193 | 3.258 | 3.353 | 3.826 | 3.353 | 3.258 | 3.193 |
| | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 |
| 339.0 | 3.193 | 3.255 | 3.350 | 3.827 | 3.350 | 3.255 | 3.193 |
| 341.1 | 3.109 | 3.160 | 3.258 | 3.745 | 3.258 | 3.160 | 3.109 |
| 343.2 | 3.016 | 3.055 | 3.155 | 3.655 | 3.155 | 3.055 | 3.016 |
| 345.3 | 2.913 | 2.938 | 3.040 | 3.556 | 3.040 | 2.938 | 2.913 |
| 347.4 | 2.799 | 2.808 | 2.913 | 3.446 | 2.913 | 2.808 | 2.799 |
| 349.5 | 2.672 | 2.664 | 2.772 | 3.324 | 2.772 | 2.664 | 2.672 |

| | | | | | | | |
|---|---|---|---|---|---|---|---|
| 351.6 | 2.533 | 2.505 | 2.617 | 3.190 | 2.617 | 2.505 | 2.533 |
| 353.7 | 2.378 | 2.329 | 2.444 | 3.040 | 2.444 | 2.329 | 2.378 |
| 355.8 | 2.207 | 2.135 | 2.254 | 2.876 | 2.254 | 2.135 | 2.207 |
| 357.9 | 2.017 | 1.920 | 2.045 | 2.693 | 2.045 | 1.920 | 2.017 |
| 360.0 | 1.808 | 1.684 | 1.813 | 2.491 | 1.813 | 1.684 | 1.808 |
| | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 |
| 360.0 | 1.808 | 2.028 | 1.940 | 1.484 | 1.940 | 2.028 | 1.808 |
| 362.0 | 1.609 | 1.803 | 1.719 | 1.291 | 1.719 | 1.803 | 1.609 |
| 364.0 | 1.427 | 1.597 | 1.518 | 1.115 | 1.518 | 1.597 | 1.427 |
| 366.0 | 1.262 | 1.409 | 1.334 | 0.956 | 1.334 | 1.409 | 1.262 |
| 368.0 | 1.112 | 1.239 | 1.168 | 0.812 | 1.168 | 1.239 | 1.112 |
| 370.0 | 0.976 | 1.084 | 1.016 | 0.681 | 1.016 | 1.084 | 0.976 |
| 372.0 | 0.852 | 0.943 | 0.878 | 0.562 | 0.878 | 0.943 | 0.852 |
| 374.0 | 0.739 | 0.814 | 0.753 | 0.454 | 0.753 | 0.814 | 0.739 |
| 376.0 | 0.637 | 0.698 | 0.639 | 0.355 | 0.639 | 0.698 | 0.637 |
| 378.0 | 0.544 | 0.594 | 0.537 | 0.265 | 0.537 | 0.594 | 0.544 |
| 380.0 | 0.460 | 0.499 | 0.444 | 0.183 | 0.444 | 0.499 | 0.460 |
| | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 |
| 380.0 | 0.460 | 0.496 | 0.442 | 0.184 | 0.442 | 0.496 | 0.460 |
| 382.0 | 0.383 | 0.410 | 0.357 | 0.110 | 0.357 | 0.410 | 0.383 |
| 384.0 | 0.313 | 0.330 | 0.280 | 0.043 | 0.280 | 0.330 | 0.313 |
| 386.0 | 0.250 | 0.258 | 0.209 | -0.018 | 0.209 | 0.258 | 0.250 |
| 388.0 | 0.192 | 0.193 | 0.145 | -0.073 | 0.145 | 0.193 | 0.192 |
| 390.0 | 0.140 | 0.133 | 0.087 | -0.124 | 0.087 | 0.133 | 0.140 |
| 392.0 | 0.092 | 0.079 | 0.034 | -0.170 | 0.034 | 0.079 | 0.092 |
| 394.0 | 0.049 | 0.030 | -0.014 | -0.211 | -0.014 | 0.030 | 0.049 |
| 396.0 | 0.010 | -0.014 | -0.058 | -0.249 | -0.058 | -0.014 | 0.010 |
| 398.0 | -0.026 | -0.055 | -0.097 | -0.283 | -0.097 | -0.055 | -0.026 |
| 400.0 | -0.058 | -0.091 | -0.133 | -0.315 | -0.133 | -0.091 | -0.058 |
| | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 |
| 400.0 | -0.058 | -0.092 | -0.134 | -0.314 | -0.134 | -0.092 | -0.058 |
| 402.0 | -0.088 | -0.125 | -0.166 | -0.343 | -0.166 | -0.125 | -0.088 |
| 404.0 | -0.114 | -0.156 | -0.196 | -0.369 | -0.196 | -0.156 | -0.114 |
| 406.0 | -0.139 | -0.183 | -0.223 | -0.392 | -0.223 | -0.183 | -0.139 |
| 408.0 | -0.161 | -0.209 | -0.248 | -0.414 | -0.248 | -0.209 | -0.161 |
| 410.0 | -0.181 | -0.231 | -0.270 | -0.433 | -0.270 | -0.231 | -0.181 |
| 412.0 | -0.199 | -0.252 | -0.290 | -0.450 | -0.290 | -0.252 | -0.199 |
| 414.0 | -0.216 | -0.271 | -0.309 | -0.466 | -0.309 | -0.271 | -0.216 |
| 416.0 | -0.231 | -0.288 | -0.325 | -0.481 | -0.325 | -0.288 | -0.231 |
| 418.0 | -0.244 | -0.304 | -0.341 | -0.494 | -0.341 | -0.304 | -0.244 |
| 420.0 | -0.257 | -0.318 | -0.354 | -0.506 | -0.354 | -0.318 | -0.257 |
| | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 |
| 420.0 | -0.257 | -0.318 | -0.355 | -0.506 | -0.355 | -0.318 | -0.257 |
| 422.0 | -0.268 | -0.331 | -0.367 | -0.517 | -0.367 | -0.331 | -0.268 |

| | | | | | | | |
|---|---|---|---|---|---|---|---|
| 424.0 | -0.278 | -0.342 | -0.378 | -0.527 | -0.378 | -0.342 | -0.278 |
| 426.0 | -0.288 | -0.353 | -0.389 | -0.536 | -0.389 | -0.353 | -0.288 |
| 428.0 | -0.296 | -0.362 | -0.398 | -0.544 | -0.398 | -0.362 | -0.296 |
| 430.0 | -0.304 | -0.371 | -0.407 | -0.551 | -0.407 | -0.371 | -0.304 |
| 432.0 | -0.311 | -0.379 | -0.414 | -0.558 | -0.414 | -0.379 | -0.311 |
| 434.0 | -0.317 | -0.386 | -0.421 | -0.564 | -0.421 | -0.386 | -0.317 |
| 436.0 | -0.323 | -0.393 | -0.428 | -0.569 | -0.428 | -0.393 | -0.323 |
| 438.0 | -0.328 | -0.399 | -0.433 | -0.574 | -0.433 | -0.399 | -0.328 |
| 440.0 | -0.333 | -0.404 | -0.439 | -0.579 | -0.439 | -0.404 | -0.333 |
| | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 |
| 440.0 | -0.333 | -0.404 | -0.439 | -0.579 | -0.439 | -0.404 | -0.333 |
| 442.0 | -0.337 | -0.409 | -0.443 | -0.583 | -0.443 | -0.409 | -0.337 |
| 444.0 | -0.341 | -0.413 | -0.448 | -0.587 | -0.448 | -0.413 | -0.341 |
| 446.0 | -0.344 | -0.417 | -0.451 | -0.590 | -0.451 | -0.417 | -0.344 |
| 448.0 | -0.347 | -0.421 | -0.455 | -0.593 | -0.455 | -0.421 | -0.347 |
| 450.0 | -0.350 | -0.424 | -0.458 | -0.596 | -0.458 | -0.424 | -0.350 |
| 452.0 | -0.353 | -0.427 | -0.461 | -0.598 | -0.461 | -0.427 | -0.353 |
| 454.0 | -0.355 | -0.429 | -0.463 | -0.600 | -0.463 | -0.429 | -0.355 |
| 456.0 | -0.357 | -0.432 | -0.466 | -0.602 | -0.466 | -0.432 | -0.357 |
| 458.0 | -0.359 | -0.434 | -0.468 | -0.604 | -0.468 | -0.434 | -0.359 |
| 460.0 | -0.360 | -0.435 | -0.469 | -0.606 | -0.469 | -0.435 | -0.360 |
| | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 |
| 460.0 | -0.360 | -0.435 | -0.469 | -0.606 | -0.469 | -0.435 | -0.360 |
| 462.0 | -0.362 | -0.437 | -0.471 | -0.607 | -0.471 | -0.437 | -0.362 |
| 464.0 | -0.363 | -0.438 | -0.472 | -0.608 | -0.472 | -0.438 | -0.363 |
| 466.0 | -0.364 | -0.440 | -0.474 | -0.609 | -0.474 | -0.440 | -0.364 |
| 468.0 | -0.365 | -0.441 | -0.475 | -0.610 | -0.475 | -0.441 | -0.365 |
| 470.0 | -0.366 | -0.441 | -0.475 | -0.611 | -0.475 | -0.441 | -0.366 |
| 472.0 | -0.366 | -0.442 | -0.476 | -0.611 | -0.476 | -0.442 | -0.366 |
| 474.0 | -0.367 | -0.443 | -0.477 | -0.612 | -0.477 | -0.443 | -0.367 |
| 476.0 | -0.367 | -0.443 | -0.477 | -0.612 | -0.477 | -0.443 | -0.367 |
| 478.0 | -0.367 | -0.443 | -0.477 | -0.612 | -0.477 | -0.443 | -0.367 |
| 480.0 | -0.367 | -0.443 | -0.477 | -0.612 | -0.477 | -0.443 | -0.367 |

## APPENDIX H

## MEDWADOWSKI'S TORSION PROBLEM

In the EBC of case study four, this material corresponds to the input model, input data, input forms, figures of output data with checks in notepad version, and excel processed output data and charts.

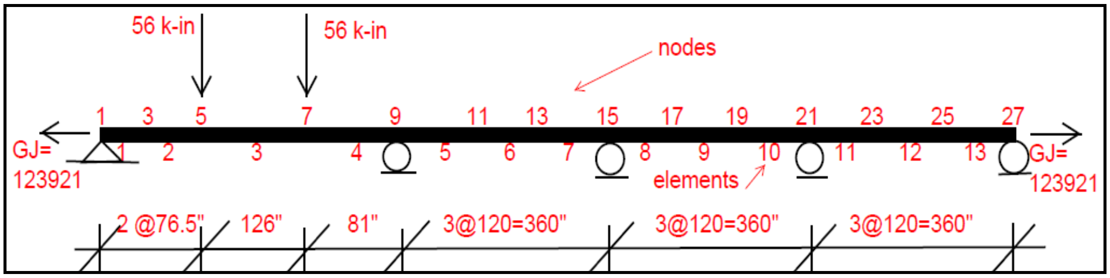
Charts are presented containing both partial and combined tresses along interest points of the beam and cross section profile. Again, the asymptotic behavior of the thin-walled beam elastic line is successfully shown in the charts, which evidences the efficiency of the high order finite element used by BMTORSWP.

Positive shear and axial stresses are assumed similar to those occurring at the cross section flanges and web when the beam undergoes bending as shown below.

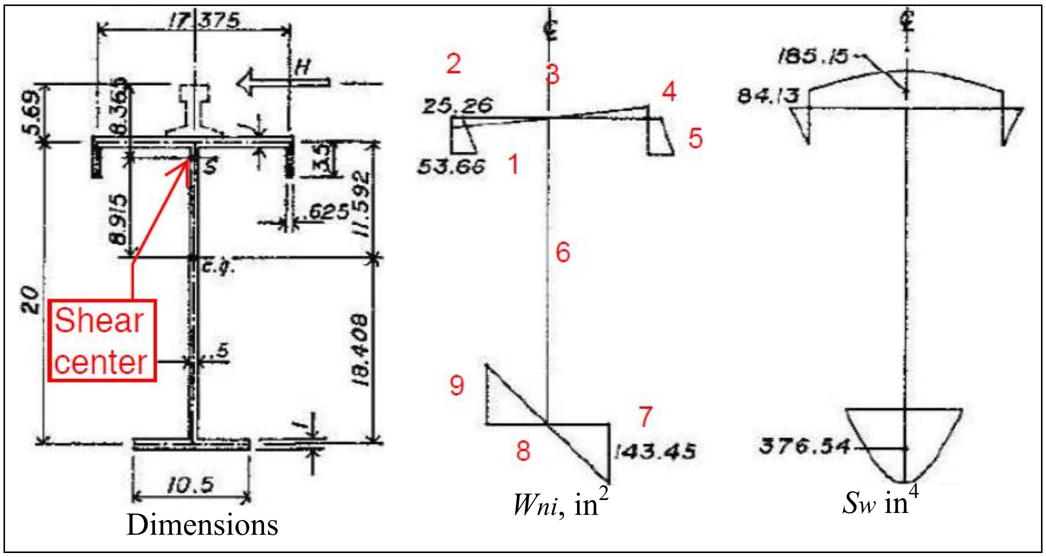
There is a match between the silhouette of BMTORSWP and Medwadowski charts. Nevertheless, Medwadowski committed a huge error in the scale of torques. He found torques 100 times larger than expected, which in terms of stresses is not acceptable at all. The reasoning to support this statement is as follows:

Given the hyperbolic nature of the functions involve in the angle of twist θ(z), each of its derivatives decreases an order of magnitude equal to the characteristic length "a". On the other hand, the ratio between bimoment B (= – ECwθ", ksi-in^2) and warping moment Tw (= – ECwθ'", ksi-in) should be in the range of "a" (√ECw/ GJ =138.5 in).

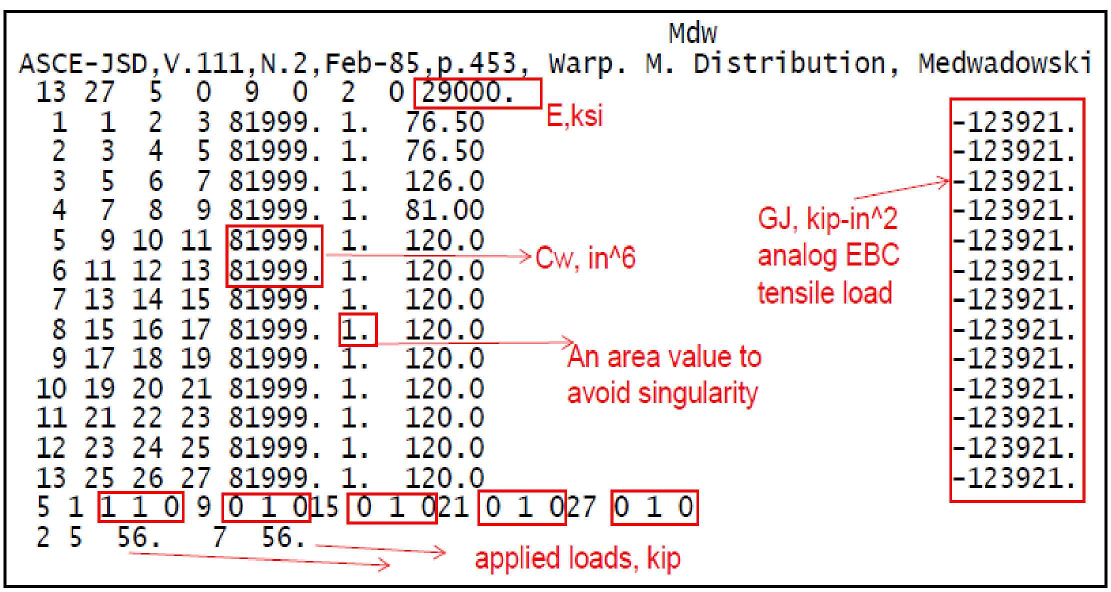
Therefore, using Medwadowski own bimoment data, the warping moment Tw is expected to be around (4000+3000)ks-in^2/138.50in ~ 50 k-in; as shown in the BMTORSWP chart.




EBC Model for BMTORSWP




Section Properties and Profile Interest Points “s”



```
                                        Mdw
ASCE-JSD,V.111,N.2,Feb-85,p.453, Warp. M. Distribution, Medwadowski
 13 27  5  0  9  0  2  0 29000.
  1  1  2  3 81999. 1.  76.50                          -123921.
  2  3  4  5 81999. 1.  76.50                          -123921.
  3  5  6  7 81999. 1.  126.0                          -123921.
  4  7  8  9 81999. 1.  81.00                          -123921.
  5  9 10 11 81999. 1.  120.0                          -123921.
  6 11 12 13 81999. 1.  120.0                          -123921.
  7 13 14 15 81999. 1.  120.0                          -123921.
  8 15 16 17 81999. 1.  120.0                          -123921.
  9 17 18 19 81999. 1.  120.0                          -123921.
 10 19 20 21 81999. 1.  120.0                          -123921.
 11 21 22 23 81999. 1.  120.0                          -123921.
 12 23 24 25 81999. 1.  120.0                          -123921.
 13 25 26 27 81999. 1.  120.0                          -123921.
 5 1 1 1 0 9 0 1 015 0 1 021 0 1 027 0 1 0
 2 5  56.   7  56.
```



Input Notepad for Medwadowski Problem

# PROGRAM BMTORSW INPUT FORM

ALPHAMERIC DESCRIPTION OF THE JOB — FILE NAME:

ASCE JSD, Vol. 111, No. 2, Feb-85, p. 453, Warpi

| NEL | NOD | NSUP | NSPD | JBW | NFX | NFY | NFZ | ELASTICITY (E11.4) |
|---|---|---|---|---|---|---|---|---|
| 13 | 27 | 5 | 0 | 9 | 0 | 2 | 0 | 29000. |

| NE | NI | NC | NJ | Cw | AREA | LENGTH | WTANG | SOIL NI | SOI |
|---|---|---|---|---|---|---|---|---|---|
| 1 | 1 | 2 | 3 | 81989. | 1. | 76.5 | | | |
| 2 | 3 | 4 | 5 | " | " | 76.5 | | | |
| 3 | 5 | 6 | 7 | " | " | 126. | | | |
| 4 | 7 | 8 | 9 | " | " | 81. | | | |
| 5 | 9 | 10 | 11 | " | " | 120. | | | |
| 6 | 11 | 12 | 13 | " | " | " | | | |
| 7 | 13 | 14 | 15 | " | " | " | | | |
| 8 | 15 | 16 | 17 | " | " | " | | | |
| 9 | 17 | 18 | 19 | " | " | " | | | |
| 10 | 19 | 20 | 21 | " | " | " | | | |
| 11 | 21 | 22 | 23 | " | " | " | | | |
| 12 | 23 | 24 | 25 | " | " | " | | | |
| 13 | 25 | 26 | 27 | " | " | " | | | |

| NF | N | KX | KY | KZ | N | KX | KY | KZ | N | KX | KY | KZ | N | KX | KY | KZ | N | KX | KY | K |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| 5 | 1 | 1 | 1 | 0 | 9 | 0 | 1 | 0 | 15 | 0 | 1 | 0 | 21 | 0 | 1 | 0 | 27 | 0 | 1 | |

| NF | N | KD | SPRING | N | KD | SPRING | N | KD |
|---|---|---|---|---|---|---|---|---|
| | | | | | | | | |
| | | | | | | | | |

| NF | N | LOAD VALUE | N | LOAD VALUE | N | LOAD VALUE | N | LOAD VALUE | N | L |
|---|---|---|---|---|---|---|---|---|---|---|
| 2 | 5 | 56. | 7 | 56. | | | | | | |
| | | | | | | | | | | |
| | | | | | | | | | | |
| | | | | | | | | | | |

| NF | ELEM | END | ROTATIONAL SPRING MODULUS | ELEM | END | ROTATIONAL SPRING MODULUS | ELEM | END | ROTATION SPRING MOD |
|---|---|---|---|---|---|---|---|---|---|
| | | | | | | | | | |
| | | | | | | | | | |
| | | | | | | | | | |

Input Form Left Side for Medwadowski Problem

AUTHOR: B. DESCHAPELLES

39 40 41 42 43 44 45 46 47 48 49 50 51 52 53 54 55 56 57 58 59 60 61 62 63 64 65 66 67 68 69 70 71 72 73 74 75 76 77 78 79 80

NAME:

Warping Moment Distribution by S. Medwadowski

GENERAL INFORMATION

PARTIAL RESTRAINT AT END ? 1 FOR YES, 0 FOR NO

| I | SOIL NJ | SOIL TI | SOIL TJ | ANGLE | - G*J | Wn @ NI | Wn @ NJ | 1 | 2 |
|---|---|---|---|---|---|---|---|---|---|
| | | | | | -123921. | | | | |
| | | | | | " | | | | |
| | | | | | " | | | | |
| | | | | | " | | | | |
| | | | | | " | | | | |
| | | | | | " | | | | |
| | | | | | " | | | | |
| | | | | | " | | | | |
| | | | | | " | | | | |
| | | | | | " | | | | |
| | | | | | " | | | | |
| | | | | | -123921. | | | | |
| | | | | | -123921. | | | | |

| KY | KZ | N | KX | KY | KZ | N | KX | KY | KZ | N | KX | KY | KZ | SUPPORTS |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| 1 | 0 | | | | | | | | | | | | | |

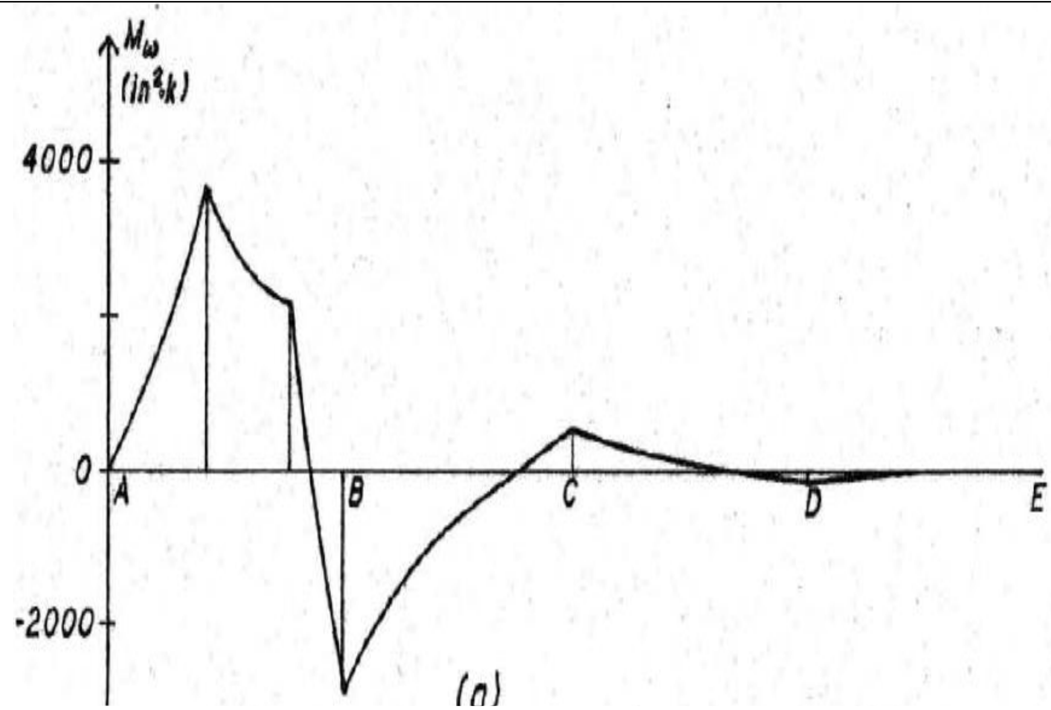
SPRING | N | KD | SPRING — SPRINGS AT SUPPORTS

N | LOAD VALUE | N | LOAD VALUE | N | LOAD VALUE — APPLIED NODAL FORCES

ROTATIONAL SPRING MODULUS | ELEM | END | ROTATIONAL SPRING MODULUS | ELEM | END | ROTATIONAL SPRING MODULUS — SPRINGS AT ELEM. END, 1 OR 2, IF ANY

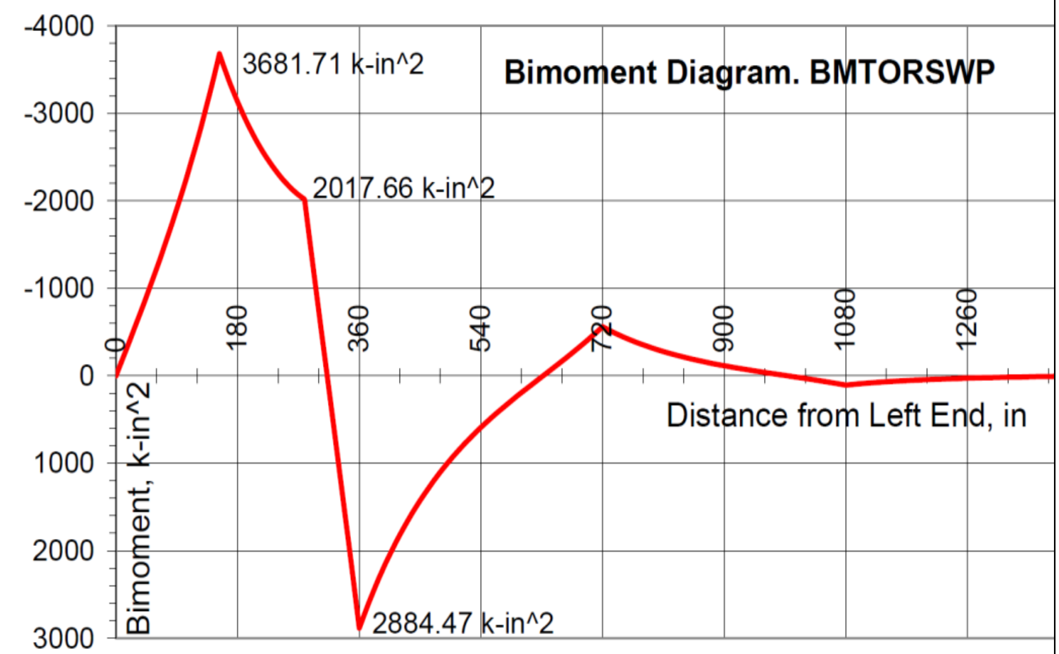
39 40 41 42 43 44 45 46 47 48 49 50 51 52 53 54 55 56 57 58 59 60 61 62 63 64 65 66 67 68 69 70 71 72 73 74 75 76 77 78 79 80

Input Form Right Side for Medwadowski Problem




Bimoment Values Obtained by Medwadowski (1985)

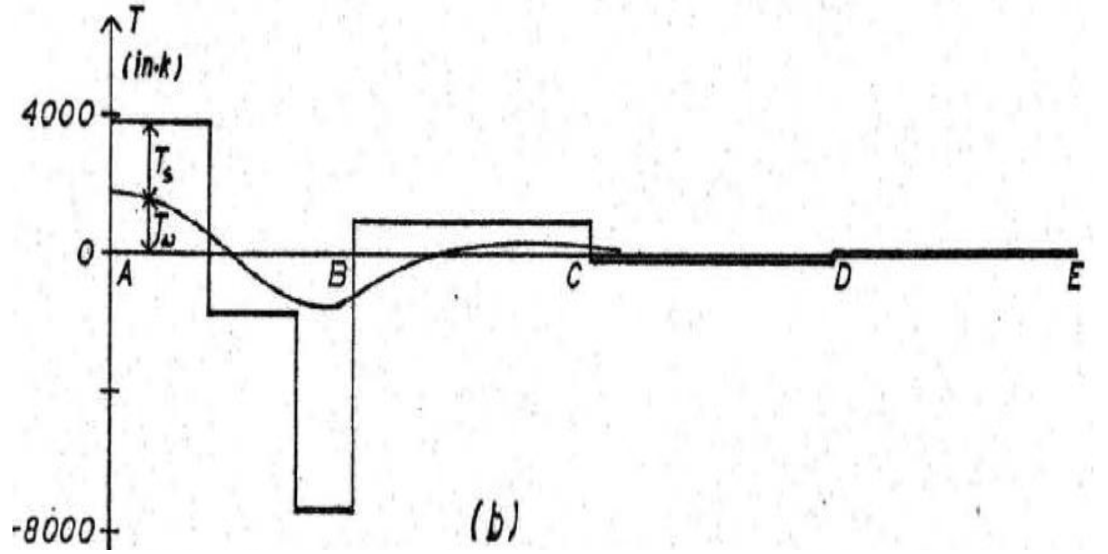
| | | | | | | | |
|---|---|---|---|---|---|---|---|
| 0 | -2885 | 2885 | 562 | -562 | -104 | 104 | 0 |



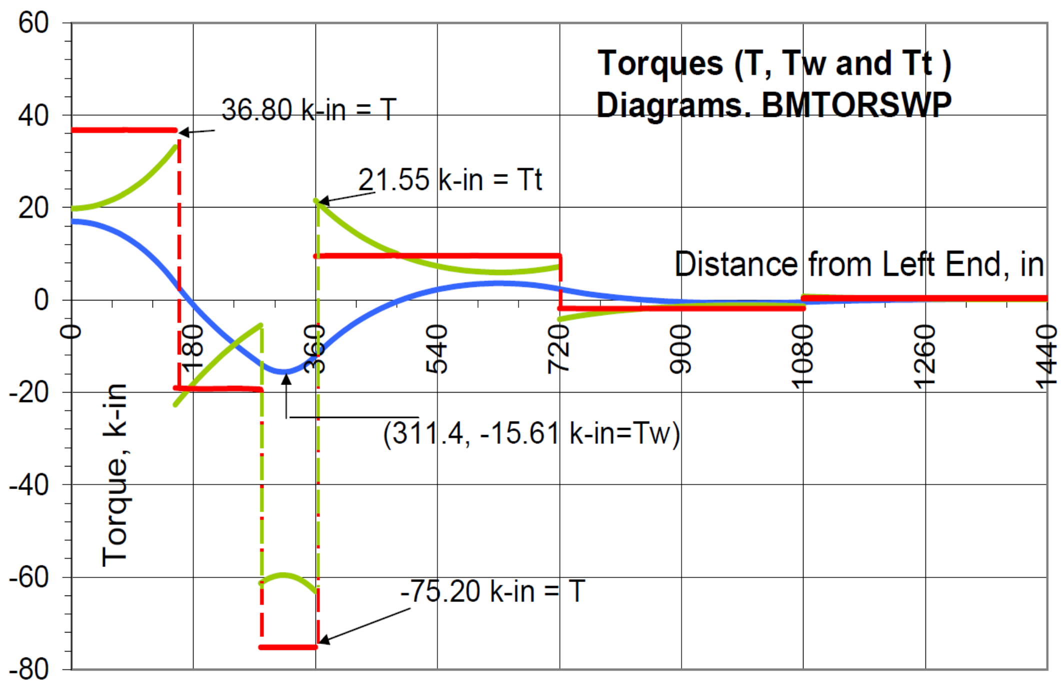

Obtained with BMTORSWP

| | | | | | | | |
|---|---|---|---|---|---|---|---|
| 0 | -2884.5 | 2884.5 | 561.5 | -561.5 | -105.4 | 105.4 | 0 |

Comparison of Bimoment Charts by Medwadowski and with BMTORSWP

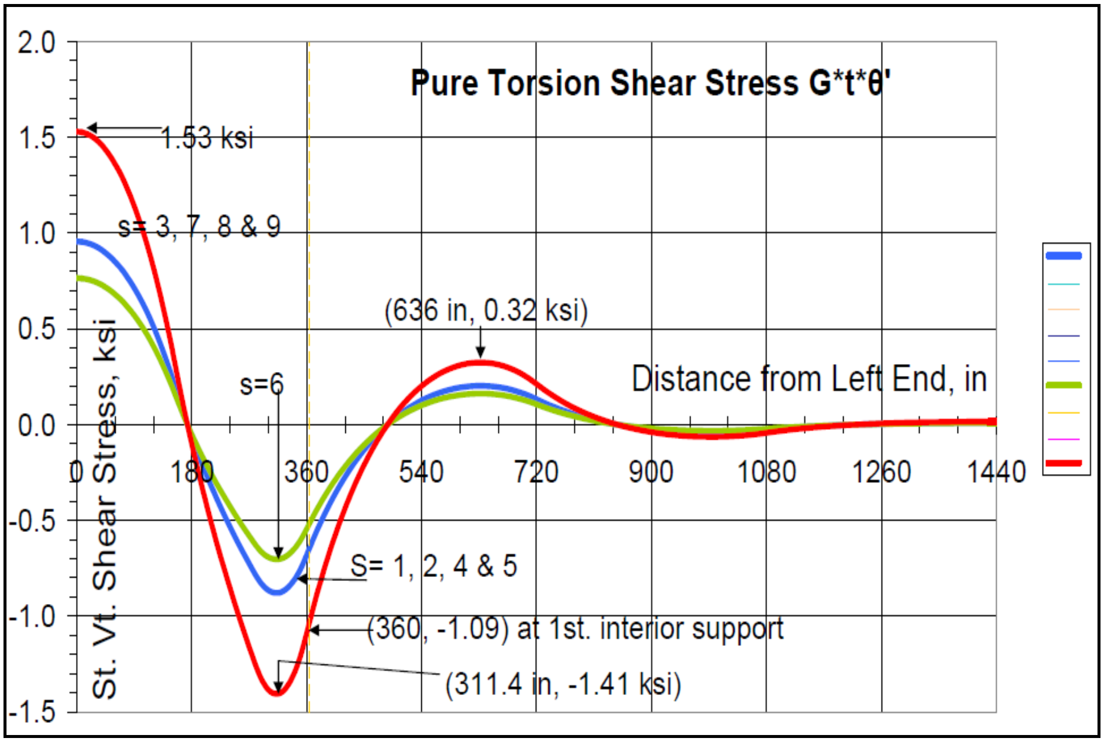



Medwadowski's with Scale Error. Given the hyperbolic nature of the functions involve in θ(z), each of its derivatives decreases an order of magnitude equal to the characteristic length "a". Thus B= -ECwθ" should be around "a" (√ECw/ GJ =138.5) times Tw = -ECwθ'". From the bimoment chart, Tw could be reasonably expected to be around (4000+3000)/138.50 ~ 50; that is, a two-figure-number as shown below

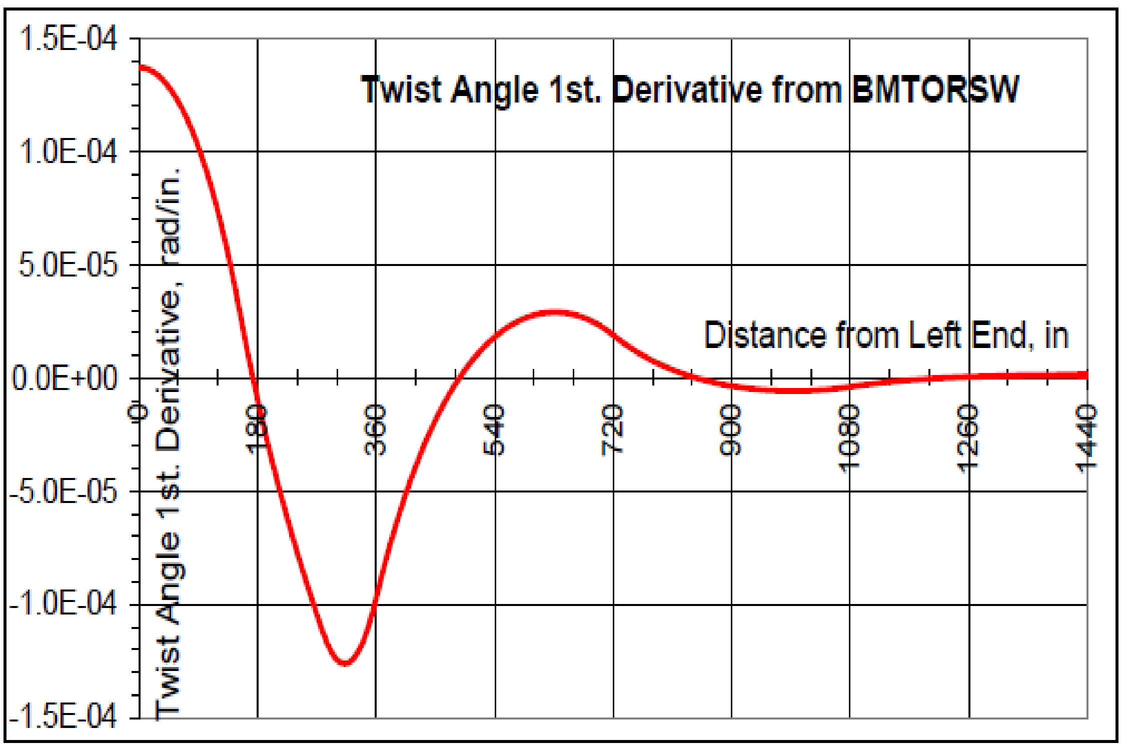



Obtained with BMTORSWP

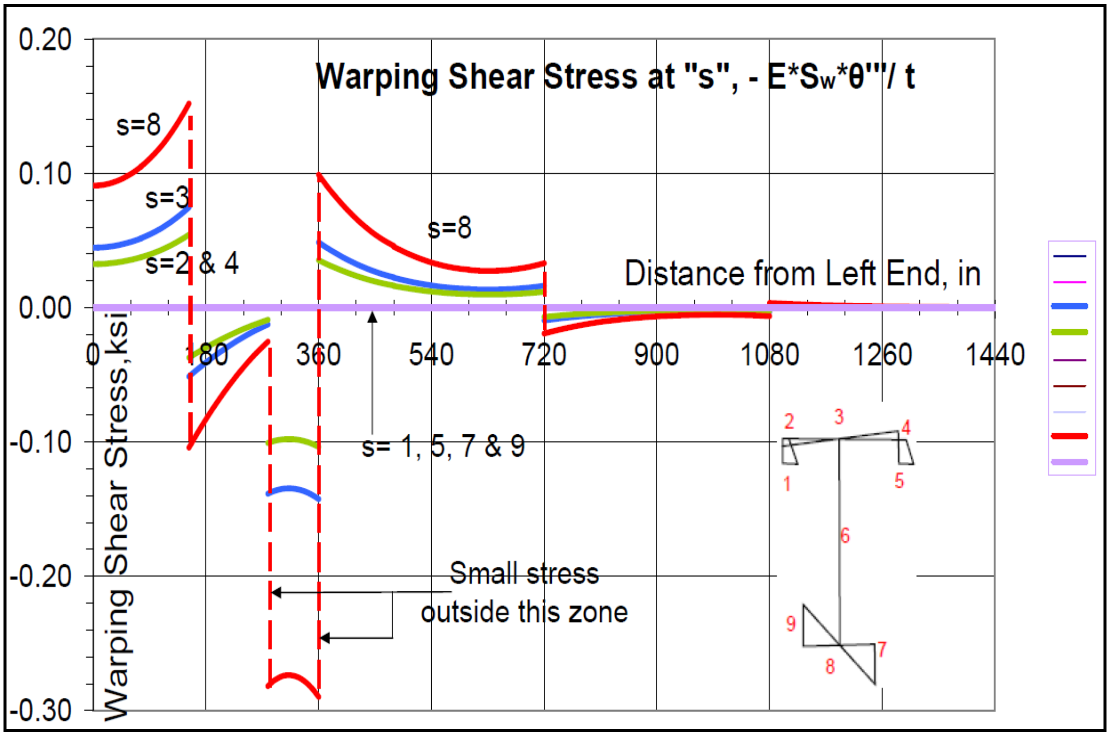
Comparison of Torque Charts by Medwadowski and BMTORSWP



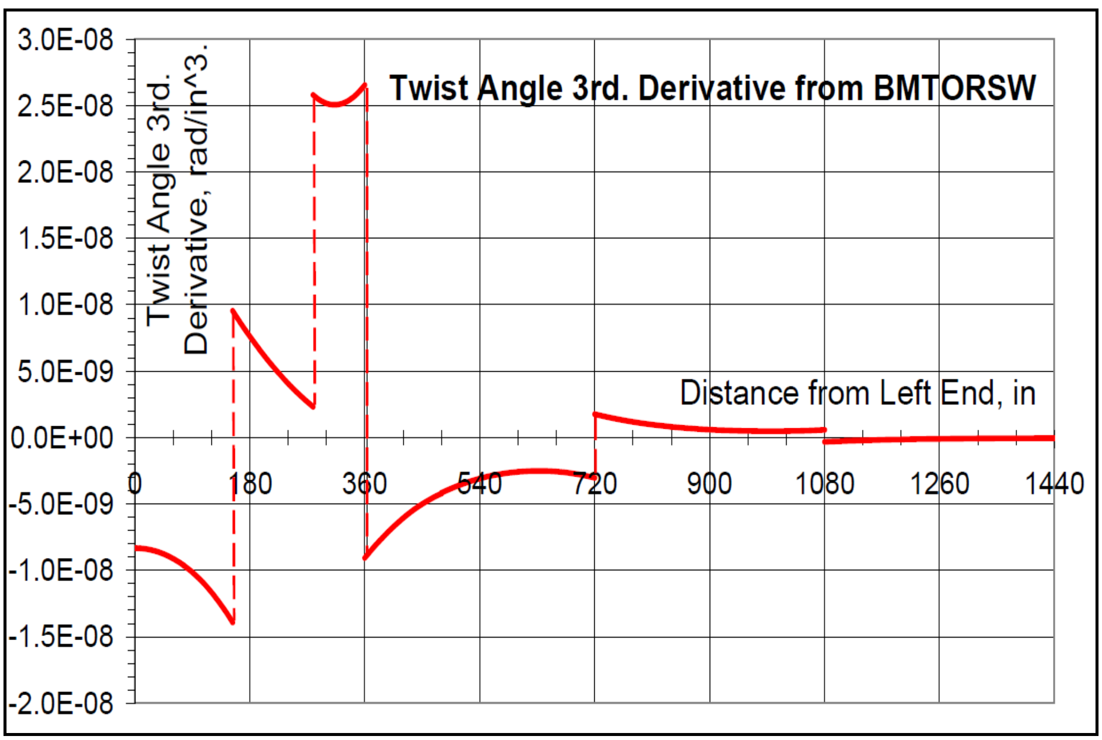
Pure Torsion Shear Stress Beam and Profile



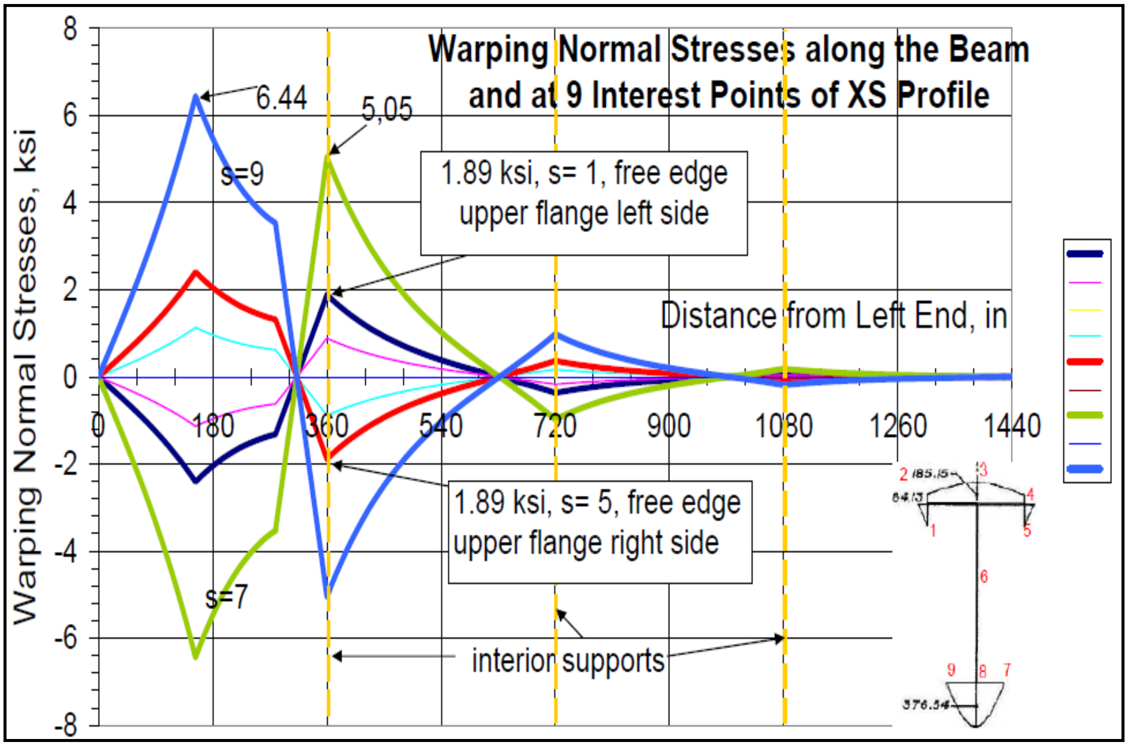
Twist Angle 1st. Derivative from BMTORSW



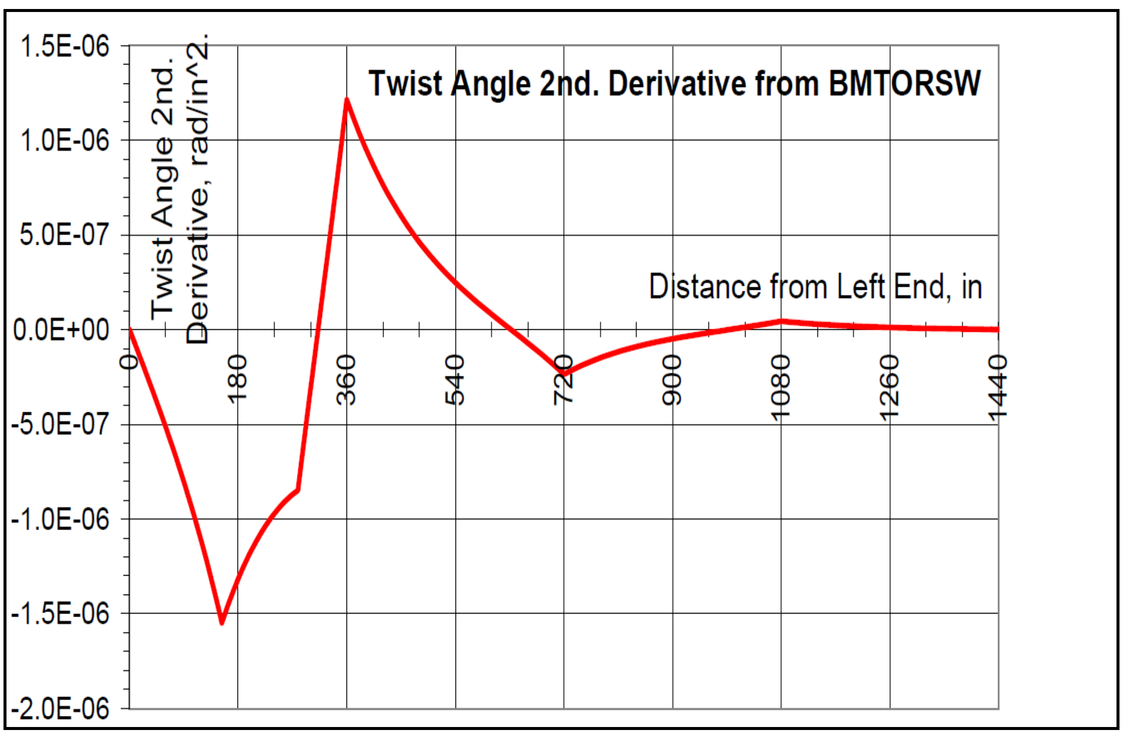
Warping Shear Stress along the Beam and Profile



Twist Angle 3rd. Derivative from BMTORSW




Warping Normal Stresses along the Beam and Profile




Twist Angle 2nd. Derivative from BMTORSW

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                                 Mdwout
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 YOU ARE USING COMPUTER PROGRAM BMTORSW,DEVELOPED BY DR. BERNARDO DESCHAPELLES
0        INPUT DATA FILE NAME IS = Mdw.txt
0                OUTPUT FILE NAME IS = Mdwout.txt
0          STORAGE FILE FOR POST-PROCESSING WITH EXCEL = Mdwgr.grf

0------------------------------------------------------------------------
 ASCE-JSD,V.111,N.2,Feb-85,p.453, Warp. M. Distribution, Medwadowski
0modulus of elasticity of the material=   29000. k/ft2
0ELEM nodes inertia length distrib. load  AXIAL  SOIL NORMAL MODULUS,Ksf  angle
      i   j  ft.4      ft     at i    at j   LOAD     1st END       2nd END    rad
  1   1   3******** 76.50  0.000  0.000********       0.0          0.0   0.000 00
  2   3   5******** 76.50  0.000  0.000********       0.0          0.0   0.000 00
  3   5   7********126.00  0.000  0.000********       0.0          0.0   0.000 00
  4   7   9******** 81.00  0.000  0.000********       0.0          0.0   0.000 00
  5   9  11********120.00  0.000  0.000********       0.0          0.0   0.000 00
  6  11  13********120.00  0.000  0.000********       0.0          0.0   0.000 00
  7  13  15********120.00  0.000  0.000********       0.0          0.0   0.000 00
  8  15  17********120.00  0.000  0.000********       0.0          0.0   0.000 00
  9  17  19********120.00  0.000  0.000********       0.0          0.0   0.000 00
 10  19  21********120.00  0.000  0.000********       0.0          0.0   0.000 00
 11  21  23********120.00  0.000  0.000********       0.0          0.0   0.000 00
 12  23  25********120.00  0.000  0.000********       0.0          0.0   0.000 00
 13  25  27********120.00  0.000  0.000********       0.0          0.0   0.000 00
0    INPUT DATA RELATED TO THE  5 SUPPORTS
  5 1 1 1 0 9 0 1 015 0 1 021 0 1 027 0 1 0
0    INPUT OF NODAL FORCES RELATED TO GLOBAL AXIS  2
          2 5  56.00 7  56.00
0      FINAL  SOLUTION   FOUND  AFTER    1  ITERATIONS
0     Output of nodal displacements in reference to  global axes
0node  displ.      displ.      displ.     node  displ.      displ.      displ.
     along  x     along  y    around  z        along  x    along  y    around  z
     or nonn1     or nonn2    or nonn 3        or nonn1    or nonn2    or nonn 3

+  1 0.0000E+00  0.0000E+00  0.1372E-03
+                                          2 0.0000E+00  0.5090E-02  0.8274E-03

+  3 0.0000E+00  0.9864E-02  0.1122E-03
+                                          4 0.0000E+00  0.1331E-01  0.4942E-03

+  5 0.0000E+00  0.1571E-01  0.2947E-04
+                                          6 0.0000E+00  0.1412E-01 -0.5240E-03

+  7 0.0000E+00  0.9604E-02 -0.1119E-03
+                                          8 0.0000E+00  0.4705E-02 -0.8189E-03

+  9 0.0000E+00  0.0000E+00 -0.9755E-04
+                                         10 0.0000E+00 -0.3503E-02 -0.4132E-03

+ 11 0.0000E+00 -0.5136E-02 -0.2877E-05
+                                         12 0.0000E+00 -0.4465E-02  0.1699E-03

+ 13 0.0000E+00 -0.3187E-02  0.2782E-04
+                                         14 0.0000E+00 -0.1507E-02  0.2718E-03

+ 15 0.0000E+00  0.0000E+00  0.1904E-04
+                                         16 0.0000E+00  0.6849E-03  0.8093E-04

+ 17 0.0000E+00  0.1006E-02  0.5994E-06
+                                         18 0.0000E+00  0.8765E-03 -0.3288E-04

+ 19 0.0000E+00  0.6284E-03 -0.5426E-05
+                                         20 0.0000E+00  0.2986E-03 -0.5356E-04
                                 Page 1
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                              Mdwout
+ 21 0.0000E+00  0.0000E+00 -0.3847E-05
+                                        22 0.0000E+00 -0.1442E-03 -0.1771E-04

+ 23 0.0000E+00 -0.2187E-03 -0.3150E-06
+                                        24 0.0000E+00 -0.2027E-03  0.5198E-05

+ 25 0.0000E+00 -0.1591E-03  0.1078E-05
+                                        26 0.0000E+00 -0.8316E-04  0.1338E-04

+ 27 0.0000E+00  0.0000E+00  0.1445E-05
0-----------------------------------------------------------------------
      OUTPUT OF SOIL REACTIONS,STRESSES AND TRANSVERSE DISPLACEMENTS
0-----------------------------------------------------------------------
O     ELEMENT   1  DISPLACEMENTS IN INCIDENCES    1   2   3
         NODE   1         0.00000E+00        0.00000E+00        0.13718E-03
         NODE   2         0.00000E+00        0.50904E-02        0.82741E-03
         NODE   3         0.00000E+00        0.98641E-02        0.11221E-03
O     FORCES ACTING ALONG THE 9 DOF
         NODE   1         0.00000E+00       -0.36788E+02       -0.25466E-10
         NODE   2         0.00000E+00       -0.40927E-11        0.00000E+00
         NODE   3         0.00000E+00        0.36788E+02       -0.15919E+04
0ELEMENT  1, FROM NODE  1, TO NODE  3 - LENGTH = 76.50 ft
0 left half of span,at tenth points of length
               span      span      span      span      span      span
+          0.0       0.1       0.2       0.3       0.4       0.5
 soil,k/ft    0.000     0.000     0.000     0.000     0.000     0.000
 shear,k     -36.79    -36.79    -36.79    -36.79    -36.79    -36.79
 bmom,kft      0.00   -151.45   -303.37   -456.21   -610.44   -766.53
 tdisp,ft   0.00000   0.00105   0.00209   0.00313   0.00416   0.00517
 axial,k       0.00 AT 1st END and      0.00 AT 2nd END
0right half of span,at tenth points of length
               span      span      span      span      span      span
+          0.5       0.6       0.7       0.8       0.9       1.0
 soil,k/ft    0.000     0.000     0.000     0.000     0.000     0.000
 shear,k     -36.79    -36.79    -36.79    -36.79    -36.79    -36.79
 bmom,kft   -766.53   -924.97  -1086.22  -1250.79  -1419.17  -1591.88
 tdisp,ft   0.00517   0.00616   0.00713   0.00807   0.00899   0.00986
 axial,k       0.00 AT 1st END and      0.00 AT 2nd END
0-----------------------------------------------------------------------
O     ELEMENT   2  DISPLACEMENTS IN INCIDENCES    3   4   5
         NODE   3         0.00000E+00        0.98641E-02        0.11221E-03
         NODE   4         0.00000E+00        0.13312E-01        0.49424E-03
         NODE   5         0.00000E+00        0.15710E-01        0.29475E-04
O     FORCES ACTING ALONG THE 9 DOF
         NODE   3         0.00000E+00       -0.36788E+02        0.15919E+04
         NODE   4         0.00000E+00       -0.26148E-11       -0.10368E-09
         NODE   5         0.00000E+00        0.36788E+02       -0.36817E+04
0ELEMENT  2, FROM NODE  3, TO NODE  5 - LENGTH = 76.50 ft
0 left half of span,at tenth points of length
               span      span      span      span      span      span
+          0.0       0.1       0.2       0.3       0.4       0.5
 soil,k/ft    0.000     0.000     0.000     0.000     0.000     0.000
 shear,k     -36.79    -36.79    -36.79    -36.79    -36.79    -36.79
 bmom,kft  -1591.88  -1769.45  -1952.42  -2141.34  -2336.79  -2539.38
 tdisp,ft   0.00986   0.01070   0.01150   0.01224   0.01294   0.01357
 axial,k       0.00 AT 1st END and      0.00 AT 2nd END
0right half of span,at tenth points of length
               span      span      span      span      span      span
+          0.5       0.6       0.7       0.8       0.9       1.0
 soil,k/ft    0.000     0.000     0.000     0.000     0.000     0.000
 shear,k     -36.79    -36.79    -36.79    -36.79    -36.79    -36.79
 bmom,kft  -2539.38  -2749.71  -2968.42  -3196.19  -3433.72  -3681.71
                                         Page 2
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(-Di + Dj)* GJ + Vj*L = Mj-Mi, where GJ=123921

.01571*123921-36.79*(76.5+76.5)= 1956.713-5628.87= -3672.157

Mj-Mi = -3681.71 OK

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                                             Mdwout
 tdisp,ft    0.01357   0.01415   0.01465   0.01509   0.01544   0.01571
 axial,k        0.00 AT 1st END and      0.00 AT 2nd END
0---------------------------------------------------------------------
O      ELEMENT   3  DISPLACEMENTS IN INCIDENCES    5   6   7
          NODE   5          0.00000E+00         0.15710E-01         0.29475E-04
          NODE   6          0.00000E+00         0.14121E-01        -0.52400E-03
          NODE   7          0.00000E+00         0.96036E-02        -0.11190E-03
O      FORCES ACTING ALONG THE 9 DOF
          NODE   5          0.00000E+00         0.19212E+02         0.36817E+04
          NODE   6          0.00000E+00         0.90949E-12        -0.58208E-10
          NODE   7          0.00000E+00        -0.19212E+02        -0.20177E+04
OELEMENT   3, FROM NODE   5, TO NODE   7 - LENGTH =126.00 ft
0 left half of span,at tenth points of length
              span      span      span      span      span      span
+          0.0       0.1       0.2       0.3       0.4       0.5
 soil,k/ft    0.000     0.000     0.000     0.000     0.000     0.000
 shear,k      19.21     19.21     19.21     19.21     19.21     19.21
 bmom,kft  -3681.71  -3408.46  -3163.42  -2944.57  -2750.11  -2578.41
 tdisp,ft   0.01571   0.01596   0.01599   0.01580   0.01541   0.01485
 axial,k        0.00 AT 1st END and      0.00 AT 2nd END
Oright half of span,at tenth points of length
              span      span      span      span      span      span
+          0.5       0.6       0.7       0.8       0.9       1.0
 soil,k/ft    0.000     0.000     0.000     0.000     0.000     0.000
 shear,k      19.21     19.21     19.21     19.21     19.21     19.21
 bmom,kft  -2578.41  -2428.06  -2297.80  -2186.57  -2093.45  -2017.66
 tdisp,ft   0.01485   0.01411   0.01320   0.01215   0.01095   0.00960
 axial,k        0.00 AT 1st END and      0.00 AT 2nd END
0---------------------------------------------------------------------
O      ELEMENT   4  DISPLACEMENTS IN INCIDENCES    7   8   9
          NODE   7          0.00000E+00         0.96036E-02        -0.11190E-03
          NODE   8          0.00000E+00         0.47054E-02        -0.81893E-03
          NODE   9          0.00000E+00         0.00000E+00        -0.97548E-04
O      FORCES ACTING ALONG THE 9 DOF
          NODE   7          0.00000E+00         0.75212E+02         0.20177E+04
          NODE   8          0.00000E+00        -0.22737E-11         0.36380E-11
          NODE   9          0.00000E+00        -0.75212E+02         0.28845E+04
OELEMENT   4, FROM NODE   7, TO NODE   9 - LENGTH = 81.00 ft
0 left half of span,at tenth points of length
              span      span      span      span      span      span
+          0.0       0.1       0.2       0.3       0.4       0.5
 soil,k/ft    0.000     0.000     0.000     0.000     0.000     0.000
 shear,k      75.21     75.21     75.21     75.21     75.21     75.21
 bmom,kft  -2017.66  -1523.93  -1035.41   -550.44    -67.34    415.52
 tdisp,ft   0.00960   0.00867   0.00770   0.00669   0.00568   0.00466
 axial,k        0.00 AT 1st END and      0.00 AT 2nd END
Oright half of span,at tenth points of length
              span      span      span      span      span      span
+          0.5       0.6       0.7       0.8       0.9       1.0
 soil,k/ft    0.000     0.000     0.000     0.000     0.000     0.000
 shear,k      75.21     75.21     75.21     75.21     75.21     75.21
 bmom,kft    415.52    899.80   1387.16   1879.27   2377.80   2884.47
 tdisp,ft   0.00466   0.00365   0.00267   0.00172   0.00083   0.00000
 axial,k        0.00 AT 1st END and      0.00 AT 2nd END
0---------------------------------------------------------------------
O      ELEMENT   5  DISPLACEMENTS IN INCIDENCES    9  10  11
          NODE   9          0.00000E+00         0.00000E+00        -0.97548E-04
          NODE  10          0.00000E+00        -0.35030E-02        -0.41323E-03
          NODE  11          0.00000E+00        -0.51357E-02        -0.28765E-05
O      FORCES ACTING ALONG THE 9 DOF
          NODE   9          0.00000E+00        -0.95721E+01        -0.28845E+04
          NODE  10          0.00000E+00         0.23093E-12        -0.14779E-11
          NODE  11          0.00000E+00         0.95721E+01         0.10994E+04
                                             Page 3
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-.0096*123921+75.21*81=4902.37

2884.47+2017.66 = 4901.66 OK

(-Di + Dj)* GJ + Vj*L = Mj-Mi, where GJ=123921

{[(-.01571+.00960)*123921 = - 757.157] + (19.21*126=2420.460) = 2420.460-757.157}= 1663.303

Mj-Mi = - 2017.66 + 3681.71 = 1664.05 OK

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(-Di + Dj)* GJ + Vj*L = Mj-Mi, where GJ=123921

-.00514*123921-9.57*120=-636.954-1148.4

=-1785.354 <OK> 1099.39-2884.47= -1785.08

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                                                    Mdwout
0ELEMENT   5, FROM NODE   9, TO NODE 11 - LENGTH =120.00 ft
0 left half of span,at tenth points of length
               span       span       span       span       span       span
+           0.0        0.1        0.2        0.3        0.4        0.5
 soil,k/ft     0.000      0.000      0.000      0.000      0.000      0.000
 shear,k       -9.57      -9.57      -9.57      -9.57      -9.57      -9.57
 bmom,kft   2884.47    2635.05    2405.41    2193.84    1998.74    1818.65
 tdisp,ft   0.00000   -0.00109   -0.00201   -0.00279   -0.00344   -0.00397
 axial,k        0.00 AT 1st END and       0.00 AT 2nd END
0right half of span,at tenth points of length
               span       span       span       span       span       span
+           0.5        0.6        0.7        0.8        0.9        1.0
 soil,k/ft     0.000      0.000      0.000      0.000      0.000      0.000
 shear,k       -9.57      -9.57      -9.57      -9.57      -9.57      -9.57
 bmom,kft   1818.65    1652.22    1498.19    1355.41    1222.81    1099.39
 tdisp,ft  -0.00397   -0.00438   -0.00470   -0.00492   -0.00507   -0.00514
 axial,k        0.00 AT 1st END and       0.00 AT 2nd END
0-----------------------------------------------------------------------
0     ELEMENT    6  DISPLACEMENTS IN INCIDENCES  11  12  13
         NODE   11          0.00000E+00         -0.51357E-02         -0.28765E-05
         NODE   12          0.00000E+00         -0.44646E-02          0.16988E-03
         NODE   13          0.00000E+00         -0.31872E-02          0.27817E-04
0     FORCES ACTING ALONG THE 9 DOF
         NODE   11          0.00000E+00         -0.95721E+01         -0.10994E+04
         NODE   12          0.00000E+00         -0.17053E-12         -0.15916E-10
         NODE   13          0.00000E+00          0.95721E+01          0.19221E+03
0ELEMENT   6, FROM NODE 11, TO NODE 13 - LENGTH =120.00 ft
0 left half of span,at tenth points of length
               span       span       span       span       span       span
+           0.0        0.1        0.2        0.3        0.4        0.5
 soil,k/ft     0.000      0.000      0.000      0.000      0.000      0.000
 shear,k       -9.57      -9.57      -9.57      -9.57      -9.57      -9.57
 bmom,kft   1099.39     984.23     876.45     775.26     679.89     589.62
 tdisp,ft  -0.00514   -0.00514   -0.00508   -0.00497   -0.00481   -0.00461
 axial,k        0.00 AT 1st END and       0.00 AT 2nd END
0right half of span,at tenth points of length
               span       span       span       span       span       span
+           0.5        0.6        0.7        0.8        0.9        1.0
 soil,k/ft     0.000      0.000      0.000      0.000      0.000      0.000
 shear,k       -9.57      -9.57      -9.57      -9.57      -9.57      -9.57
 bmom,kft    589.62     503.78     421.73     342.84     266.52     192.21
 tdisp,ft  -0.00461   -0.00438   -0.00412   -0.00383   -0.00351   -0.00319
 axial,k        0.00 AT 1st END and       0.00 AT 2nd END
0-----------------------------------------------------------------------
0     ELEMENT    7  DISPLACEMENTS IN INCIDENCES  13  14  15
         NODE   13          0.00000E+00         -0.31872E-02          0.27817E-04
         NODE   14          0.00000E+00         -0.15069E-02          0.27182E-03
         NODE   15          0.00000E+00          0.00000E+00          0.19041E-04
0     FORCES ACTING ALONG THE 9 DOF
         NODE   13          0.00000E+00         -0.95721E+01         -0.19221E+03
         NODE   14          0.00000E+00         -0.28422E-13          0.50022E-11
         NODE   15          0.00000E+00          0.95721E+01         -0.56149E+03
0ELEMENT   7, FROM NODE 13, TO NODE 15 - LENGTH =120.00 ft
0 left half of span,at tenth points of length
               span       span       span       span       span       span
+           0.0        0.1        0.2        0.3        0.4        0.5
 soil,k/ft     0.000      0.000      0.000      0.000      0.000      0.000
 shear,k       -9.57      -9.57      -9.57      -9.57      -9.57      -9.57
 bmom,kft    192.21     119.34      47.36     -24.25     -96.05    -168.58
 tdisp,ft  -0.00319   -0.00285   -0.00250   -0.00215   -0.00181   -0.00146
 axial,k        0.00 AT 1st END and       0.00 AT 2nd END
0right half of span,at tenth points of length
               span       span       span       span       span       span
                                        Page 4
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+          0.5       0.6       0.7       0.8       0.9       1.0
 soil,k/ft    0.000     0.000     0.000     0.000     0.000     0.000
 shear,k      -9.57     -9.57     -9.57     -9.57     -9.57     -9.57
 bmom,kft   -168.58   -242.36   -317.97   -395.97   -476.94   -561.49
 tdisp,ft  -0.00146  -0.00113  -0.00082  -0.00052  -0.00024   0.00000
 axial,k       0.00 AT 1st END and      0.00 AT 2nd END
0-----------------------------------------------------------------------
O     ELEMENT   8  DISPLACEMENTS IN INCIDENCES  15  16  17
         NODE  15          0.00000E+00          0.00000E+00          0.19041E-04
         NODE  16          0.00000E+00          0.68492E-03          0.80928E-04
         NODE  17          0.00000E+00          0.10055E-02          0.59935E-06
O     FORCES ACTING ALONG THE 9 DOF
         NODE  15          0.00000E+00          0.18526E+01          0.56149E+03
         NODE  16          0.00000E+00          0.37303E-13          0.17053E-11
         NODE  17          0.00000E+00         -0.18526E+01         -0.21457E+03
0ELEMENT  8, FROM NODE 15, TO NODE 17 - LENGTH =120.00 ft
0 left half of span,at tenth points of length
               span      span      span      span      span      span
+          0.0       0.1       0.2       0.3       0.4       0.5
 soil,k/ft    0.000     0.000     0.000     0.000     0.000     0.000
 shear,k       1.85      1.85      1.85      1.85      1.85      1.85
 bmom,kft   -561.49   -512.98   -468.33   -427.20   -389.28   -354.27
 tdisp,ft   0.00000   0.00021   0.00039   0.00055   0.00067   0.00078
 axial,k       0.00 AT 1st END and      0.00 AT 2nd END
0right half of span,at tenth points of length
               span      span      span      span      span      span
+          0.5       0.6       0.7       0.8       0.9       1.0
 soil,k/ft    0.000     0.000     0.000     0.000     0.000     0.000
 shear,k       1.85      1.85      1.85      1.85      1.85      1.85
 bmom,kft   -354.27   -321.93   -292.01   -264.27   -238.53   -214.57
 tdisp,ft   0.00078   0.00086   0.00092   0.00096   0.00099   0.00101
 axial,k       0.00 AT 1st END and      0.00 AT 2nd END
0-----------------------------------------------------------------------
O     ELEMENT   9  DISPLACEMENTS IN INCIDENCES  17  18  19
         NODE  17          0.00000E+00          0.10055E-02          0.59935E-06
         NODE  18          0.00000E+00          0.87648E-03         -0.32878E-04
         NODE  19          0.00000E+00          0.62840E-03         -0.54263E-05
O     FORCES ACTING ALONG THE 9 DOF
         NODE  17          0.00000E+00          0.18526E+01          0.21457E+03
         NODE  18          0.00000E+00          0.14211E-13          0.90949E-12
         NODE  19          0.00000E+00         -0.18526E+01         -0.38994E+02
0ELEMENT  9, FROM NODE 17, TO NODE 19 - LENGTH =120.00 ft
0 left half of span,at tenth points of length
               span      span      span      span      span      span
+          0.0       0.1       0.2       0.3       0.4       0.5
 soil,k/ft    0.000     0.000     0.000     0.000     0.000     0.000
 shear,k       1.85      1.85      1.85      1.85      1.85      1.85
 bmom,kft   -214.57   -192.22   -171.32   -151.71   -133.23   -115.75
 tdisp,ft   0.00101   0.00101   0.00100   0.00097   0.00094   0.00091
 axial,k       0.00 AT 1st END and      0.00 AT 2nd END
0right half of span,at tenth points of length
               span      span      span      span      span      span
+          0.5       0.6       0.7       0.8       0.9       1.0
 soil,k/ft    0.000     0.000     0.000     0.000     0.000     0.000
 shear,k       1.85      1.85      1.85      1.85      1.85      1.85
 bmom,kft   -115.75    -99.15    -83.28    -68.05    -53.32    -38.99
 tdisp,ft   0.00091   0.00086   0.00081   0.00075   0.00069   0.00063
 axial,k       0.00 AT 1st END and      0.00 AT 2nd END
0-----------------------------------------------------------------------
O     ELEMENT  10  DISPLACEMENTS IN INCIDENCES  19  20  21
         NODE  19          0.00000E+00          0.62840E-03         -0.54263E-05
         NODE  20          0.00000E+00          0.29860E-03         -0.53560E-04
         NODE  21          0.00000E+00          0.00000E+00         -0.38472E-05
```



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Mdwout

```
0     FORCES ACTING ALONG THE 9 DOF
         NODE  19         0.00000E+00        0.18526E+01        0.38994E+02
         NODE  20         0.00000E+00        0.35527E-13       -0.34106E-12
         NODE  21         0.00000E+00       -0.18526E+01        0.10544E+03
0ELEMENT 10, FROM NODE 19, TO NODE 21 - LENGTH =120.00 ft
0 left half of span,at tenth points of length
              span      span      span      span      span      span
+          0.0       0.1       0.2       0.3       0.4       0.5
 soil,k/ft    0.000     0.000     0.000     0.000     0.000     0.000
 shear,k       1.85      1.85      1.85      1.85      1.85      1.85
 bmom,kft    -38.99    -24.96    -11.12      2.65     16.43     30.33
 tdisp,ft   0.00063   0.00056   0.00049   0.00043   0.00036   0.00029
 axial,k       0.00 AT 1st END and      0.00 AT 2nd END
0right half of span,at tenth points of length
              span      span      span      span      span      span
+          0.5       0.6       0.7       0.8       0.9       1.0
 soil,k/ft    0.000     0.000     0.000     0.000     0.000     0.000
 shear,k       1.85      1.85      1.85      1.85      1.85      1.85
 bmom,kft     30.33     44.47     58.93     73.84     89.31    105.44
 tdisp,ft   0.00029   0.00023   0.00016   0.00010   0.00005   0.00000
 axial,k       0.00 AT 1st END and      0.00 AT 2nd END
0-----------------------------------------------------------------------
0     ELEMENT  11  DISPLACEMENTS IN INCIDENCES  21  22  23
         NODE  21         0.00000E+00        0.00000E+00       -0.38472E-05
         NODE  22         0.00000E+00       -0.14424E-03       -0.17711E-04
         NODE  23         0.00000E+00       -0.21871E-03       -0.31503E-06
0     FORCES ACTING ALONG THE 9 DOF
         NODE  21         0.00000E+00       -0.29290E+00       -0.10544E+03
         NODE  22         0.00000E+00        0.22649E-13        0.26290E-12
         NODE  23         0.00000E+00        0.29290E+00        0.43193E+02
0ELEMENT 11, FROM NODE 21, TO NODE 23 - LENGTH =120.00 ft
0 left half of span,at tenth points of length
              span      span      span      span      span      span
+          0.0       0.1       0.2       0.3       0.4       0.5
 soil,k/ft    0.000     0.000     0.000     0.000     0.000     0.000
 shear,k      -0.29     -0.29     -0.29     -0.29     -0.29     -0.29
 bmom,kft    105.44     96.59     88.47     81.00     74.15     67.85
 tdisp,ft   0.00000  -0.00004  -0.00008  -0.00011  -0.00014  -0.00016
 axial,k       0.00 AT 1st END and      0.00 AT 2nd END
0right half of span,at tenth points of length
              span      span      span      span      span      span
+          0.5       0.6       0.7       0.8       0.9       1.0
 soil,k/ft    0.000     0.000     0.000     0.000     0.000     0.000
 shear,k      -0.29     -0.29     -0.29     -0.29     -0.29     -0.29
 bmom,kft     67.85     62.07     56.74     51.85     47.34     43.19
 tdisp,ft  -0.00016  -0.00018  -0.00019  -0.00021  -0.00021  -0.00022
 axial,k       0.00 AT 1st END and      0.00 AT 2nd END
0-----------------------------------------------------------------------
0     ELEMENT  12  DISPLACEMENTS IN INCIDENCES  23  24  25
         NODE  23         0.00000E+00       -0.21871E-03       -0.31503E-06
         NODE  24         0.00000E+00       -0.20265E-03        0.51981E-05
         NODE  25         0.00000E+00       -0.15908E-03        0.10782E-05
0     FORCES ACTING ALONG THE 9 DOF
         NODE  23         0.00000E+00       -0.29290E+00       -0.43193E+02
         NODE  24         0.00000E+00       -0.21316E-13        0.51159E-12
         NODE  25         0.00000E+00        0.29290E+00        0.15434E+02
0ELEMENT 12, FROM NODE 23, TO NODE 25 - LENGTH =120.00 ft
0 left half of span,at tenth points of length
              span      span      span      span      span      span
+          0.0       0.1       0.2       0.3       0.4       0.5
 soil,k/ft    0.000     0.000     0.000     0.000     0.000     0.000
 shear,k      -0.29     -0.29     -0.29     -0.29     -0.29     -0.29
 bmom,kft     43.19     39.37     35.84     32.58     29.56     26.76
```

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```
                                   Mdwout
 tdisp,ft  -0.00022  -0.00022  -0.00022  -0.00022  -0.00022  -0.00021
 axial,k       0.00 AT 1st END and      0.00 AT 2nd END
0right half of span,at tenth points of length
               span      span      span      span      span      span
+          0.5       0.6       0.7       0.8       0.9       1.0
 soil,k/ft    0.000     0.000     0.000     0.000     0.000     0.000
 shear,k      -0.29     -0.29     -0.29     -0.29     -0.29     -0.29
 bmom,kft     26.76     24.17     21.76     19.51     17.41     15.43
 tdisp,ft  -0.00021  -0.00020  -0.00019  -0.00018  -0.00017  -0.00016
 axial,k       0.00 AT 1st END and      0.00 AT 2nd END
0-----------------------------------------------------------------------
O     ELEMENT  13  DISPLACEMENTS IN INCIDENCES  25  26  27
         NODE  25          0.00000E+00         -0.15908E-03         0.10782E-05
         NODE  26          0.00000E+00         -0.83163E-04         0.13384E-04
         NODE  27          0.00000E+00          0.00000E+00         0.14450E-05
O     FORCES ACTING ALONG THE 9 DOF
         NODE  25          0.00000E+00         -0.29290E+00        -0.15434E+02
         NODE  26          0.00000E+00          0.88818E-14         0.56843E-13
         NODE  27          0.00000E+00          0.29290E+00         0.17053E-12
0ELEMENT 13, FROM NODE 25, TO NODE 27 - LENGTH =120.00 ft
0 left half of span,at tenth points of length
               span      span      span      span      span      span
+          0.0       0.1       0.2       0.3       0.4       0.5
 soil,k/ft    0.000     0.000     0.000     0.000     0.000     0.000
 shear,k      -0.29     -0.29     -0.29     -0.29     -0.29     -0.29
 bmom,kft     15.43     13.58     11.82     10.16      8.57      7.05
 tdisp,ft  -0.00016  -0.00015  -0.00013  -0.00012  -0.00010  -0.00008
 axial,k       0.00 AT 1st END and      0.00 AT 2nd END
0right half of span,at tenth points of length
               span      span      span      span      span      span
+          0.5       0.6       0.7       0.8       0.9       1.0
 soil,k/ft    0.000     0.000     0.000     0.000     0.000     0.000
 shear,k      -0.29     -0.29     -0.29     -0.29     -0.29     -0.29
 bmom,kft      7.05      5.57      4.14      2.75      1.37      0.00
 tdisp,ft  -0.00008  -0.00007  -0.00005  -0.00003  -0.00002   0.00000
 axial,k       0.00 AT 1st END and      0.00 AT 2nd END
1
```

(-Di + Dj)* GJ + Vj*L = Mj-Mi, where GJ=123921

-(-0.00016)*123921 - 0.29* 120 = 19.827 - 3.48 = 16.347

bmom= 15.43 ~ 16.347 OK

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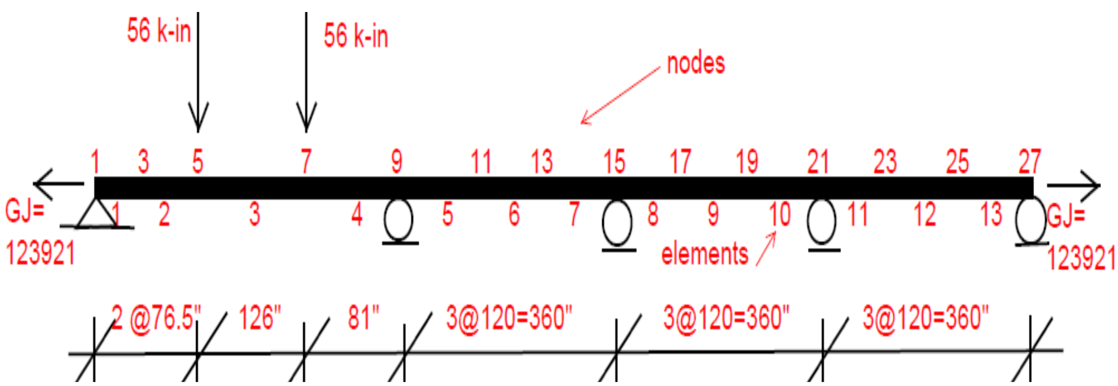


Equivalent Beam Column Analog to the TWB

Table of Twist angle and Derivatives

| | z | θ | T(z) | B(z) | θ' | θ'' | θ''' |
|---|---|---|---|---|---|---|---|
| 1 | | | | | | | |
| | 0 | 0.000E+00 | -36.7876 | 0 | 1.372E-04 | 1.590E-11 | -8.325E-09 |
| | 7.65 | 1.000E-03 | -36.7876 | -151.4528 | 1.369E-04 | -6.369E-08 | -8.335E-09 |
| | 15.3 | 2.100E-03 | -36.7876 | -303.3677 | 1.362E-04 | -1.276E-07 | -8.372E-09 |
| | 22.95 | 3.100E-03 | -36.7876 | -456.2081 | 1.350E-04 | -1.919E-07 | -8.435E-09 |
| | 30.6 | 4.200E-03 | -36.7876 | -610.4401 | 1.333E-04 | -2.567E-07 | -8.524E-09 |
| | 38.25 | 5.200E-03 | -36.7876 | -766.5342 | 1.311E-04 | -3.223E-07 | -8.640E-09 |
| | 45.9 | 6.200E-03 | -36.7876 | -924.9667 | 1.283E-04 | -3.890E-07 | -8.783E-09 |
| | 53.55 | 7.100E-03 | -36.7876 | -1086.221 | 1.251E-04 | -4.568E-07 | -8.952E-09 |
| | 61.2 | 8.100E-03 | -36.7876 | -1250.788 | 1.213E-04 | -5.260E-07 | -9.147E-09 |
| | 68.85 | 9.000E-03 | -36.7876 | -1419.171 | 1.171E-04 | -5.968E-07 | -9.369E-09 |
| | 76.5 | 9.900E-03 | -36.7876 | -1591.884 | 1.122E-04 | -6.694E-07 | -9.617E-09 |
| 2 | | | | | | | |
| | 76.5 | 9.900E-03 | -36.7876 | -1591.884 | 1.122E-04 | -6.694E-07 | -9.637E-09 |
| | 84.15 | 1.070E-02 | -36.7876 | -1769.452 | 1.068E-04 | -7.441E-07 | -9.909E-09 |
| | 91.8 | 1.150E-02 | -36.7876 | -1952.418 | 1.008E-04 | -8.211E-07 | -1.022E-08 |
| | 99.45 | 1.220E-02 | -36.7876 | -2141.34 | 9.424E-05 | -9.005E-07 | -1.056E-08 |
| | 107.1 | 1.290E-02 | -36.7876 | -2336.794 | 8.703E-05 | -9.827E-07 | -1.093E-08 |
| | 114.75 | 1.360E-02 | -36.7876 | -2539.377 | 7.919E-05 | -1.068E-06 | -1.134E-08 |
| | 122.4 | 1.410E-02 | -36.7876 | -2749.706 | 7.069E-05 | -1.156E-06 | -1.179E-08 |
| | 130.05 | 1.470E-02 | -36.7876 | -2968.422 | 6.149E-05 | -1.248E-06 | -1.227E-08 |
| | 137.7 | 1.510E-02 | -36.7876 | -3196.194 | 5.158E-05 | -1.344E-06 | -1.278E-08 |
| | 145.35 | 1.540E-02 | -36.7876 | -3433.716 | 4.092E-05 | -1.444E-06 | -1.333E-08 |
| | 153 | 1.570E-02 | -36.7876 | -3681.713 | 2.947E-05 | -1.548E-06 | -1.392E-08 |
| 3 | | | | | | | |
| | 153 | 1.570E-02 | 19.2124 | -3681.713 | 2.947E-05 | -1.548E-06 | 9.540E-09 |
| | 165.6 | 1.600E-02 | 19.2124 | -3408.456 | 1.070E-05 | -1.433E-06 | 8.616E-09 |
| | 178.2 | 1.600E-02 | 19.2124 | -3163.419 | -6.696E-06 | -1.330E-06 | 7.736E-09 |
| | 190.8 | 1.580E-02 | 19.2124 | -2944.575 | -2.287E-05 | -1.238E-06 | 6.901E-09 |
| | 203.4 | 1.540E-02 | 19.2124 | -2750.11 | -3.794E-05 | -1.156E-06 | 6.111E-09 |
| | 216 | 1.480E-02 | 19.2124 | -2578.411 | -5.205E-05 | -1.084E-06 | 5.366E-09 |
| | 228.6 | 1.410E-02 | 19.2124 | -2428.056 | -6.530E-05 | -1.021E-06 | 4.666E-09 |
| | 241.2 | 1.320E-02 | 19.2124 | -2297.802 | -7.781E-05 | -9.663E-07 | 4.011E-09 |
| | 253.8 | 1.210E-02 | 19.2124 | -2186.571 | -8.969E-05 | -9.197E-07 | 3.401E-09 |
| | 266.4 | 1.090E-02 | 19.2124 | -2093.447 | -1.010E-04 | -8.804E-07 | 2.836E-09 |
| | 279 | 9.600E-03 | 19.2124 | -2017.656 | -1.119E-04 | -8.480E-07 | 2.315E-09 |
| 4 | | | | | | | |
| | 279 | 9.600E-03 | 75.2124 | -2017.656 | -1.119E-04 | -8.485E-07 | 2.580E-08 |
| | 287.1 | 8.700E-03 | 75.2124 | -1523.927 | -1.179E-04 | -6.409E-07 | 2.548E-08 |
| | 295.2 | 7.700E-03 | 75.2124 | -1035.411 | -1.223E-04 | -4.354E-07 | 2.526E-08 |
| | 303.3 | 6.700E-03 | 75.2124 | -550.4354 | -1.250E-04 | -2.315E-07 | 2.511E-08 |
| | 311.4 | 5.700E-03 | 75.2124 | -67.3426 | -1.260E-04 | -2.832E-08 | 2.506E-08 |
| | 319.5 | 4.700E-03 | 75.2124 | 415.5199 | -1.255E-04 | 1.747E-07 | 2.509E-08 |

| | | | | | | | |
|---|---|---|---|---|---|---|---|
| | 327.6 | 3.600E-03 | 75.2124 | 899.8037 | -1.232E-04 | 3.784E-07 | 2.521E-08 |
| | z | θ | T(z) | B(z) | θ' | θ'' | θ''' |
| | 335.7 | 2.700E-03 | 75.2124 | 1387.164 | -1.193E-04 | 5.833E-07 | 2.541E-08 |
| | 343.8 | 1.700E-03 | 75.2124 | 1879.27 | -1.138E-04 | 7.903E-07 | 2.570E-08 |
| | 351.9 | 8.000E-04 | 75.2124 | 2377.802 | -1.065E-04 | 9.999E-07 | 2.608E-08 |
| | 360 | 0.000E+00 | 75.2124 | 2884.467 | -9.755E-05 | 1.213E-06 | 2.654E-08 |
| 5 | | | | | | | |
| | 360 | 0.000E+00 | -9.5721 | 2884.467 | -9.755E-05 | 1.213E-06 | -9.062E-09 |
| | 372 | -1.100E-03 | -9.5721 | 2635.046 | -8.363E-05 | 1.108E-06 | -8.370E-09 |
| | 384 | -2.000E-03 | -9.5721 | 2405.411 | -7.092E-05 | 1.012E-06 | -7.725E-09 |
| | 396 | -2.800E-03 | -9.5721 | 2193.841 | -5.932E-05 | 9.226E-07 | -7.125E-09 |
| | 408 | -3.400E-03 | -9.5721 | 1998.744 | -4.875E-05 | 8.405E-07 | -6.571E-09 |
| | 420 | -4.000E-03 | -9.5721 | 1818.654 | -3.913E-05 | 7.647E-07 | -6.064E-09 |
| | 432 | -4.400E-03 | -9.5721 | 1652.219 | -3.037E-05 | 6.947E-07 | -5.602E-09 |
| | 444 | -4.700E-03 | -9.5721 | 1498.189 | -2.243E-05 | 6.301E-07 | -5.186E-09 |
| | 456 | -4.900E-03 | -9.5721 | 1355.409 | -1.523E-05 | 5.701E-07 | -4.817E-09 |
| | 468 | -5.100E-03 | -9.5721 | 1222.808 | -8.731E-06 | 5.143E-07 | -4.493E-09 |
| | 480 | -5.100E-03 | -9.5721 | 1099.39 | -2.877E-06 | 4.621E-07 | -4.215E-09 |
| 6 | | | | | | | |
| | 480 | -5.100E-03 | -9.5721 | 1099.39 | -2.877E-06 | 4.622E-07 | -4.159E-09 |
| | 492 | -5.100E-03 | -9.5721 | 984.2255 | 2.377E-06 | 4.139E-07 | -3.897E-09 |
| | 504 | -5.100E-03 | -9.5721 | 876.4516 | 7.069E-06 | 3.686E-07 | -3.658E-09 |
| | 516 | -5.000E-03 | -9.5721 | 775.2594 | 1.123E-05 | 3.260E-07 | -3.443E-09 |
| | 528 | -4.800E-03 | -9.5721 | 679.8887 | 1.490E-05 | 2.859E-07 | -3.250E-09 |
| | 540 | -4.600E-03 | -9.5721 | 589.6228 | 1.810E-05 | 2.479E-07 | -3.082E-09 |
| | 552 | -4.400E-03 | -9.5721 | 503.7838 | 2.086E-05 | 2.118E-07 | -2.936E-09 |
| | 564 | -4.100E-03 | -9.5721 | 421.7273 | 2.320E-05 | 1.774E-07 | -2.814E-09 |
| | 576 | -3.800E-03 | -9.5721 | 342.8376 | 2.512E-05 | 1.442E-07 | -2.715E-09 |
| | 588 | -3.500E-03 | -9.5721 | 266.5228 | 2.666E-05 | 1.121E-07 | -2.640E-09 |
| | 600 | -3.200E-03 | -9.5721 | 192.2095 | 2.782E-05 | 8.075E-08 | -2.588E-09 |
| 7 | | | | | | | |
| | 600 | -3.200E-03 | -9.5721 | 192.2095 | 2.782E-05 | 8.085E-08 | -2.578E-09 |
| | 612 | -2.800E-03 | -9.5721 | 119.3392 | 2.860E-05 | 5.018E-08 | -2.536E-09 |
| | 624 | -2.500E-03 | -9.5721 | 47.365 | 2.902E-05 | 1.991E-08 | -2.513E-09 |
| | 636 | -2.200E-03 | -9.5721 | -24.2538 | 2.908E-05 | -1.020E-08 | -2.509E-09 |
| | 648 | -1.800E-03 | -9.5721 | -96.0547 | 2.878E-05 | -4.039E-08 | -2.525E-09 |
| | 660 | -1.500E-03 | -9.5721 | -168.5768 | 2.811E-05 | -7.088E-08 | -2.560E-09 |
| | 672 | -1.100E-03 | -9.5721 | -242.3646 | 2.707E-05 | -1.019E-07 | -2.615E-09 |
| | 684 | -8.000E-04 | -9.5721 | -317.9721 | 2.566E-05 | -1.337E-07 | -2.689E-09 |
| | 696 | -5.000E-04 | -9.5721 | -395.9673 | 2.386E-05 | -1.665E-07 | -2.782E-09 |
| | 708 | -2.000E-04 | -9.5721 | -476.9359 | 2.166E-05 | -2.006E-07 | -2.895E-09 |
| | 720 | 0.000E+00 | -9.5721 | -561.4858 | 1.904E-05 | -2.361E-07 | -3.028E-09 |
| 8 | | | | | | | |
| | 720 | 0.000E+00 | 1.8526 | -561.4858 | 1.904E-05 | -2.361E-07 | 1.762E-09 |
| | 732 | 2.000E-04 | 1.8526 | -512.9839 | 1.633E-05 | -2.157E-07 | 1.628E-09 |

| | | | | | | | |
|---|---|---|---|---|---|---|---|
| | 744 | 4.000E-04 | 1.8526 | -468.334 | 1.386E-05 | -1.970E-07 | 1.502E-09 |
| | 756 | 5.000E-04 | 1.8526 | -427.2012 | 1.160E-05 | -1.797E-07 | 1.385E-09 |
| | z | θ | T(z) | B(z) | θ' | θ'' | θ''' |
| | 768 | 7.000E-04 | 1.8526 | -389.2762 | 9.540E-06 | -1.637E-07 | 1.277E-09 |
| | 780 | 8.000E-04 | 1.8526 | -354.274 | 7.666E-06 | -1.490E-07 | 1.178E-09 |
| | 792 | 9.000E-04 | 1.8526 | -321.9316 | 5.961E-06 | -1.354E-07 | 1.088E-09 |
| | 804 | 9.000E-04 | 1.8526 | -292.0065 | 4.413E-06 | -1.228E-07 | 1.007E-09 |
| | 816 | 1.000E-03 | 1.8526 | -264.274 | 3.010E-06 | -1.112E-07 | 9.354E-10 |
| | 828 | 1.000E-03 | 1.8526 | -238.5263 | 1.742E-06 | -1.003E-07 | 8.723E-10 |
| | 840 | 1.000E-03 | 1.8526 | -214.5698 | 5.994E-07 | -9.018E-08 | 8.181E-10 |
| 9 | | | | | | | |
| | 840 | 1.000E-03 | 1.8526 | -214.5698 | 5.994E-07 | -9.021E-08 | 8.072E-10 |
| | 852 | 1.000E-03 | 1.8526 | -192.2241 | -4.264E-07 | -8.084E-08 | 7.560E-10 |
| | 864 | 1.000E-03 | 1.8526 | -171.3219 | -1.343E-06 | -7.205E-08 | 7.093E-10 |
| | 876 | 1.000E-03 | 1.8526 | -151.7062 | -2.158E-06 | -6.380E-08 | 6.672E-10 |
| | 888 | 9.000E-04 | 1.8526 | -133.2296 | -2.876E-06 | -5.602E-08 | 6.295E-10 |
| | 900 | 9.000E-04 | 1.8526 | -115.7534 | -3.504E-06 | -4.867E-08 | 5.964E-10 |
| | 912 | 9.000E-04 | 1.8526 | -99.1463 | -4.046E-06 | -4.169E-08 | 5.678E-10 |
| | 924 | 8.000E-04 | 1.8526 | -83.2836 | -4.506E-06 | -3.503E-08 | 5.438E-10 |
| | 936 | 8.000E-04 | 1.8526 | -68.0463 | -4.887E-06 | -2.862E-08 | 5.242E-10 |
| | 948 | 7.000E-04 | 1.8526 | -53.32 | -5.194E-06 | -2.243E-08 | 5.092E-10 |
| | 960 | 6.000E-04 | 1.8526 | -38.9942 | -5.426E-06 | -1.638E-08 | 4.987E-10 |
| 10 | | | | | | | |
| | 960 | 6.000E-04 | 1.8526 | -38.9942 | -5.426E-06 | -1.640E-08 | 4.967E-10 |
| | 972 | 6.000E-04 | 1.8526 | -24.9611 | -5.588E-06 | -1.050E-08 | 4.881E-10 |
| | 984 | 5.000E-04 | 1.8526 | -11.1154 | -5.679E-06 | -4.673E-09 | 4.831E-10 |
| | 996 | 4.000E-04 | 1.8526 | 2.6469 | -5.700E-06 | 1.114E-09 | 4.819E-10 |
| | 1008 | 4.000E-04 | 1.8526 | 16.429 | -5.652E-06 | 6.908E-09 | 4.844E-10 |
| | 1020 | 3.000E-04 | 1.8526 | 30.3345 | -5.534E-06 | 1.275E-08 | 4.906E-10 |
| | 1032 | 2.000E-04 | 1.8526 | 44.4677 | -5.345E-06 | 1.870E-08 | 5.006E-10 |
| | 1044 | 2.000E-04 | 1.8526 | 58.9348 | -5.085E-06 | 2.478E-08 | 5.143E-10 |
| | 1056 | 1.000E-04 | 1.8526 | 73.8445 | -4.750E-06 | 3.106E-08 | 5.316E-10 |
| | 1068 | 0.000E+00 | 1.8526 | 89.3086 | -4.338E-06 | 3.756E-08 | 5.527E-10 |
| | 1080 | 0.000E+00 | 1.8526 | 105.4434 | -3.847E-06 | 4.434E-08 | 5.776E-10 |
| 11 | | | | | | | |
| | 1080 | 0.000E+00 | -0.2929 | 105.4434 | -3.847E-06 | 4.433E-08 | -3.219E-10 |
| | 1092 | 0.000E+00 | -0.2929 | 96.5919 | -3.338E-06 | 4.062E-08 | -2.966E-10 |
| | 1104 | -1.000E-04 | -0.2929 | 88.4658 | -2.871E-06 | 3.721E-08 | -2.729E-10 |
| | 1116 | -1.000E-04 | -0.2929 | 81.0039 | -2.444E-06 | 3.407E-08 | -2.508E-10 |
| | 1128 | -1.000E-04 | -0.2929 | 74.1504 | -2.053E-06 | 3.118E-08 | -2.303E-10 |
| | 1140 | -2.000E-04 | -0.2929 | 67.8535 | -1.694E-06 | 2.853E-08 | -2.115E-10 |
| | 1152 | -2.000E-04 | -0.2929 | 62.0661 | -1.367E-06 | 2.610E-08 | -1.942E-10 |
| | 1164 | -2.000E-04 | -0.2929 | 56.7448 | -1.067E-06 | 2.386E-08 | -1.785E-10 |
| | 1176 | -2.000E-04 | -0.2929 | 51.8495 | -7.935E-07 | 2.181E-08 | -1.644E-10 |
| | 1188 | -2.000E-04 | -0.2929 | 47.3436 | -5.433E-07 | 1.991E-08 | -1.520E-10 |

| | | | | | | | |
|---|---|---|---|---|---|---|---|
| | 1200 | -2.000E-04 | -0.2929 | 43.1933 | -3.150E-07 | 1.815E-08 | -1.411E-10 |
| 12 | | | | | | | |
| | 1200 | -2.000E-04 | -0.2929 | 43.1933 | -3.150E-07 | 1.816E-08 | -1.389E-10 |
| | z | θ | T(z) | B(z) | θ' | θ'' | θ''' |
| | 1212 | -2.000E-04 | -0.2929 | 39.3672 | -1.069E-07 | 1.656E-08 | -1.285E-10 |
| | 1224 | -2.000E-04 | -0.2929 | 35.8367 | 8.279E-08 | 1.507E-08 | -1.189E-10 |
| | 1236 | -2.000E-04 | -0.2929 | 32.5754 | 2.553E-07 | 1.370E-08 | -1.100E-10 |
| | 1248 | -2.000E-04 | -0.2929 | 29.5586 | 4.120E-07 | 1.243E-08 | -1.018E-10 |
| | 1260 | -2.000E-04 | -0.2929 | 26.7638 | 5.540E-07 | 1.125E-08 | -9.430E-11 |
| | 1272 | -2.000E-04 | -0.2929 | 24.17 | 6.824E-07 | 1.016E-08 | -8.752E-11 |
| | 1284 | -2.000E-04 | -0.2929 | 21.7576 | 7.982E-07 | 9.150E-09 | -8.146E-11 |
| | 1296 | -2.000E-04 | -0.2929 | 19.5086 | 9.023E-07 | 8.205E-09 | -7.611E-11 |
| | 1308 | -2.000E-04 | -0.2929 | 17.4061 | 9.954E-07 | 7.321E-09 | -7.148E-11 |
| | 1320 | -2.000E-04 | -0.2929 | 15.4343 | 1.078E-06 | 6.487E-09 | -6.757E-11 |
| 13 | | | | | | | |
| | 1320 | -2.000E-04 | -0.2929 | 15.4343 | 1.078E-06 | 6.489E-09 | -6.679E-11 |
| | 1332 | -1.000E-04 | -0.2929 | 13.5784 | 1.151E-06 | 5.710E-09 | -6.312E-11 |
| | 1344 | -1.000E-04 | -0.2929 | 11.8244 | 1.215E-06 | 4.973E-09 | -5.985E-11 |
| | 1356 | -1.000E-04 | -0.2929 | 10.1592 | 1.271E-06 | 4.272E-09 | -5.698E-11 |
| | 1368 | -1.000E-04 | -0.2929 | 8.5704 | 1.318E-06 | 3.604E-09 | -5.451E-11 |
| | 1380 | -1.000E-04 | -0.2929 | 7.0458 | 1.357E-06 | 2.963E-09 | -5.243E-11 |
| | 1392 | -1.000E-04 | -0.2929 | 5.5742 | 1.389E-06 | 2.344E-09 | -5.075E-11 |
| | 1404 | -1.000E-04 | -0.2929 | 4.1444 | 1.414E-06 | 1.743E-09 | -4.947E-11 |
| | 1416 | 0.000E+00 | -0.2929 | 2.7458 | 1.431E-06 | 1.155E-09 | -4.858E-11 |
| | 1428 | 0.000E+00 | -0.2929 | 1.3678 | 1.442E-06 | 5.754E-10 | -4.810E-11 |
| | 1440 | 0.000E+00 | -0.2929 | 0 | 1.445E-06 | -8.881E-13 | -4.801E-11 |

Table of Pure Torsion Shear Stresses along “z” and “s”

| Pure Torsion Shear Stresses G*t*θ' | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|
| “t”= | 0.625 | 0.625 | 1 | 0.625 | 0.625 | 0.5 | 1 | 1 | 1 |
| "s"= | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 |
| z | | | | | | | | | |
| 0 | 0.96 | 0.96 | 1.53 | 0.96 | 0.96 | 0.77 | 1.53 | 1.53 | 1.53 |
| 7.65 | 0.95 | 0.95 | 1.53 | 0.95 | 0.95 | 0.76 | 1.53 | 1.53 | 1.53 |
| 15.3 | 0.95 | 0.95 | 1.52 | 0.95 | 0.95 | 0.76 | 1.52 | 1.52 | 1.52 |
| 22.95 | 0.94 | 0.94 | 1.51 | 0.94 | 0.94 | 0.75 | 1.51 | 1.51 | 1.51 |
| 30.6 | 0.93 | 0.93 | 1.49 | 0.93 | 0.93 | 0.74 | 1.49 | 1.49 | 1.49 |
| 38.25 | 0.91 | 0.91 | 1.46 | 0.91 | 0.91 | 0.73 | 1.46 | 1.46 | 1.46 |
| 45.9 | 0.89 | 0.89 | 1.43 | 0.89 | 0.89 | 0.72 | 1.43 | 1.43 | 1.43 |
| 53.55 | 0.87 | 0.87 | 1.40 | 0.87 | 0.87 | 0.70 | 1.40 | 1.40 | 1.40 |
| 61.2 | 0.85 | 0.85 | 1.35 | 0.85 | 0.85 | 0.68 | 1.35 | 1.35 | 1.35 |
| 68.85 | 0.82 | 0.82 | 1.31 | 0.82 | 0.82 | 0.65 | 1.31 | 1.31 | 1.31 |
| 76.5 | 0.78 | 0.78 | 1.25 | 0.78 | 0.78 | 0.63 | 1.25 | 1.25 | 1.25 |
| | | | | | | | | | |

| | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|
| 76.5 | 0.78 | 0.78 | 1.25 | 0.78 | 0.78 | 0.63 | 1.25 | 1.25 | 1.25 |
| 84.15 | 0.74 | 0.74 | 1.19 | 0.74 | 0.74 | 0.60 | 1.19 | 1.19 | 1.19 |
| 91.8 | 0.70 | 0.70 | 1.12 | 0.70 | 0.70 | 0.56 | 1.12 | 1.12 | 1.12 |
| 99.45 | 0.66 | 0.66 | 1.05 | 0.66 | 0.66 | 0.53 | 1.05 | 1.05 | 1.05 |
| "s"= | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 |
| z | | | | | | | | | |
| 107.1 | 0.61 | 0.61 | 0.97 | 0.61 | 0.61 | 0.49 | 0.97 | 0.97 | 0.97 |
| 114.75 | 0.55 | 0.55 | 0.88 | 0.55 | 0.55 | 0.44 | 0.88 | 0.88 | 0.88 |
| 122.4 | 0.49 | 0.49 | 0.79 | 0.49 | 0.49 | 0.39 | 0.79 | 0.79 | 0.79 |
| 130.05 | 0.43 | 0.43 | 0.69 | 0.43 | 0.43 | 0.34 | 0.69 | 0.69 | 0.69 |
| 137.7 | 0.36 | 0.36 | 0.58 | 0.36 | 0.36 | 0.29 | 0.58 | 0.58 | 0.58 |
| 145.35 | 0.29 | 0.29 | 0.46 | 0.29 | 0.29 | 0.23 | 0.46 | 0.46 | 0.46 |
| 153 | 0.21 | 0.21 | 0.33 | 0.21 | 0.21 | 0.16 | 0.33 | 0.33 | 0.33 |
| | | | | | | | | | |
| 153 | 0.21 | 0.21 | 0.33 | 0.21 | 0.21 | 0.16 | 0.33 | 0.33 | 0.33 |
| 165.6 | 0.07 | 0.07 | 0.12 | 0.07 | 0.07 | 0.06 | 0.12 | 0.12 | 0.12 |
| 178.2 | -0.05 | -0.05 | -0.07 | -0.05 | -0.05 | -0.04 | -0.07 | -0.07 | -0.07 |
| 190.8 | -0.16 | -0.16 | -0.26 | -0.16 | -0.16 | -0.13 | -0.26 | -0.26 | -0.26 |
| 203.4 | -0.26 | -0.26 | -0.42 | -0.26 | -0.26 | -0.21 | -0.42 | -0.42 | -0.42 |
| 216 | -0.36 | -0.36 | -0.58 | -0.36 | -0.36 | -0.29 | -0.58 | -0.58 | -0.58 |
| 228.6 | -0.46 | -0.46 | -0.73 | -0.46 | -0.46 | -0.36 | -0.73 | -0.73 | -0.73 |
| 241.2 | -0.54 | -0.54 | -0.87 | -0.54 | -0.54 | -0.43 | -0.87 | -0.87 | -0.87 |
| 253.8 | -0.63 | -0.63 | -1.00 | -0.63 | -0.63 | -0.50 | -1.00 | -1.00 | -1.00 |
| 266.4 | -0.70 | -0.70 | -1.13 | -0.70 | -0.70 | -0.56 | -1.13 | -1.13 | -1.13 |
| 279 | -0.78 | -0.78 | -1.25 | -0.78 | -0.78 | -0.62 | -1.25 | -1.25 | -1.25 |
| | | | | | | | | | |
| 279 | -0.78 | -0.78 | -1.25 | -0.78 | -0.78 | -0.62 | -1.25 | -1.25 | -1.25 |
| 287.1 | -0.82 | -0.82 | -1.32 | -0.82 | -0.82 | -0.66 | -1.32 | -1.32 | -1.32 |
| 295.2 | -0.85 | -0.85 | -1.36 | -0.85 | -0.85 | -0.68 | -1.36 | -1.36 | -1.36 |
| 303.3 | -0.87 | -0.87 | -1.39 | -0.87 | -0.87 | -0.70 | -1.39 | -1.39 | -1.39 |
| 311.4 | -0.88 | -0.88 | -1.41 | -0.88 | -0.88 | -0.70 | -1.41 | -1.41 | -1.41 |
| 319.5 | -0.87 | -0.87 | -1.40 | -0.87 | -0.87 | -0.70 | -1.40 | -1.40 | -1.40 |
| 327.6 | -0.86 | -0.86 | -1.37 | -0.86 | -0.86 | -0.69 | -1.37 | -1.37 | -1.37 |
| 335.7 | -0.83 | -0.83 | -1.33 | -0.83 | -0.83 | -0.67 | -1.33 | -1.33 | -1.33 |
| 343.8 | -0.79 | -0.79 | -1.27 | -0.79 | -0.79 | -0.63 | -1.27 | -1.27 | -1.27 |
| 351.9 | -0.74 | -0.74 | -1.19 | -0.74 | -0.74 | -0.59 | -1.19 | -1.19 | -1.19 |
| 360 | -0.68 | -0.68 | -1.09 | -0.68 | -0.68 | -0.54 | -1.09 | -1.09 | -1.09 |
| | | | | | | | | | |
| 360 | -0.68 | -0.68 | -1.09 | -0.68 | -0.68 | -0.54 | -1.09 | -1.09 | -1.09 |
| 372 | -0.58 | -0.58 | -0.93 | -0.58 | -0.58 | -0.47 | -0.93 | -0.93 | -0.93 |
| 384 | -0.49 | -0.49 | -0.79 | -0.49 | -0.49 | -0.40 | -0.79 | -0.79 | -0.79 |
| 396 | -0.41 | -0.41 | -0.66 | -0.41 | -0.41 | -0.33 | -0.66 | -0.66 | -0.66 |
| 408 | -0.34 | -0.34 | -0.54 | -0.34 | -0.34 | -0.27 | -0.54 | -0.54 | -0.54 |
| 420 | -0.27 | -0.27 | -0.44 | -0.27 | -0.27 | -0.22 | -0.44 | -0.44 | -0.44 |
| 432 | -0.21 | -0.21 | -0.34 | -0.21 | -0.21 | -0.17 | -0.34 | -0.34 | -0.34 |

| | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|
| 444 | -0.16 | -0.16 | -0.25 | -0.16 | -0.16 | -0.13 | -0.25 | -0.25 | -0.25 |
| 456 | -0.11 | -0.11 | -0.17 | -0.11 | -0.11 | -0.08 | -0.17 | -0.17 | -0.17 |
| 468 | -0.06 | -0.06 | -0.10 | -0.06 | -0.06 | -0.05 | -0.10 | -0.10 | -0.10 |
| 480 | -0.02 | -0.02 | -0.03 | -0.02 | -0.02 | -0.02 | -0.03 | -0.03 | -0.03 |
| | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 |
| "s"= | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 |
| z | | | | | | | | | |
| 480 | -0.02 | -0.02 | -0.03 | -0.02 | -0.02 | -0.02 | -0.03 | -0.03 | -0.03 |
| 492 | 0.02 | 0.02 | 0.03 | 0.02 | 0.02 | 0.01 | 0.03 | 0.03 | 0.03 |
| 504 | 0.05 | 0.05 | 0.08 | 0.05 | 0.05 | 0.04 | 0.08 | 0.08 | 0.08 |
| 516 | 0.08 | 0.08 | 0.13 | 0.08 | 0.08 | 0.06 | 0.13 | 0.13 | 0.13 |
| 528 | 0.10 | 0.10 | 0.17 | 0.10 | 0.10 | 0.08 | 0.17 | 0.17 | 0.17 |
| 540 | 0.13 | 0.13 | 0.20 | 0.13 | 0.13 | 0.10 | 0.20 | 0.20 | 0.20 |
| 552 | 0.15 | 0.15 | 0.23 | 0.15 | 0.15 | 0.12 | 0.23 | 0.23 | 0.23 |
| 564 | 0.16 | 0.16 | 0.26 | 0.16 | 0.16 | 0.13 | 0.26 | 0.26 | 0.26 |
| 576 | 0.18 | 0.18 | 0.28 | 0.18 | 0.18 | 0.14 | 0.28 | 0.28 | 0.28 |
| 588 | 0.19 | 0.19 | 0.30 | 0.19 | 0.19 | 0.15 | 0.30 | 0.30 | 0.30 |
| 600 | 0.19 | 0.19 | 0.31 | 0.19 | 0.19 | 0.16 | 0.31 | 0.31 | 0.31 |
| | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 |
| 600 | 0.19 | 0.19 | 0.31 | 0.19 | 0.19 | 0.16 | 0.31 | 0.31 | 0.31 |
| 612 | 0.20 | 0.20 | 0.32 | 0.20 | 0.20 | 0.16 | 0.32 | 0.32 | 0.32 |
| 624 | 0.20 | 0.20 | 0.32 | 0.20 | 0.20 | 0.16 | 0.32 | 0.32 | 0.32 |
| 636 | 0.20 | 0.20 | 0.32 | 0.20 | 0.20 | 0.16 | 0.32 | 0.32 | 0.32 |
| 648 | 0.20 | 0.20 | 0.32 | 0.20 | 0.20 | 0.16 | 0.32 | 0.32 | 0.32 |
| 660 | 0.20 | 0.20 | 0.31 | 0.20 | 0.20 | 0.16 | 0.31 | 0.31 | 0.31 |
| 672 | 0.19 | 0.19 | 0.30 | 0.19 | 0.19 | 0.15 | 0.30 | 0.30 | 0.30 |
| 684 | 0.18 | 0.18 | 0.29 | 0.18 | 0.18 | 0.14 | 0.29 | 0.29 | 0.29 |
| 696 | 0.17 | 0.17 | 0.27 | 0.17 | 0.17 | 0.13 | 0.27 | 0.27 | 0.27 |
| 708 | 0.15 | 0.15 | 0.24 | 0.15 | 0.15 | 0.12 | 0.24 | 0.24 | 0.24 |
| 720 | 0.13 | 0.13 | 0.21 | 0.13 | 0.13 | 0.11 | 0.21 | 0.21 | 0.21 |
| | | | | | | | | | |
| 720 | 0.13 | 0.13 | 0.21 | 0.13 | 0.13 | 0.11 | 0.21 | 0.21 | 0.21 |
| 732 | 0.11 | 0.11 | 0.18 | 0.11 | 0.11 | 0.09 | 0.18 | 0.18 | 0.18 |
| 744 | 0.10 | 0.10 | 0.15 | 0.10 | 0.10 | 0.08 | 0.15 | 0.15 | 0.15 |
| 756 | 0.08 | 0.08 | 0.13 | 0.08 | 0.08 | 0.06 | 0.13 | 0.13 | 0.13 |
| 768 | 0.07 | 0.07 | 0.11 | 0.07 | 0.07 | 0.05 | 0.11 | 0.11 | 0.11 |
| 780 | 0.05 | 0.05 | 0.09 | 0.05 | 0.05 | 0.04 | 0.09 | 0.09 | 0.09 |
| 792 | 0.04 | 0.04 | 0.07 | 0.04 | 0.04 | 0.03 | 0.07 | 0.07 | 0.07 |
| 804 | 0.03 | 0.03 | 0.05 | 0.03 | 0.03 | 0.02 | 0.05 | 0.05 | 0.05 |
| 816 | 0.02 | 0.02 | 0.03 | 0.02 | 0.02 | 0.02 | 0.03 | 0.03 | 0.03 |
| 828 | 0.01 | 0.01 | 0.02 | 0.01 | 0.01 | 0.01 | 0.02 | 0.02 | 0.02 |
| 840 | 0.00 | 0.00 | 0.01 | 0.00 | 0.00 | 0.00 | 0.01 | 0.01 | 0.01 |
| | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 |
| 840 | 0.00 | 0.00 | 0.01 | 0.00 | 0.00 | 0.00 | 0.01 | 0.01 | 0.01 |
| 852 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 |

| | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|
| 864 | -0.01 | -0.01 | -0.01 | -0.01 | -0.01 | -0.01 | -0.01 | -0.01 | -0.01 |
| 876 | -0.02 | -0.02 | -0.02 | -0.02 | -0.02 | -0.01 | -0.02 | -0.02 | -0.02 |
| 888 | -0.02 | -0.02 | -0.03 | -0.02 | -0.02 | -0.02 | -0.03 | -0.03 | -0.03 |
| 900 | -0.02 | -0.02 | -0.04 | -0.02 | -0.02 | -0.02 | -0.04 | -0.04 | -0.04 |
| 912 | -0.03 | -0.03 | -0.05 | -0.03 | -0.03 | -0.02 | -0.05 | -0.05 | -0.05 |
| 924 | -0.03 | -0.03 | -0.05 | -0.03 | -0.03 | -0.03 | -0.05 | -0.05 | -0.05 |
| "s"= | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 |
| z | | | | | | | | | |
| 936 | -0.03 | -0.03 | -0.05 | -0.03 | -0.03 | -0.03 | -0.05 | -0.05 | -0.05 |
| 948 | -0.04 | -0.04 | -0.06 | -0.04 | -0.04 | -0.03 | -0.06 | -0.06 | -0.06 |
| 960 | -0.04 | -0.04 | -0.06 | -0.04 | -0.04 | -0.03 | -0.06 | -0.06 | -0.06 |
| | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 |
| 960 | -0.04 | -0.04 | -0.06 | -0.04 | -0.04 | -0.03 | -0.06 | -0.06 | -0.06 |
| 972 | -0.04 | -0.04 | -0.06 | -0.04 | -0.04 | -0.03 | -0.06 | -0.06 | -0.06 |
| 984 | -0.04 | -0.04 | -0.06 | -0.04 | -0.04 | -0.03 | -0.06 | -0.06 | -0.06 |
| 996 | -0.04 | -0.04 | -0.06 | -0.04 | -0.04 | -0.03 | -0.06 | -0.06 | -0.06 |
| 1008 | -0.04 | -0.04 | -0.06 | -0.04 | -0.04 | -0.03 | -0.06 | -0.06 | -0.06 |
| 1020 | -0.04 | -0.04 | -0.06 | -0.04 | -0.04 | -0.03 | -0.06 | -0.06 | -0.06 |
| 1032 | -0.04 | -0.04 | -0.06 | -0.04 | -0.04 | -0.03 | -0.06 | -0.06 | -0.06 |
| 1044 | -0.04 | -0.04 | -0.06 | -0.04 | -0.04 | -0.03 | -0.06 | -0.06 | -0.06 |
| 1056 | -0.03 | -0.03 | -0.05 | -0.03 | -0.03 | -0.03 | -0.05 | -0.05 | -0.05 |
| 1068 | -0.03 | -0.03 | -0.05 | -0.03 | -0.03 | -0.02 | -0.05 | -0.05 | -0.05 |
| 1080 | -0.03 | -0.03 | -0.04 | -0.03 | -0.03 | -0.02 | -0.04 | -0.04 | -0.04 |
| | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 |
| 1080 | -0.03 | -0.03 | -0.04 | -0.03 | -0.03 | -0.02 | -0.04 | -0.04 | -0.04 |
| 1092 | -0.02 | -0.02 | -0.04 | -0.02 | -0.02 | -0.02 | -0.04 | -0.04 | -0.04 |
| 1104 | -0.02 | -0.02 | -0.03 | -0.02 | -0.02 | -0.02 | -0.03 | -0.03 | -0.03 |
| 1116 | -0.02 | -0.02 | -0.03 | -0.02 | -0.02 | -0.01 | -0.03 | -0.03 | -0.03 |
| 1128 | -0.01 | -0.01 | -0.02 | -0.01 | -0.01 | -0.01 | -0.02 | -0.02 | -0.02 |
| 1140 | -0.01 | -0.01 | -0.02 | -0.01 | -0.01 | -0.01 | -0.02 | -0.02 | -0.02 |
| 1152 | -0.01 | -0.01 | -0.02 | -0.01 | -0.01 | -0.01 | -0.02 | -0.02 | -0.02 |
| 1164 | -0.01 | -0.01 | -0.01 | -0.01 | -0.01 | -0.01 | -0.01 | -0.01 | -0.01 |
| 1176 | -0.01 | -0.01 | -0.01 | -0.01 | -0.01 | 0.00 | -0.01 | -0.01 | -0.01 |
| 1188 | 0.00 | 0.00 | -0.01 | 0.00 | 0.00 | 0.00 | -0.01 | -0.01 | -0.01 |
| 1200 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 |
| | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 |
| 1200 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 |
| 1212 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 |
| 1224 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 |
| 1236 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 |
| 1248 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 |
| 1260 | 0.00 | 0.00 | 0.01 | 0.00 | 0.00 | 0.00 | 0.01 | 0.01 | 0.01 |
| 1272 | 0.00 | 0.00 | 0.01 | 0.00 | 0.00 | 0.00 | 0.01 | 0.01 | 0.01 |
| 1284 | 0.01 | 0.01 | 0.01 | 0.01 | 0.01 | 0.00 | 0.01 | 0.01 | 0.01 |
| 1296 | 0.01 | 0.01 | 0.01 | 0.01 | 0.01 | 0.01 | 0.01 | 0.01 | 0.01 |

| | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|
| 1308 | 0.01 | 0.01 | 0.01 | 0.01 | 0.01 | 0.01 | 0.01 | 0.01 | 0.01 |
| 1320 | 0.01 | 0.01 | 0.01 | 0.01 | 0.01 | 0.01 | 0.01 | 0.01 | 0.01 |
| | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 |
| 1320 | 0.01 | 0.01 | 0.01 | 0.01 | 0.01 | 0.01 | 0.01 | 0.01 | 0.01 |
| 1332 | 0.01 | 0.01 | 0.01 | 0.01 | 0.01 | 0.01 | 0.01 | 0.01 | 0.01 |
| 1344 | 0.01 | 0.01 | 0.01 | 0.01 | 0.01 | 0.01 | 0.01 | 0.01 | 0.01 |
| 1356 | 0.01 | 0.01 | 0.01 | 0.01 | 0.01 | 0.01 | 0.01 | 0.01 | 0.01 |
| "s"= | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 |
| z | | | | | | | | | |
| 1368 | 0.01 | 0.01 | 0.01 | 0.01 | 0.01 | 0.01 | 0.01 | 0.01 | 0.01 |
| 1380 | 0.01 | 0.01 | 0.02 | 0.01 | 0.01 | 0.01 | 0.02 | 0.02 | 0.02 |
| 1392 | 0.01 | 0.01 | 0.02 | 0.01 | 0.01 | 0.01 | 0.02 | 0.02 | 0.02 |
| 1404 | 0.01 | 0.01 | 0.02 | 0.01 | 0.01 | 0.01 | 0.02 | 0.02 | 0.02 |
| 1416 | 0.01 | 0.01 | 0.02 | 0.01 | 0.01 | 0.01 | 0.02 | 0.02 | 0.02 |
| 1428 | 0.01 | 0.01 | 0.02 | 0.01 | 0.01 | 0.01 | 0.02 | 0.02 | 0.02 |
| 1440 | 0.01 | 0.01 | 0.02 | 0.01 | 0.01 | 0.01 | 0.02 | 0.02 | 0.02 |

Table of Warping Shear Stress along “z” and “s”

| Warping Shear Stress at "s" (- E*Sw*θ'''/ t) | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|
| Sw | 0.0 | 84.13 | 185.15 | 84.13 | 0.0 | 0.0 | 0.0 | 376.54 | 0.0 |
| “s”= | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 |
| z | | | | | | | | | |
| 0 | 0 | 0.03 | 0.04 | 0.03 | 0 | 0 | 0 | 0.09 | 0 |
| 7.65 | 0 | 0.03 | 0.04 | 0.03 | 0 | 0 | 0 | 0.09 | 0 |
| 15.3 | 0 | 0.03 | 0.04 | 0.03 | 0 | 0 | 0 | 0.09 | 0 |
| 22.95 | 0 | 0.03 | 0.05 | 0.03 | 0 | 0 | 0 | 0.09 | 0 |
| 30.6 | 0 | 0.03 | 0.05 | 0.03 | 0 | 0 | 0 | 0.09 | 0 |
| 38.25 | 0 | 0.03 | 0.05 | 0.03 | 0 | 0 | 0 | 0.09 | 0 |
| 45.9 | 0 | 0.03 | 0.05 | 0.03 | 0 | 0 | 0 | 0.10 | 0 |
| 53.55 | 0 | 0.03 | 0.05 | 0.03 | 0 | 0 | 0 | 0.10 | 0 |
| 61.2 | 0 | 0.04 | 0.05 | 0.04 | 0 | 0 | 0 | 0.10 | 0 |
| 68.85 | 0 | 0.04 | 0.05 | 0.04 | 0 | 0 | 0 | 0.10 | 0 |
| 76.5 | 0 | 0.04 | 0.05 | 0.04 | 0 | 0 | 0 | 0.11 | 0 |
| | | | | | | | | | |
| 76.5 | 0 | 0.04 | 0.05 | 0.04 | 0 | 0 | 0 | 0.11 | 0 |
| 84.15 | 0 | 0.04 | 0.05 | 0.04 | 0 | 0 | 0 | 0.11 | 0 |
| 91.8 | 0 | 0.04 | 0.05 | 0.04 | 0 | 0 | 0 | 0.11 | 0 |
| 99.45 | 0 | 0.04 | 0.06 | 0.04 | 0 | 0 | 0 | 0.12 | 0 |
| 107.1 | 0 | 0.04 | 0.06 | 0.04 | 0 | 0 | 0 | 0.12 | 0 |
| 114.75 | 0 | 0.04 | 0.06 | 0.04 | 0 | 0 | 0 | 0.12 | 0 |
| 122.4 | 0 | 0.05 | 0.06 | 0.05 | 0 | 0 | 0 | 0.13 | 0 |
| 130.05 | 0 | 0.05 | 0.07 | 0.05 | 0 | 0 | 0 | 0.13 | 0 |
| 137.7 | 0 | 0.05 | 0.07 | 0.05 | 0 | 0 | 0 | 0.14 | 0 |
| 145.35 | 0 | 0.05 | 0.07 | 0.05 | 0 | 0 | 0 | 0.15 | 0 |
| 153 | 0 | 0.05 | 0.07 | 0.05 | 0 | 0 | 0 | 0.15 | 0 |
| | | | | | | | | | |
| 153 | 0 | -0.04 | -0.05 | -0.04 | 0 | 0 | 0 | -0.10 | 0 |
| 165.6 | 0 | -0.03 | -0.05 | -0.03 | 0 | 0 | 0 | -0.09 | 0 |
| 178.2 | 0 | -0.03 | -0.04 | -0.03 | 0 | 0 | 0 | -0.08 | 0 |
| 190.8 | 0 | -0.03 | -0.04 | -0.03 | 0 | 0 | 0 | -0.08 | 0 |
| 203.4 | 0 | -0.02 | -0.03 | -0.02 | 0 | 0 | 0 | -0.07 | 0 |
| 216 | 0 | -0.02 | -0.03 | -0.02 | 0 | 0 | 0 | -0.06 | 0 |
| 228.6 | 0 | -0.02 | -0.03 | -0.02 | 0 | 0 | 0 | -0.05 | 0 |
| 228.6 | 0 | -0.02 | -0.03 | -0.02 | 0 | 0 | 0 | -0.05 | 0 |
| “s”= | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 |
| z | | | | | | | | | |
| 241.2 | 0 | -0.02 | -0.02 | -0.02 | 0 | 0 | 0 | -0.04 | 0 |
| 253.8 | 0 | -0.01 | -0.02 | -0.01 | 0 | 0 | 0 | -0.04 | 0 |
| 266.4 | 0 | -0.01 | -0.02 | -0.01 | 0 | 0 | 0 | -0.03 | 0 |
| 279 | 0 | -0.01 | -0.01 | -0.01 | 0 | 0 | 0 | -0.03 | 0 |
| | | | | | | | | | |
| 279 | 0 | -0.10 | -0.14 | -0.10 | 0 | 0 | 0 | -0.28 | 0 |
| 287.1 | 0 | -0.10 | -0.14 | -0.10 | 0 | 0 | 0 | -0.28 | 0 |

| | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|
| 295.2 | 0 | -0.10 | -0.14 | -0.10 | 0 | 0 | 0 | -0.28 | 0 |
| 303.3 | 0 | -0.10 | -0.13 | -0.10 | 0 | 0 | 0 | -0.27 | 0 |
| 311.4 | 0 | -0.10 | -0.13 | -0.10 | 0 | 0 | 0 | -0.27 | 0 |
| 319.5 | 0 | -0.10 | -0.13 | -0.10 | 0 | 0 | 0 | -0.27 | 0 |
| 327.6 | 0 | -0.10 | -0.14 | -0.10 | 0 | 0 | 0 | -0.28 | 0 |
| 335.7 | 0 | -0.10 | -0.14 | -0.10 | 0 | 0 | 0 | -0.28 | 0 |
| 343.8 | 0 | -0.10 | -0.14 | -0.10 | 0 | 0 | 0 | -0.28 | 0 |
| 351.9 | 0 | -0.10 | -0.14 | -0.10 | 0 | 0 | 0 | -0.28 | 0 |
| 360 | 0 | -0.10 | -0.14 | -0.10 | 0 | 0 | 0 | -0.29 | 0 |
| | | | | | | | | | |
| 360 | 0 | 0.04 | 0.05 | 0.04 | 0 | 0 | 0 | 0.10 | 0 |
| 372 | 0 | 0.03 | 0.04 | 0.03 | 0 | 0 | 0 | 0.09 | 0 |
| 384 | 0 | 0.03 | 0.04 | 0.03 | 0 | 0 | 0 | 0.08 | 0 |
| 396 | 0 | 0.03 | 0.04 | 0.03 | 0 | 0 | 0 | 0.08 | 0 |
| 408 | 0 | 0.03 | 0.04 | 0.03 | 0 | 0 | 0 | 0.07 | 0 |
| 420 | 0 | 0.02 | 0.03 | 0.02 | 0 | 0 | 0 | 0.07 | 0 |
| 432 | 0 | 0.02 | 0.03 | 0.02 | 0 | 0 | 0 | 0.06 | 0 |
| 444 | 0 | 0.02 | 0.03 | 0.02 | 0 | 0 | 0 | 0.06 | 0 |
| 456 | 0 | 0.02 | 0.03 | 0.02 | 0 | 0 | 0 | 0.05 | 0 |
| 468 | 0 | 0.02 | 0.02 | 0.02 | 0 | 0 | 0 | 0.05 | 0 |
| 480 | 0 | 0.02 | 0.02 | 0.02 | 0 | 0 | 0 | 0.05 | 0 |
| | | | | | | | | | |
| 480 | 0 | 0.02 | 0.02 | 0.02 | 0 | 0 | 0 | 0.05 | 0 |
| 492 | 0 | 0.02 | 0.02 | 0.02 | 0 | 0 | 0 | 0.04 | 0 |
| 504 | 0 | 0.01 | 0.02 | 0.01 | 0 | 0 | 0 | 0.04 | 0 |
| 516 | 0 | 0.01 | 0.02 | 0.01 | 0 | 0 | 0 | 0.04 | 0 |
| 528 | 0 | 0.01 | 0.02 | 0.01 | 0 | 0 | 0 | 0.04 | 0 |
| 540 | 0 | 0.01 | 0.02 | 0.01 | 0 | 0 | 0 | 0.03 | 0 |
| 552 | 0 | 0.01 | 0.02 | 0.01 | 0 | 0 | 0 | 0.03 | 0 |
| 564 | 0 | 0.01 | 0.02 | 0.01 | 0 | 0 | 0 | 0.03 | 0 |
| 576 | 0 | 0.01 | 0.01 | 0.01 | 0 | 0 | 0 | 0.03 | 0 |
| 588 | 0 | 0.01 | 0.01 | 0.01 | 0 | 0 | 0 | 0.03 | 0 |
| 600 | 0 | 0.01 | 0.01 | 0.01 | 0 | 0 | 0 | 0.03 | 0 |
| | | | | | | | | | |
| 600 | 0 | 0.01 | 0.01 | 0.01 | 0 | 0 | 0 | 0.03 | 0 |
| 612 | 0 | 0.01 | 0.01 | 0.01 | 0 | 0 | 0 | 0.03 | 0 |
| 624 | 0 | 0.01 | 0.01 | 0.01 | 0 | 0 | 0 | 0.03 | 0 |
| 636 | 0 | 0.01 | 0.01 | 0.01 | 0 | 0 | 0 | 0.03 | 0 |
| 636 | 0 | 0.01 | 0.01 | 0.01 | 0 | 0 | 0 | 0.03 | 0 |
| | | | | | | | | | |
| “s”= z | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 |
| 648 | 0 | 0.01 | 0.01 | 0.01 | 0 | 0 | 0 | 0.03 | 0 |
| 660 | 0 | 0.01 | 0.01 | 0.01 | 0 | 0 | 0 | 0.03 | 0 |
| 672 | 0 | 0.01 | 0.01 | 0.01 | 0 | 0 | 0 | 0.03 | 0 |
| 684 | 0 | 0.01 | 0.01 | 0.01 | 0 | 0 | 0 | 0.03 | 0 |

| | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|
| 696 | 0 | 0.01 | 0.01 | 0.01 | 0 | 0 | 0 | 0.03 | 0 |
| 708 | 0 | 0.01 | 0.02 | 0.01 | 0 | 0 | 0 | 0.03 | 0 |
| 720 | 0 | 0.01 | 0.02 | 0.01 | 0 | 0 | 0 | 0.03 | 0 |
| | | | | | | | | | |
| 720 | 0 | -0.01 | -0.01 | -0.01 | 0 | 0 | 0 | -0.02 | 0 |
| 732 | 0 | -0.01 | -0.01 | -0.01 | 0 | 0 | 0 | -0.02 | 0 |
| 744 | 0 | -0.01 | -0.01 | -0.01 | 0 | 0 | 0 | -0.02 | 0 |
| 756 | 0 | -0.01 | -0.01 | -0.01 | 0 | 0 | 0 | -0.02 | 0 |
| 768 | 0 | 0.00 | -0.01 | 0.00 | 0 | 0 | 0 | -0.01 | 0 |
| 780 | 0 | 0.00 | -0.01 | 0.00 | 0 | 0 | 0 | -0.01 | 0 |
| 792 | 0 | 0.00 | -0.01 | 0.00 | 0 | 0 | 0 | -0.01 | 0 |
| 804 | 0 | 0.00 | -0.01 | 0.00 | 0 | 0 | 0 | -0.01 | 0 |
| 816 | 0 | 0.00 | -0.01 | 0.00 | 0 | 0 | 0 | -0.01 | 0 |
| 828 | 0 | 0.00 | 0.00 | 0.00 | 0 | 0 | 0 | -0.01 | 0 |
| 840 | 0 | 0.00 | 0.00 | 0.00 | 0 | 0 | 0 | -0.01 | 0 |
| | | | | | | | | | |
| 840 | 0 | 0.00 | 0.00 | 0.00 | 0 | 0 | 0 | -0.01 | 0 |
| 852 | 0 | 0.00 | 0.00 | 0.00 | 0 | 0 | 0 | -0.01 | 0 |
| 864 | 0 | 0.00 | 0.00 | 0.00 | 0 | 0 | 0 | -0.01 | 0 |
| 876 | 0 | 0.00 | 0.00 | 0.00 | 0 | 0 | 0 | -0.01 | 0 |
| 888 | 0 | 0.00 | 0.00 | 0.00 | 0 | 0 | 0 | -0.01 | 0 |
| 900 | 0 | 0.00 | 0.00 | 0.00 | 0 | 0 | 0 | -0.01 | 0 |
| 912 | 0 | 0.00 | 0.00 | 0.00 | 0 | 0 | 0 | -0.01 | 0 |
| 924 | 0 | 0.00 | 0.00 | 0.00 | 0 | 0 | 0 | -0.01 | 0 |
| 936 | 0 | 0.00 | 0.00 | 0.00 | 0 | 0 | 0 | -0.01 | 0 |
| 948 | 0 | 0.00 | 0.00 | 0.00 | 0 | 0 | 0 | -0.01 | 0 |
| 960 | 0 | 0.00 | 0.00 | 0.00 | 0 | 0 | 0 | -0.01 | 0 |
| | | | | | | | | | |
| 960 | 0 | 0.00 | 0.00 | 0.00 | 0 | 0 | 0 | -0.01 | 0 |
| 972 | 0 | 0.00 | 0.00 | 0.00 | 0 | 0 | 0 | -0.01 | 0 |
| 984 | 0 | 0.00 | 0.00 | 0.00 | 0 | 0 | 0 | -0.01 | 0 |
| 996 | 0 | 0.00 | 0.00 | 0.00 | 0 | 0 | 0 | -0.01 | 0 |
| 1008 | 0 | 0.00 | 0.00 | 0.00 | 0 | 0 | 0 | -0.01 | 0 |
| 1020 | 0 | 0.00 | 0.00 | 0.00 | 0 | 0 | 0 | -0.01 | 0 |
| 1032 | 0 | 0.00 | 0.00 | 0.00 | 0 | 0 | 0 | -0.01 | 0 |
| 1044 | 0 | 0.00 | 0.00 | 0.00 | 0 | 0 | 0 | -0.01 | 0 |
| 1056 | 0 | 0.00 | 0.00 | 0.00 | 0 | 0 | 0 | -0.01 | 0 |
| 1068 | 0 | 0.00 | 0.00 | 0.00 | 0 | 0 | 0 | -0.01 | 0 |
| 1080 | 0 | 0.00 | 0.00 | 0.00 | 0 | 0 | 0 | -0.01 | 0 |
| | | | | | | | | | |
| 1080 | 0 | 0.00 | 0.00 | 0.00 | 0 | 0 | 0 | 0.00 | 0 |
| 1092 | 0 | 0.00 | 0.00 | 0.00 | 0 | 0 | 0 | 0.00 | 0 |
| | | | | | | | | | |
| “s”= z | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 |
| 1104 | 0 | 0.00 | 0.00 | 0.00 | 0 | 0 | 0 | 0.00 | 0 |

| | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|
| 1116 | 0 | 0.00 | 0.00 | 0.00 | 0 | 0 | 0 | 0.00 | 0 |
| 1128 | 0 | 0.00 | 0.00 | 0.00 | 0 | 0 | 0 | 0.00 | 0 |
| 1140 | 0 | 0.00 | 0.00 | 0.00 | 0 | 0 | 0 | 0.00 | 0 |
| 1152 | 0 | 0.00 | 0.00 | 0.00 | 0 | 0 | 0 | 0.00 | 0 |
| 1164 | 0 | 0.00 | 0.00 | 0.00 | 0 | 0 | 0 | 0.00 | 0 |
| 1176 | 0 | 0.00 | 0.00 | 0.00 | 0 | 0 | 0 | 0.00 | 0 |
| 1188 | 0 | 0.00 | 0.00 | 0.00 | 0 | 0 | 0 | 0.00 | 0 |
| 1200 | 0 | 0.00 | 0.00 | 0.00 | 0 | 0 | 0 | 0.00 | 0 |
| | 0 | 0.00 | 0.00 | 0.00 | 0 | 0 | 0 | 0.00 | 0 |
| 1200 | 0 | 0.00 | 0.00 | 0.00 | 0 | 0 | 0 | 0.00 | 0 |
| 1212 | 0 | 0.00 | 0.00 | 0.00 | 0 | 0 | 0 | 0.00 | 0 |
| 1224 | 0 | 0.00 | 0.00 | 0.00 | 0 | 0 | 0 | 0.00 | 0 |
| 1236 | 0 | 0.00 | 0.00 | 0.00 | 0 | 0 | 0 | 0.00 | 0 |
| 1248 | 0 | 0.00 | 0.00 | 0.00 | 0 | 0 | 0 | 0.00 | 0 |
| 1260 | 0 | 0.00 | 0.00 | 0.00 | 0 | 0 | 0 | 0.00 | 0 |
| 1272 | 0 | 0.00 | 0.00 | 0.00 | 0 | 0 | 0 | 0.00 | 0 |
| 1284 | 0 | 0.00 | 0.00 | 0.00 | 0 | 0 | 0 | 0.00 | 0 |
| 1296 | 0 | 0.00 | 0.00 | 0.00 | 0 | 0 | 0 | 0.00 | 0 |
| 1308 | 0 | 0.00 | 0.00 | 0.00 | 0 | 0 | 0 | 0.00 | 0 |
| 1320 | 0 | 0.00 | 0.00 | 0.00 | 0 | 0 | 0 | 0.00 | 0 |
| | | | | | | | | | |
| 1320 | 0 | 0.00 | 0.00 | 0.00 | 0 | 0 | 0 | 0.00 | 0 |
| 1332 | 0 | 0.00 | 0.00 | 0.00 | 0 | 0 | 0 | 0.00 | 0 |
| 1344 | 0 | 0.00 | 0.00 | 0.00 | 0 | 0 | 0 | 0.00 | 0 |
| 1356 | 0 | 0.00 | 0.00 | 0.00 | 0 | 0 | 0 | 0.00 | 0 |
| 1368 | 0 | 0.00 | 0.00 | 0.00 | 0 | 0 | 0 | 0.00 | 0 |
| 1380 | 0 | 0.00 | 0.00 | 0.00 | 0 | 0 | 0 | 0.00 | 0 |
| 1392 | 0 | 0.00 | 0.00 | 0.00 | 0 | 0 | 0 | 0.00 | 0 |
| 1404 | 0 | 0.00 | 0.00 | 0.00 | 0 | 0 | 0 | 0.00 | 0 |
| 1416 | 0 | 0.00 | 0.00 | 0.00 | 0 | 0 | 0 | 0.00 | 0 |
| 1428 | 0 | 0.00 | 0.00 | 0.00 | 0 | 0 | 0 | 0.00 | 0 |
| 1440 | 0 | 0.00 | 0.00 | 0.00 | 0 | 0 | 0 | 0.00 | 0 |

Table of Warping Normal Stresses along “z” and “s”

| Warping Normal Stresses, E*Wn*θ'' | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|
| Wn | 53.66 | 25.26 | 0.00 | - 25.26 | - 53.66 | 0.00 | 143.45 | 0 | -144 |
| “s”= | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 |
| z | | | | | | | | | |
| 0 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 |
| 7.65 | -0.10 | -0.05 | 0.00 | 0.05 | 0.10 | 0.00 | -0.26 | 0.00 | 0.27 |
| 15.3 | -0.20 | -0.09 | 0.00 | 0.09 | 0.20 | 0.00 | -0.53 | 0.00 | 0.53 |
| 22.95 | -0.30 | -0.14 | 0.00 | 0.14 | 0.30 | 0.00 | -0.80 | 0.00 | 0.80 |
| 30.6 | -0.40 | -0.19 | 0.00 | 0.19 | 0.40 | 0.00 | -1.07 | 0.00 | 1.07 |
| 38.25 | -0.50 | -0.24 | 0.00 | 0.24 | 0.50 | 0.00 | -1.34 | 0.00 | 1.34 |
| “s”= | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 |

| z | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|
| 45.9 | -0.61 | -0.28 | 0.00 | 0.28 | 0.61 | 0.00 | -1.62 | 0.00 | 1.62 |
| 53.55 | -0.71 | -0.33 | 0.00 | 0.33 | 0.71 | 0.00 | -1.90 | 0.00 | 1.90 |
| 61.2 | -0.82 | -0.39 | 0.00 | 0.39 | 0.82 | 0.00 | -2.19 | 0.00 | 2.19 |
| 68.85 | -0.93 | -0.44 | 0.00 | 0.44 | 0.93 | 0.00 | -2.48 | 0.00 | 2.48 |
| 76.5 | -1.04 | -0.49 | 0.00 | 0.49 | 1.04 | 0.00 | -2.78 | 0.00 | 2.79 |
| | | | | | | | | | |
| 76.5 | -1.04 | -0.49 | 0.00 | 0.49 | 1.04 | 0.00 | -2.78 | 0.00 | 2.79 |
| 84.15 | -1.16 | -0.55 | 0.00 | 0.55 | 1.16 | 0.00 | -3.10 | 0.00 | 3.10 |
| 91.8 | -1.28 | -0.60 | 0.00 | 0.60 | 1.28 | 0.00 | -3.42 | 0.00 | 3.42 |
| 99.45 | -1.40 | -0.66 | 0.00 | 0.66 | 1.40 | 0.00 | -3.75 | 0.00 | 3.75 |
| 107.1 | -1.53 | -0.72 | 0.00 | 0.72 | 1.53 | 0.00 | -4.09 | 0.00 | 4.09 |
| 114.75 | -1.66 | -0.78 | 0.00 | 0.78 | 1.66 | 0.00 | -4.44 | 0.00 | 4.45 |
| 122.4 | -1.80 | -0.85 | 0.00 | 0.85 | 1.80 | 0.00 | -4.81 | 0.00 | 4.81 |
| 130.05 | -1.94 | -0.91 | 0.00 | 0.91 | 1.94 | 0.00 | -5.19 | 0.00 | 5.20 |
| 137.7 | -2.09 | -0.98 | 0.00 | 0.98 | 2.09 | 0.00 | -5.59 | 0.00 | 5.60 |
| 145.35 | -2.25 | -1.06 | 0.00 | 1.06 | 2.25 | 0.00 | -6.01 | 0.00 | 6.01 |
| 153 | -2.41 | -1.13 | 0.00 | 1.13 | 2.41 | 0.00 | -6.44 | 0.00 | 6.44 |
| | | | | | | | | | |
| 153 | -2.41 | -1.13 | 0.00 | 1.13 | 2.41 | 0.00 | -6.44 | 0.00 | 6.44 |
| 165.6 | -2.23 | -1.05 | 0.00 | 1.05 | 2.23 | 0.00 | -5.96 | 0.00 | 5.97 |
| 178.2 | -2.07 | -0.97 | 0.00 | 0.97 | 2.07 | 0.00 | -5.53 | 0.00 | 5.54 |
| 190.8 | -1.93 | -0.91 | 0.00 | 0.91 | 1.93 | 0.00 | -5.15 | 0.00 | 5.15 |
| 203.4 | -1.80 | -0.85 | 0.00 | 0.85 | 1.80 | 0.00 | -4.81 | 0.00 | 4.81 |
| 216 | -1.69 | -0.79 | 0.00 | 0.79 | 1.69 | 0.00 | -4.51 | 0.00 | 4.51 |
| 228.6 | -1.59 | -0.75 | 0.00 | 0.75 | 1.59 | 0.00 | -4.25 | 0.00 | 4.25 |
| 241.2 | -1.50 | -0.71 | 0.00 | 0.71 | 1.50 | 0.00 | -4.02 | 0.00 | 4.02 |
| 253.8 | -1.43 | -0.67 | 0.00 | 0.67 | 1.43 | 0.00 | -3.83 | 0.00 | 3.83 |
| 266.4 | -1.37 | -0.64 | 0.00 | 0.64 | 1.37 | 0.00 | -3.66 | 0.00 | 3.67 |
| 279 | -1.32 | -0.62 | 0.00 | 0.62 | 1.32 | 0.00 | -3.53 | 0.00 | 3.53 |
| | | | | | | | | | |
| 279 | -1.32 | -0.62 | 0.00 | 0.62 | 1.32 | 0.00 | -3.53 | 0.00 | 3.53 |
| 287.1 | -1.00 | -0.47 | 0.00 | 0.47 | 1.00 | 0.00 | -2.67 | 0.00 | 2.67 |
| 295.2 | -0.68 | -0.32 | 0.00 | 0.32 | 0.68 | 0.00 | -1.81 | 0.00 | 1.81 |
| 303.3 | -0.36 | -0.17 | 0.00 | 0.17 | 0.36 | 0.00 | -0.96 | 0.00 | 0.96 |
| 311.4 | -0.04 | -0.02 | 0.00 | 0.02 | 0.04 | 0.00 | -0.12 | 0.00 | 0.12 |
| 319.5 | 0.27 | 0.13 | 0.00 | -0.13 | -0.27 | 0.00 | 0.73 | 0.00 | -0.73 |
| 327.6 | 0.59 | 0.28 | 0.00 | -0.28 | -0.59 | 0.00 | 1.57 | 0.00 | -1.58 |
| 335.7 | 0.91 | 0.43 | 0.00 | -0.43 | -0.91 | 0.00 | 2.43 | 0.00 | -2.43 |
| 343.8 | 1.23 | 0.58 | 0.00 | -0.58 | -1.23 | 0.00 | 3.29 | 0.00 | -3.29 |
| 351.9 | 1.56 | 0.73 | 0.00 | -0.73 | -1.56 | 0.00 | 4.16 | 0.00 | -4.16 |
| 360 | 1.89 | 0.89 | 0.00 | -0.89 | -1.89 | 0.00 | 5.05 | 0.00 | -5.05 |
| | | | | | | | | | |
| 360 | 1.89 | 0.89 | 0.00 | -0.89 | -1.89 | 0.00 | 5.05 | 0.00 | -5.05 |
| 372 | 1.72 | 0.81 | 0.00 | -0.81 | -1.72 | 0.00 | 4.61 | 0.00 | -4.61 |

| | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|
| 384 | 1.57 | 0.74 | 0.00 | -0.74 | -1.57 | 0.00 | 4.21 | 0.00 | -4.21 |
| “s”= | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 |
| z | | | | | | | | | |
| 396 | 1.44 | 0.68 | 0.00 | -0.68 | -1.44 | 0.00 | 3.84 | 0.00 | -3.84 |
| 408 | 1.31 | 0.62 | 0.00 | -0.62 | -1.31 | 0.00 | 3.50 | 0.00 | -3.50 |
| 420 | 1.19 | 0.56 | 0.00 | -0.56 | -1.19 | 0.00 | 3.18 | 0.00 | -3.18 |
| 432 | 1.08 | 0.51 | 0.00 | -0.51 | -1.08 | 0.00 | 2.89 | 0.00 | -2.89 |
| 444 | 0.98 | 0.46 | 0.00 | -0.46 | -0.98 | 0.00 | 2.62 | 0.00 | -2.62 |
| 456 | 0.89 | 0.42 | 0.00 | -0.42 | -0.89 | 0.00 | 2.37 | 0.00 | -2.37 |
| 468 | 0.80 | 0.38 | 0.00 | -0.38 | -0.80 | 0.00 | 2.14 | 0.00 | -2.14 |
| 480 | 0.72 | 0.34 | 0.00 | -0.34 | -0.72 | 0.00 | 1.92 | 0.00 | -1.92 |
| | | | | | | | | | |
| 480 | 0.72 | 0.34 | 0.00 | -0.34 | -0.72 | 0.00 | 1.92 | 0.00 | -1.92 |
| 492 | 0.64 | 0.30 | 0.00 | -0.30 | -0.64 | 0.00 | 1.72 | 0.00 | -1.72 |
| 504 | 0.57 | 0.27 | 0.00 | -0.27 | -0.57 | 0.00 | 1.53 | 0.00 | -1.53 |
| 516 | 0.51 | 0.24 | 0.00 | -0.24 | -0.51 | 0.00 | 1.36 | 0.00 | -1.36 |
| 528 | 0.44 | 0.21 | 0.00 | -0.21 | -0.44 | 0.00 | 1.19 | 0.00 | -1.19 |
| 540 | 0.39 | 0.18 | 0.00 | -0.18 | -0.39 | 0.00 | 1.03 | 0.00 | -1.03 |
| 552 | 0.33 | 0.16 | 0.00 | -0.16 | -0.33 | 0.00 | 0.88 | 0.00 | -0.88 |
| 564 | 0.28 | 0.13 | 0.00 | -0.13 | -0.28 | 0.00 | 0.74 | 0.00 | -0.74 |
| 576 | 0.22 | 0.11 | 0.00 | -0.11 | -0.22 | 0.00 | 0.60 | 0.00 | -0.60 |
| 588 | 0.17 | 0.08 | 0.00 | -0.08 | -0.17 | 0.00 | 0.47 | 0.00 | -0.47 |
| 600 | 0.13 | 0.06 | 0.00 | -0.06 | -0.13 | 0.00 | 0.34 | 0.00 | -0.34 |
| | | | | | | | | | |
| 600 | 0.13 | 0.06 | 0.00 | -0.06 | -0.13 | 0.00 | 0.34 | 0.00 | -0.34 |
| 612 | 0.08 | 0.04 | 0.00 | -0.04 | -0.08 | 0.00 | 0.21 | 0.00 | -0.21 |
| 624 | 0.03 | 0.01 | 0.00 | -0.01 | -0.03 | 0.00 | 0.08 | 0.00 | -0.08 |
| 636 | -0.02 | -0.01 | 0.00 | 0.01 | 0.02 | 0.00 | -0.04 | 0.00 | 0.04 |
| 648 | -0.06 | -0.03 | 0.00 | 0.03 | 0.06 | 0.00 | -0.17 | 0.00 | 0.17 |
| 660 | -0.11 | -0.05 | 0.00 | 0.05 | 0.11 | 0.00 | -0.29 | 0.00 | 0.30 |
| 672 | -0.16 | -0.07 | 0.00 | 0.07 | 0.16 | 0.00 | -0.42 | 0.00 | 0.42 |
| 684 | -0.21 | -0.10 | 0.00 | 0.10 | 0.21 | 0.00 | -0.56 | 0.00 | 0.56 |
| 696 | -0.26 | -0.12 | 0.00 | 0.12 | 0.26 | 0.00 | -0.69 | 0.00 | 0.69 |
| 708 | -0.31 | -0.15 | 0.00 | 0.15 | 0.31 | 0.00 | -0.83 | 0.00 | 0.84 |
| 720 | -0.37 | -0.17 | 0.00 | 0.17 | 0.37 | 0.00 | -0.98 | 0.00 | 0.98 |
| | | | | | | | | | |
| 720 | -0.37 | -0.17 | 0.00 | 0.17 | 0.37 | 0.00 | -0.98 | 0.00 | 0.98 |
| 732 | -0.34 | -0.16 | 0.00 | 0.16 | 0.34 | 0.00 | -0.90 | 0.00 | 0.90 |
| 744 | -0.31 | -0.14 | 0.00 | 0.14 | 0.31 | 0.00 | -0.82 | 0.00 | 0.82 |
| 756 | -0.28 | -0.13 | 0.00 | 0.13 | 0.28 | 0.00 | -0.75 | 0.00 | 0.75 |
| 768 | -0.25 | -0.12 | 0.00 | 0.12 | 0.25 | 0.00 | -0.68 | 0.00 | 0.68 |
| 780 | -0.23 | -0.11 | 0.00 | 0.11 | 0.23 | 0.00 | -0.62 | 0.00 | 0.62 |
| 792 | -0.21 | -0.10 | 0.00 | 0.10 | 0.21 | 0.00 | -0.56 | 0.00 | 0.56 |
| 804 | -0.19 | -0.09 | 0.00 | 0.09 | 0.19 | 0.00 | -0.51 | 0.00 | 0.51 |
| 816 | -0.17 | -0.08 | 0.00 | 0.08 | 0.17 | 0.00 | -0.46 | 0.00 | 0.46 |

| | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|
| 828 | -0.16 | -0.07 | 0.00 | 0.07 | 0.16 | 0.00 | -0.42 | 0.00 | 0.42 |
| 840 | -0.14 | -0.07 | 0.00 | 0.07 | 0.14 | 0.00 | -0.38 | 0.00 | 0.38 |
| 840 | -0.14 | -0.07 | 0.00 | 0.07 | 0.14 | 0.00 | -0.38 | 0.00 | 0.38 |
| “s”= | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 |
| z | | | | | | | | | |
| 852 | -0.13 | -0.06 | 0.00 | 0.06 | 0.13 | 0.00 | -0.34 | 0.00 | 0.34 |
| 864 | -0.11 | -0.05 | 0.00 | 0.05 | 0.11 | 0.00 | -0.30 | 0.00 | 0.30 |
| 876 | -0.10 | -0.05 | 0.00 | 0.05 | 0.10 | 0.00 | -0.27 | 0.00 | 0.27 |
| 888 | -0.09 | -0.04 | 0.00 | 0.04 | 0.09 | 0.00 | -0.23 | 0.00 | 0.23 |
| 900 | -0.08 | -0.04 | 0.00 | 0.04 | 0.08 | 0.00 | -0.20 | 0.00 | 0.20 |
| 912 | -0.06 | -0.03 | 0.00 | 0.03 | 0.06 | 0.00 | -0.17 | 0.00 | 0.17 |
| 924 | -0.05 | -0.03 | 0.00 | 0.03 | 0.05 | 0.00 | -0.15 | 0.00 | 0.15 |
| 936 | -0.04 | -0.02 | 0.00 | 0.02 | 0.04 | 0.00 | -0.12 | 0.00 | 0.12 |
| 948 | -0.03 | -0.02 | 0.00 | 0.02 | 0.03 | 0.00 | -0.09 | 0.00 | 0.09 |
| 960 | -0.03 | -0.01 | 0.00 | 0.01 | 0.03 | 0.00 | -0.07 | 0.00 | 0.07 |
| | | | | | | | | | |
| 960 | -0.03 | -0.01 | 0.00 | 0.01 | 0.03 | 0.00 | -0.07 | 0.00 | 0.07 |
| 972 | -0.02 | -0.01 | 0.00 | 0.01 | 0.02 | 0.00 | -0.04 | 0.00 | 0.04 |
| 984 | -0.01 | 0.00 | 0.00 | 0.00 | 0.01 | 0.00 | -0.02 | 0.00 | 0.02 |
| 996 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 |
| 1008 | 0.01 | 0.01 | 0.00 | -0.01 | -0.01 | 0.00 | 0.03 | 0.00 | -0.03 |
| 1020 | 0.02 | 0.01 | 0.00 | -0.01 | -0.02 | 0.00 | 0.05 | 0.00 | -0.05 |
| 1032 | 0.03 | 0.01 | 0.00 | -0.01 | -0.03 | 0.00 | 0.08 | 0.00 | -0.08 |
| 1044 | 0.04 | 0.02 | 0.00 | -0.02 | -0.04 | 0.00 | 0.10 | 0.00 | -0.10 |
| 1056 | 0.05 | 0.02 | 0.00 | -0.02 | -0.05 | 0.00 | 0.13 | 0.00 | -0.13 |
| 1068 | 0.06 | 0.03 | 0.00 | -0.03 | -0.06 | 0.00 | 0.16 | 0.00 | -0.16 |
| 1080 | 0.07 | 0.03 | 0.00 | -0.03 | -0.07 | 0.00 | 0.18 | 0.00 | -0.18 |
| | | | | | | | | | |
| 1080 | 0.07 | 0.03 | 0.00 | -0.03 | -0.07 | 0.00 | 0.18 | 0.00 | -0.18 |
| 1092 | 0.06 | 0.03 | 0.00 | -0.03 | -0.06 | 0.00 | 0.17 | 0.00 | -0.17 |
| 1104 | 0.06 | 0.03 | 0.00 | -0.03 | -0.06 | 0.00 | 0.15 | 0.00 | -0.15 |
| 1116 | 0.05 | 0.02 | 0.00 | -0.02 | -0.05 | 0.00 | 0.14 | 0.00 | -0.14 |
| 1128 | 0.05 | 0.02 | 0.00 | -0.02 | -0.05 | 0.00 | 0.13 | 0.00 | -0.13 |
| 1140 | 0.04 | 0.02 | 0.00 | -0.02 | -0.04 | 0.00 | 0.12 | 0.00 | -0.12 |
| 1152 | 0.04 | 0.02 | 0.00 | -0.02 | -0.04 | 0.00 | 0.11 | 0.00 | -0.11 |
| 1164 | 0.04 | 0.02 | 0.00 | -0.02 | -0.04 | 0.00 | 0.10 | 0.00 | -0.10 |
| 1176 | 0.03 | 0.02 | 0.00 | -0.02 | -0.03 | 0.00 | 0.09 | 0.00 | -0.09 |
| 1188 | 0.03 | 0.01 | 0.00 | -0.01 | -0.03 | 0.00 | 0.08 | 0.00 | -0.08 |
| 1200 | 0.03 | 0.01 | 0.00 | -0.01 | -0.03 | 0.00 | 0.08 | 0.00 | -0.08 |
| | | | | | | | | | |
| 1200 | 0.03 | 0.01 | 0.00 | -0.01 | -0.03 | 0.00 | 0.08 | 0.00 | -0.08 |
| 1212 | 0.03 | 0.01 | 0.00 | -0.01 | -0.03 | 0.00 | 0.07 | 0.00 | -0.07 |
| 1224 | 0.02 | 0.01 | 0.00 | -0.01 | -0.02 | 0.00 | 0.06 | 0.00 | -0.06 |
| 1236 | 0.02 | 0.01 | 0.00 | -0.01 | -0.02 | 0.00 | 0.06 | 0.00 | -0.06 |
| 1248 | 0.02 | 0.01 | 0.00 | -0.01 | -0.02 | 0.00 | 0.05 | 0.00 | -0.05 |

| | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|
| 1260 | 0.02 | 0.01 | 0.00 | -0.01 | -0.02 | 0.00 | 0.05 | 0.00 | -0.05 |
| 1272 | 0.02 | 0.01 | 0.00 | -0.01 | -0.02 | 0.00 | 0.04 | 0.00 | -0.04 |
| 1284 | 0.01 | 0.01 | 0.00 | -0.01 | -0.01 | 0.00 | 0.04 | 0.00 | -0.04 |
| 1296 | 0.01 | 0.01 | 0.00 | -0.01 | -0.01 | 0.00 | 0.03 | 0.00 | -0.03 |
| 1308 | 0.01 | 0.01 | 0.00 | -0.01 | -0.01 | 0.00 | 0.03 | 0.00 | -0.03 |
| “s”= | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 |
| z | | | | | | | | | |
| 1320 | 0.01 | 0.00 | 0.00 | 0.00 | -0.01 | 0.00 | 0.03 | 0.00 | -0.03 |
| | | | | | | | | | |
| 1320 | 0.01 | 0.00 | 0.00 | 0.00 | -0.01 | 0.00 | 0.03 | 0.00 | -0.03 |
| 1332 | 0.01 | 0.00 | 0.00 | 0.00 | -0.01 | 0.00 | 0.02 | 0.00 | -0.02 |
| 1344 | 0.01 | 0.00 | 0.00 | 0.00 | -0.01 | 0.00 | 0.02 | 0.00 | -0.02 |
| 1356 | 0.01 | 0.00 | 0.00 | 0.00 | -0.01 | 0.00 | 0.02 | 0.00 | -0.02 |
| 1368 | 0.01 | 0.00 | 0.00 | 0.00 | -0.01 | 0.00 | 0.01 | 0.00 | -0.02 |
| 1380 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.01 | 0.00 | -0.01 |
| 1392 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.01 | 0.00 | -0.01 |
| 1404 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.01 | 0.00 | -0.01 |
| 1416 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 |
| 1428 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 |
| 1440 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 |

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